

# Phases of dense and hot matter in chiral models

*Alessandro Drago - Ferrara*

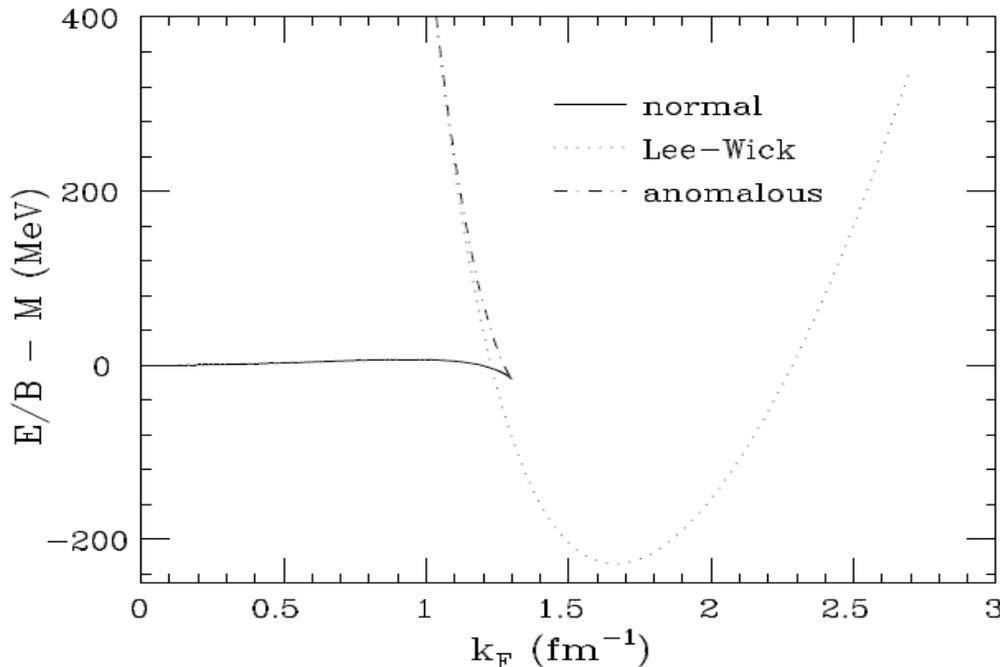
*Luca Bonanno - Frankfurt*

*Valentina Mantovani Sarti - Ferrara*

- Chiral lagrangian for **nuclear** matter
  - Linear  $\sigma$ -model fails
  - Non-linear  $\sigma$ -model (but with a scalar field...)
  - Scaled linear models
- Scaled chiral nuclear model at finite  $\rho$  and T  
[Phys. Rev.Lett. 99 \(2007\) 242301](#); [Phys.Rev.C 79 \(2009\) 045801](#)
  - Similarities with McLerran-Pisarski large  $N_c$  scheme
- Solitons from a scaled chiral **quark** model
- Wigner-Seitz lattice of solitons
  - Linear  $\sigma$ -model fails
  - Non-linear  $\sigma$ -model (without a scalar field, Glendenning)
  - Scaled linear models: without ([arXiv:1109.5399](#))  
and with vector mesons

# Linear $\sigma$ -models based on the “Mexican-hat” potential

$$\begin{aligned} \mathcal{L} = & \bar{\psi} [i\gamma_\mu \partial^\mu - g_v \gamma_\mu V^\mu - g_\pi (\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi})] \psi \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) - \frac{1}{4} \lambda (\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 \\ & - \frac{1}{4} (\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + \frac{1}{2} m_v^2 V_\mu V^\mu + \epsilon \sigma . \end{aligned}$$



Furnstahl, Serot, Tang NPA598 (1996) 539

The ground state is not the normal solution, but the Lee-Wick one, having effective nucleon mass  $M^* = 0$

# Non-linear $\sigma$ -model with a scalar field

## Furnstahl, Serot, Tang 1996

- Furnstahl and Serot 1993 conclude that the failure of many chiral models is due to the restrictions on the scalar field dynamics imposed by the “Mexican hat” potential. The problem is not alleviated by introducing scaled versions of the “Mexican hat”.
- FST 1996 use a non-linear realization of chiral symmetry in which a scalar-isoscalar effective field is introduced, as a chiral singlet, to simulate intermediate range attraction.
- The dynamics of the chiral singlet field is no-more regulated by the “Mexican hat” potential.

# The chiral-dilaton model

Carter, Ellis, Heide, Rudaz (Univ. Minnesota)

- On scale and Chiral Symmetry in Nuclear Matter PLB 282 (1992) 271
- Implications of a Modified Glueball Potential for Nuclear Matter PLB 293 (1992) 870
- An Effective Lagrangian with Broken Scale and Chiral Symmetry:
  - Applied to Nuclear Matter and Finite Nuclei NPA 571 (1994) 713
  - Pion phenomenology NPA 603 (1996) 367
  - Mesons at Finite Temperature NPA 618 (1997) 317
  - Nucleons and Mesons at Finite Temperature NPA 628 (1998) 325

Other approaches to linear chiral lagrangians in nuclear physics developed by the Frankfurt group, also extended to  $SU(3)_f$

# Chiral lagrangians in Nuclear Physics

Problem: the linear sigma model fails to yield saturation.  
It provides chiral symmetry restoration ( $m_N = 0$ ) already at low density  
due to the form of the meson self-interaction.

***Some physical ingredient is missing***

In QCD, scale symmetry is broken by trace anomaly.

This mechanism is responsible for the existence of  $\Lambda_{\text{QCD}}$  parameter,  
which sets the scale of hadron masses and radii.

In an effective model, the **QCD trace anomaly** is reproduced at a mean field level,  
by introducing a scalar field, **the dilaton field** (Schechter 1980), so that

$$\Theta_{\mu}^{\mu} = 4\mathcal{E}(\phi / \phi_0)^4$$

Heide, Rudaz and Ellis 1992 modifies the dilaton potential by including chiral fields

$$\mathcal{V} = B\phi^4 \left( \ln \frac{\phi}{\phi_0} - \frac{1}{4} \right) - \frac{1}{2} B \delta \phi^4 \ln \frac{\sigma^2 + \pi^2}{\sigma_0^2}$$

# The chiral dilaton model

An effective lagrangian with broken scale and chiral symmetry

(Nucl.Phys.A628:325,1998)

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi} \cdot \partial^\mu\boldsymbol{\pi} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}G_{\omega\phi}\phi^2\omega_\mu\omega^\mu$$

$$+ [(G_4)^2\omega_\mu\omega^\mu]^2 + \bar{N} \left[ \gamma^\mu (i\partial_\mu - g_\omega\omega_\mu) - g\sqrt{\sigma^2 + \boldsymbol{\pi}^2} \right] N - \mathcal{V},$$

$$\mathcal{V} = B\phi^4 \left( \ln \frac{\phi}{\phi_0} - \frac{1}{4} \right) - \frac{1}{2}B\delta\phi^4 \ln \frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2} + \frac{1}{2}B\delta\zeta^2\phi^2 \left[ \sigma^2 + \boldsymbol{\pi}^2 - \frac{\phi^2}{2\zeta^2} \right]$$

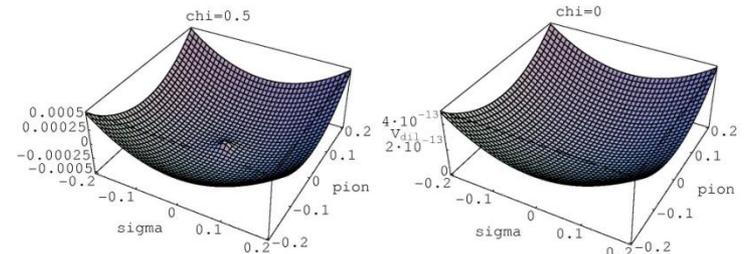
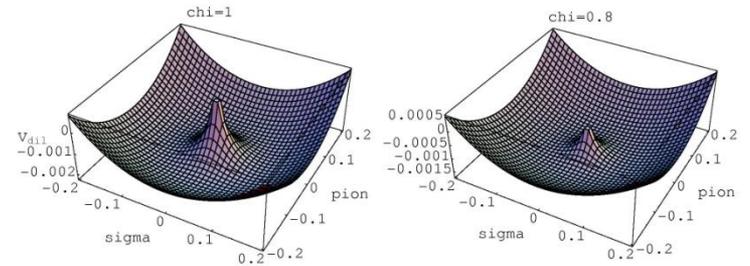
$$- \frac{1}{4}\epsilon'_1 \left( \frac{\phi}{\phi_0} \right)^2 \left[ \frac{4\sigma}{\sigma_0} - 2 \left( \frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2} \right) - \left( \frac{\phi}{\phi_0} \right)^2 \right] - \frac{3}{4}\epsilon'_1.$$

Dilaton potential with  $\epsilon'=0$

$$\Theta_\mu^\mu = 4\varepsilon(\phi/\phi_0)^4$$

$$\varepsilon = -B\phi_0^4(1-\delta)/4$$

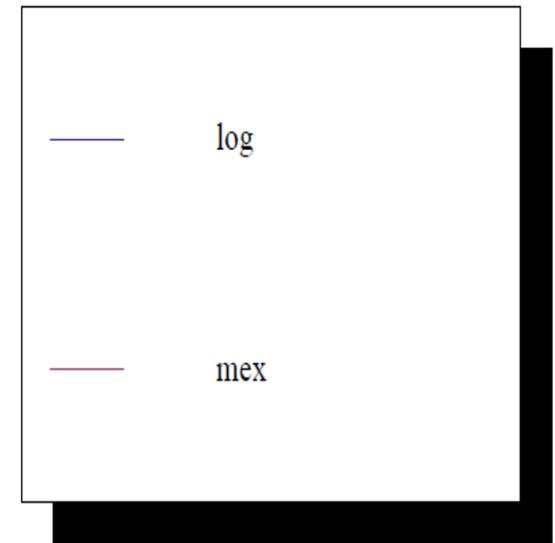
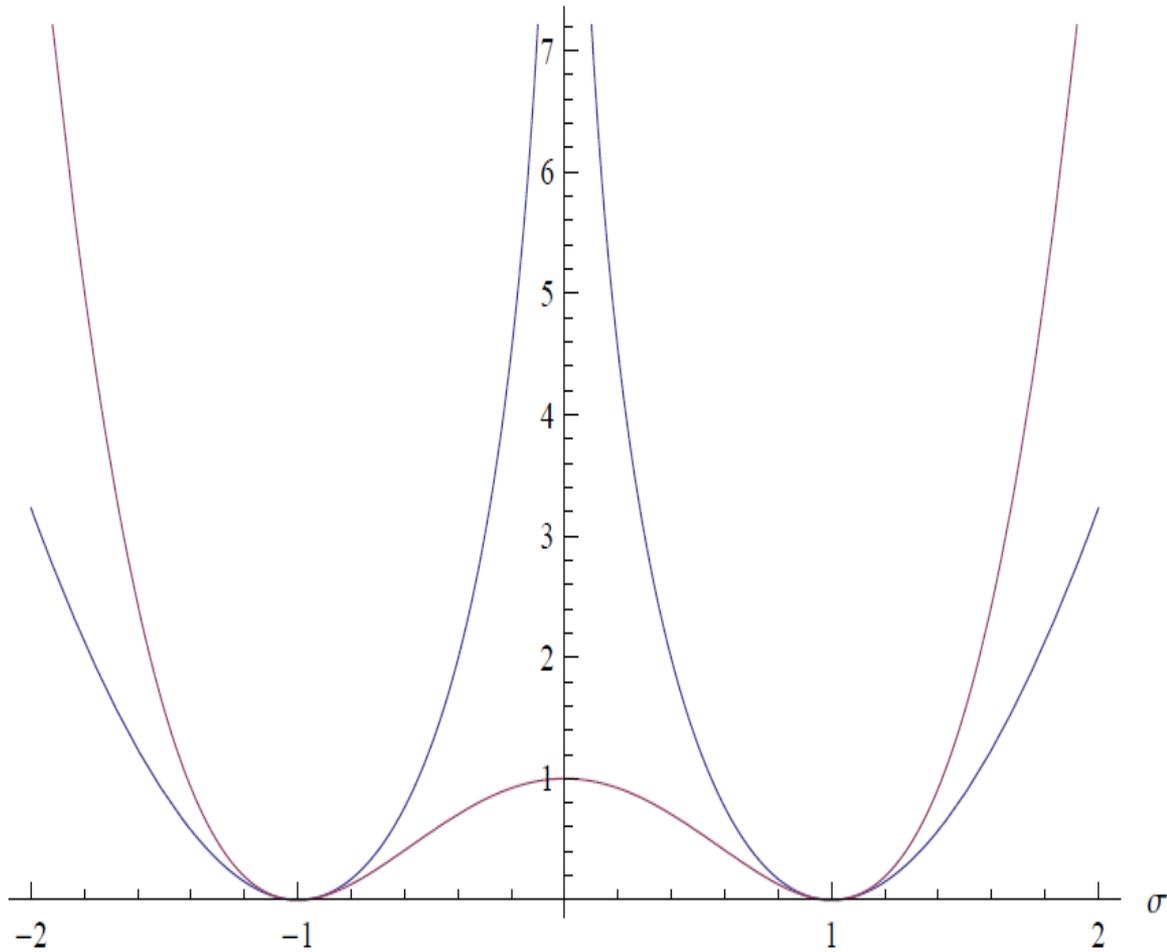
$\delta=4/33$  from QCD  $\beta$  function



When scale invariance is restored also

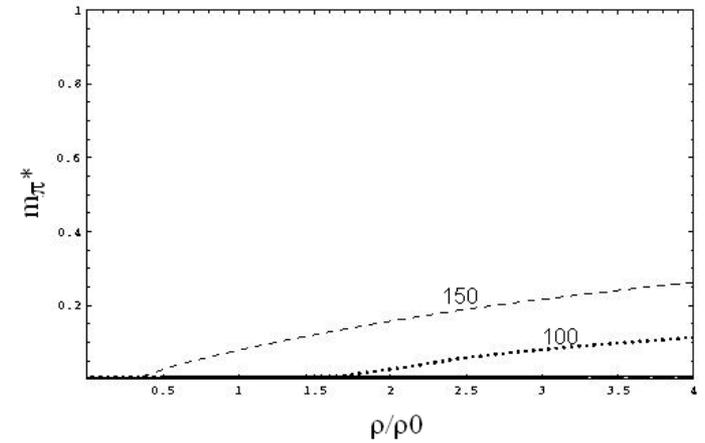
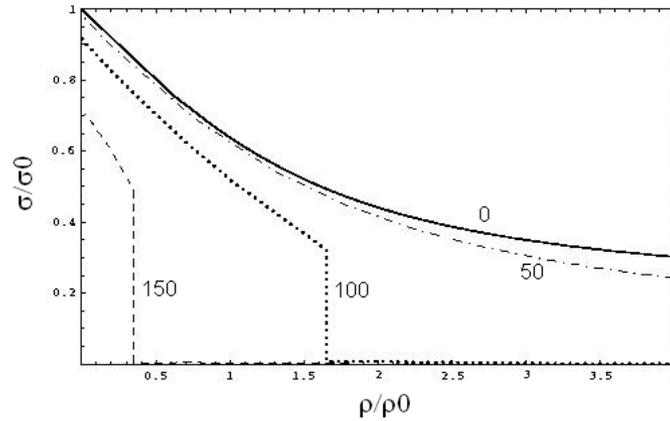
chiral symmetry is restored

# Mexican hat vs Log potential



# Chiral symmetry restoration at high $\rho$ and finite T

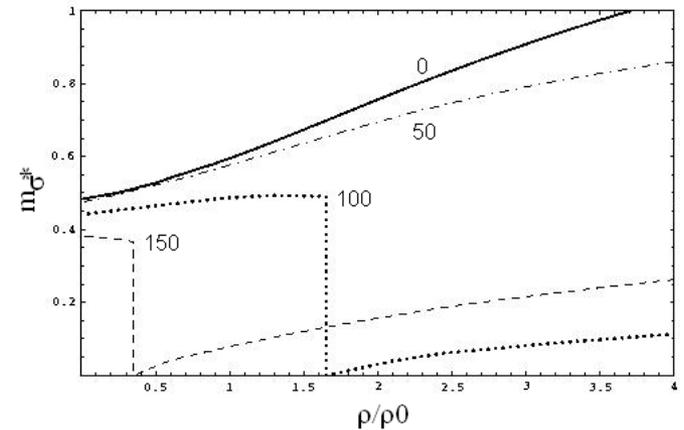
Simmetric nuclear matter in the **chiral limit**  $\rightarrow m_\pi = 0$



Chiral Symmetry restoration:

$$\langle \sigma \rangle = 0$$

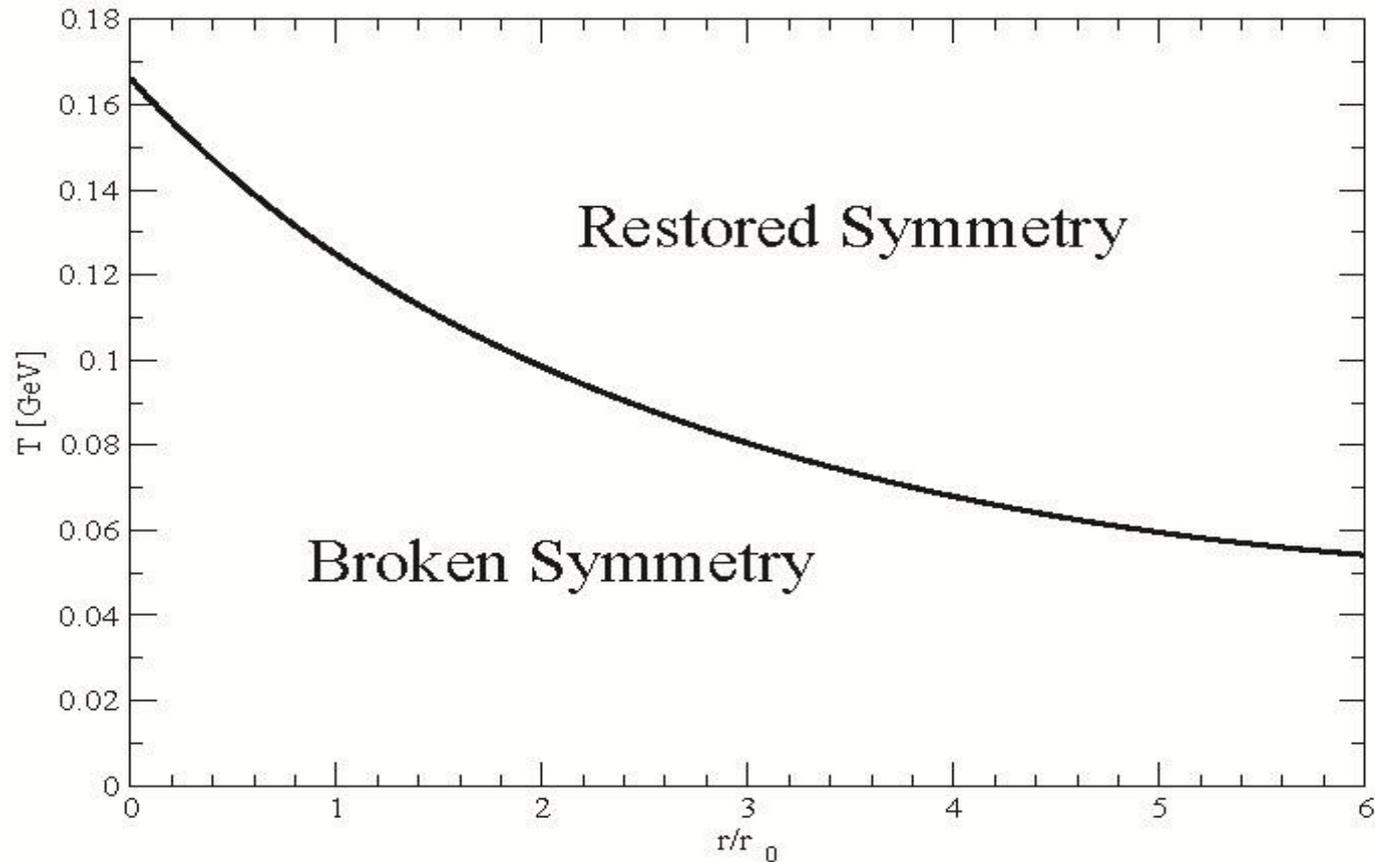
$$m_\sigma^* = m_\pi^*$$



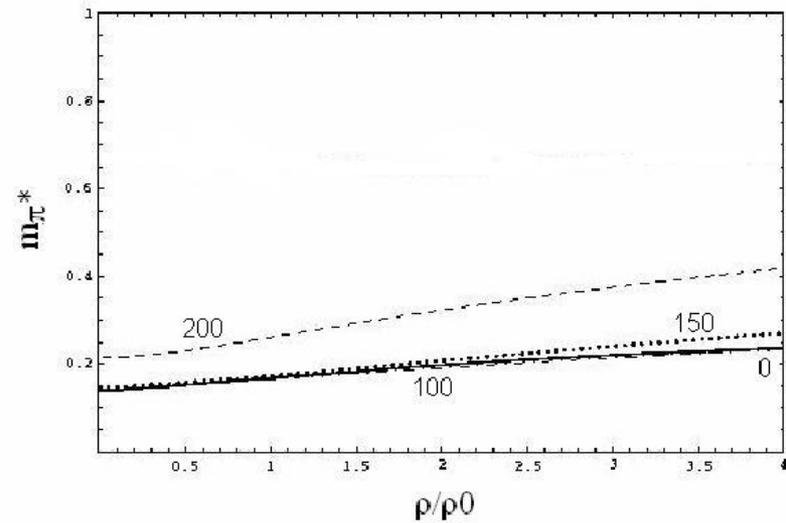
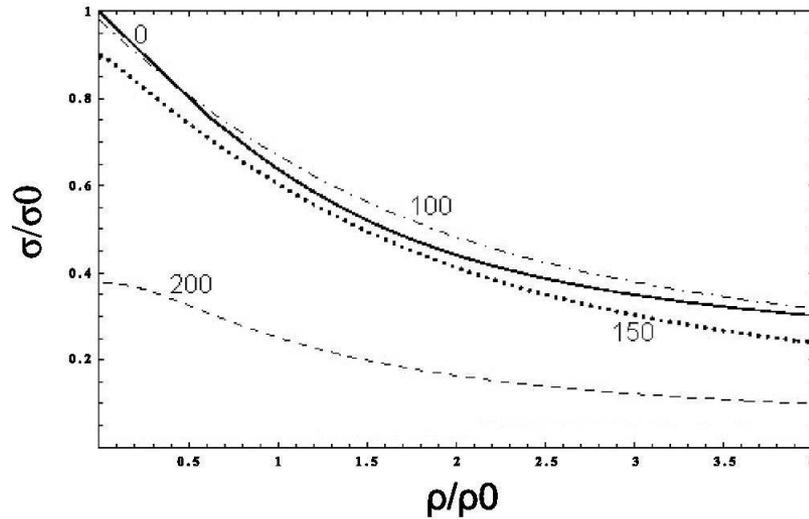
First order transition,  $\sigma$  discontinuous

# Chiral transition in the $\rho$ -T plane

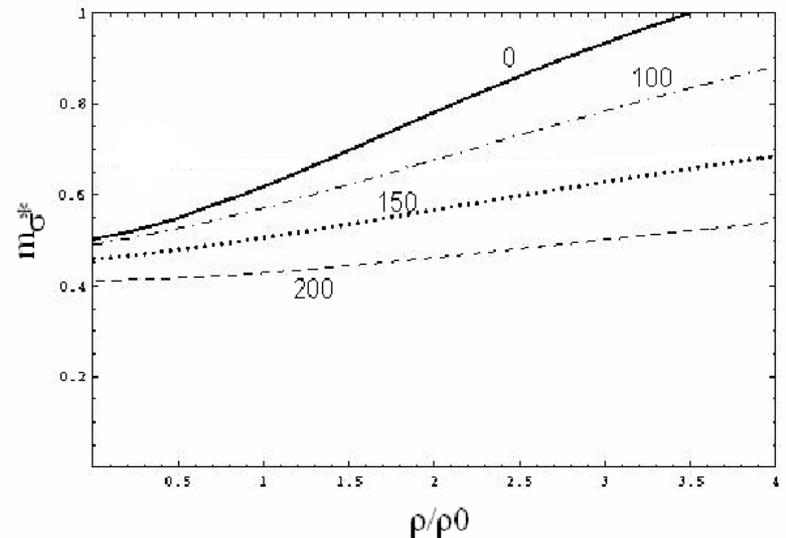
chiral limit  $m_\pi = 0$



# Symmetry broken case ( $m_\pi = 138$ MeV)

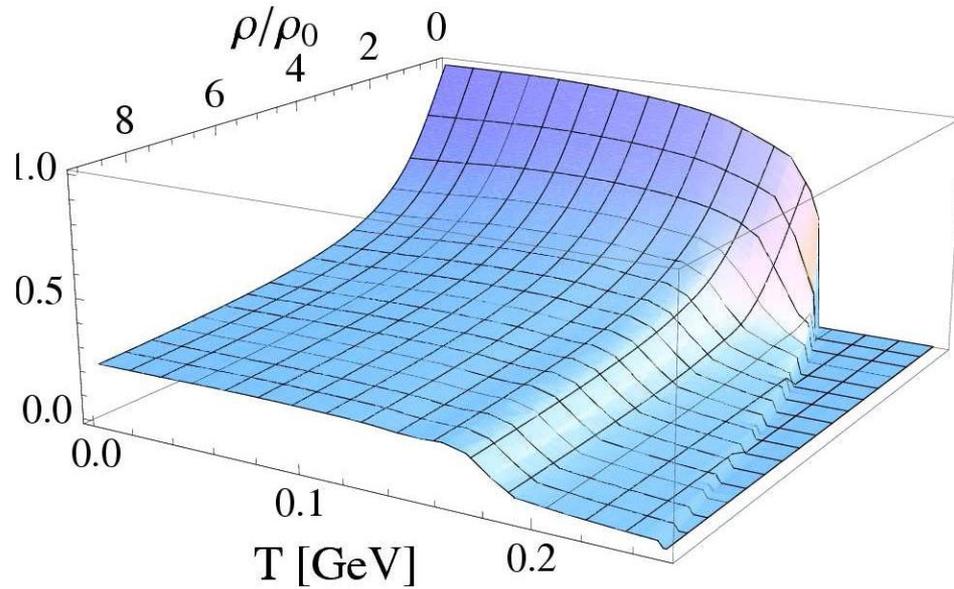


If an explicit breaking of chiral symmetry is introduced, there is not a phase transition but a crossover.

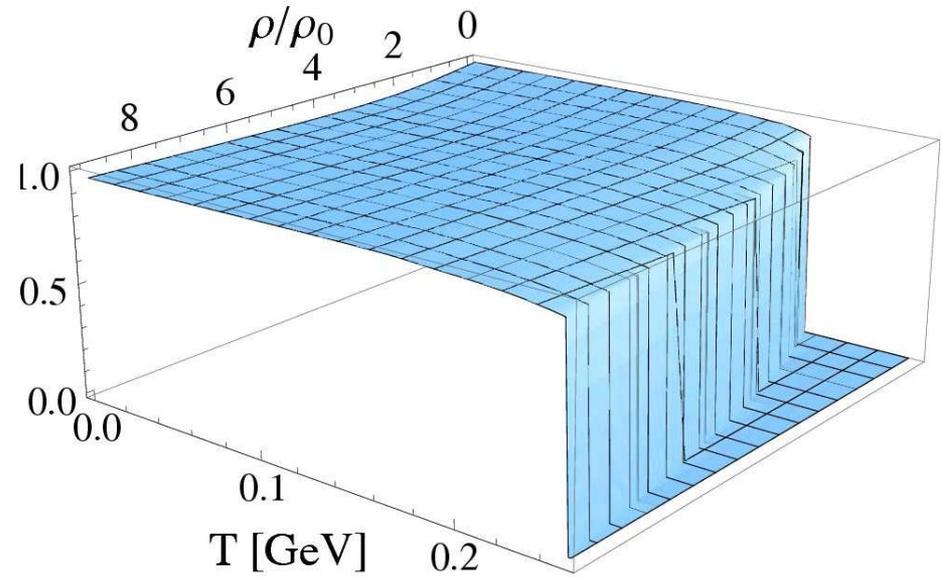


*Luca Bonanno, 0909.0924*

chiral condensate and dilaton at finite  $\rho$  and  $T$



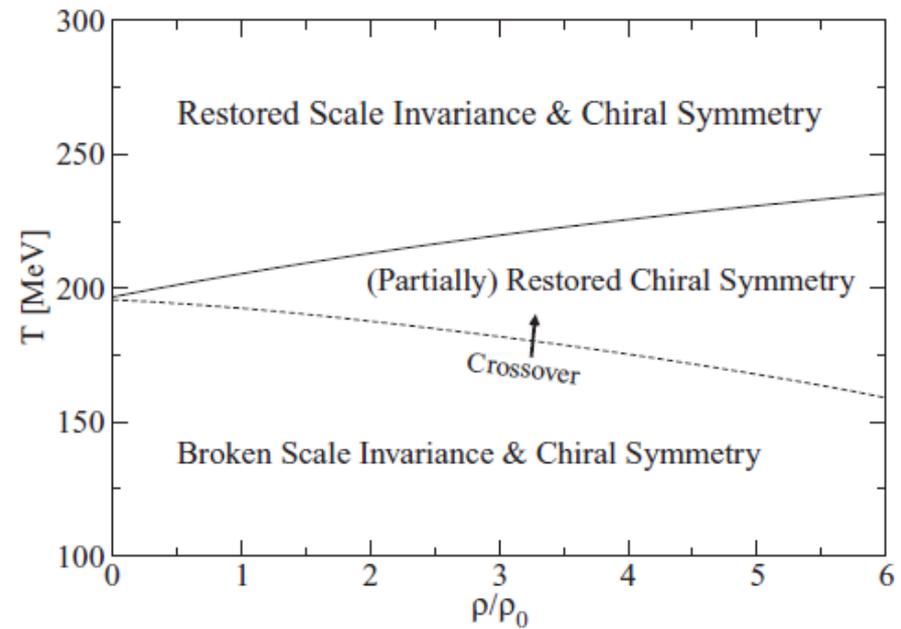
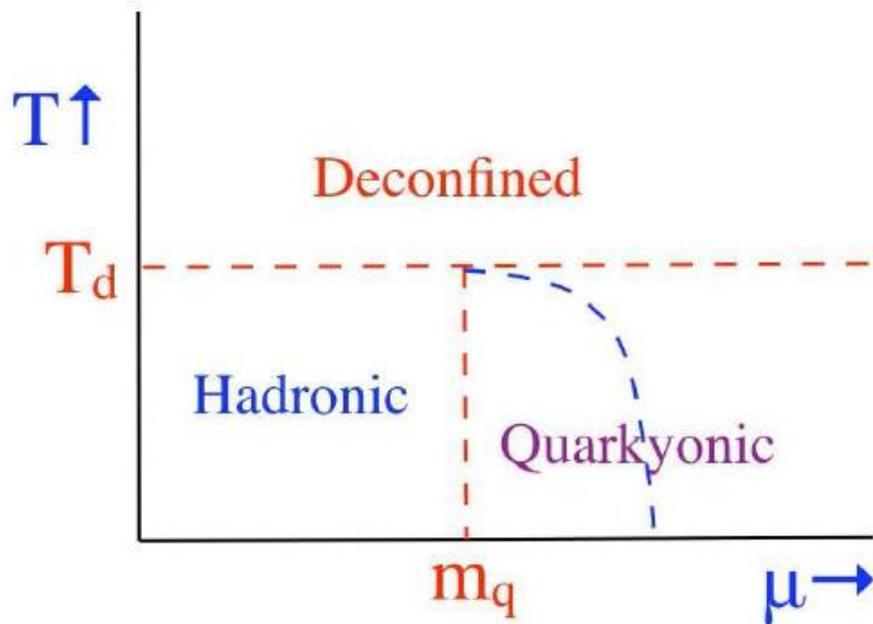
$\sigma/\sigma_0$



$\phi/\phi_0$

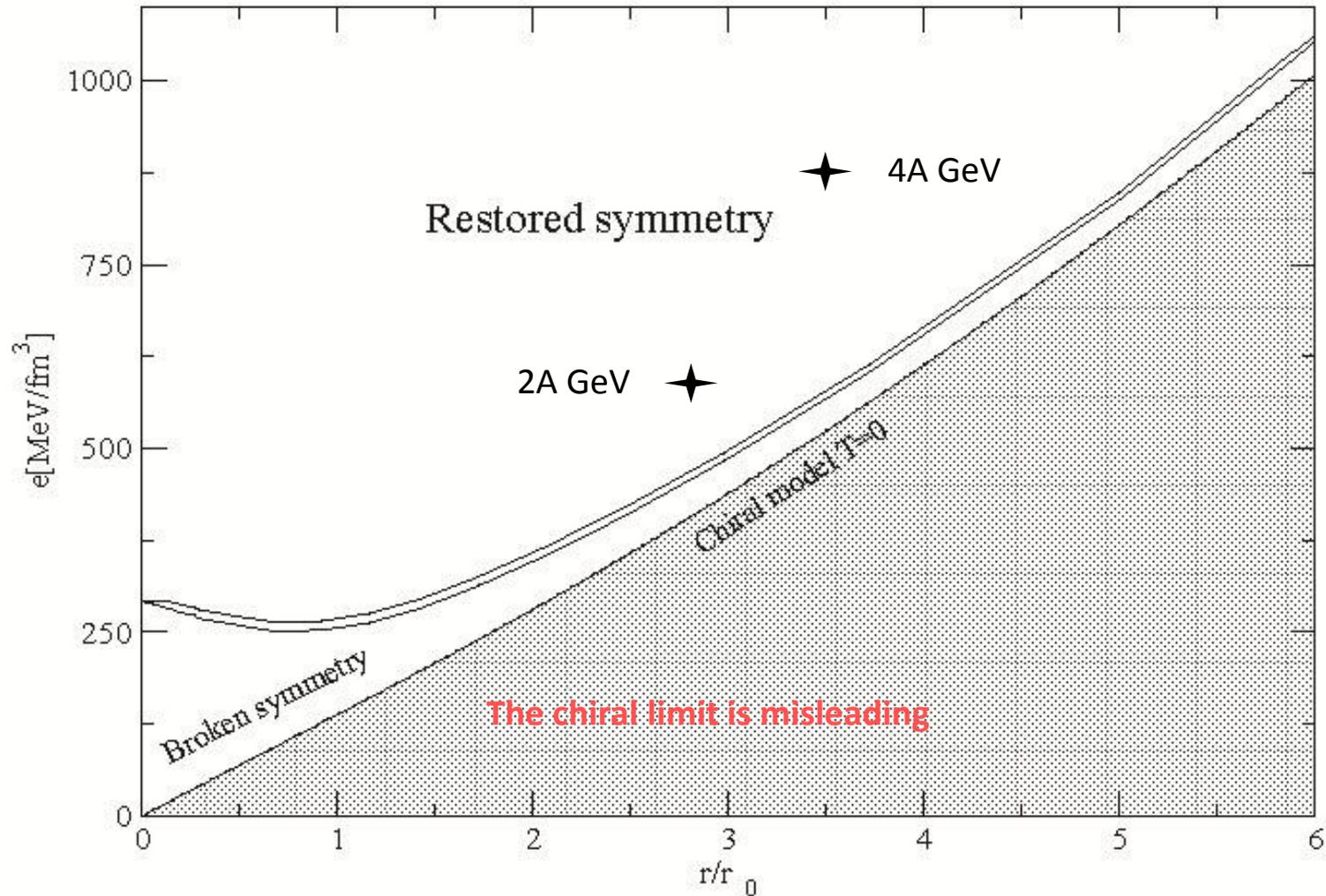
# McLerran and Pisarski 2007

## *Phases of dense quarks at large $N_c$*

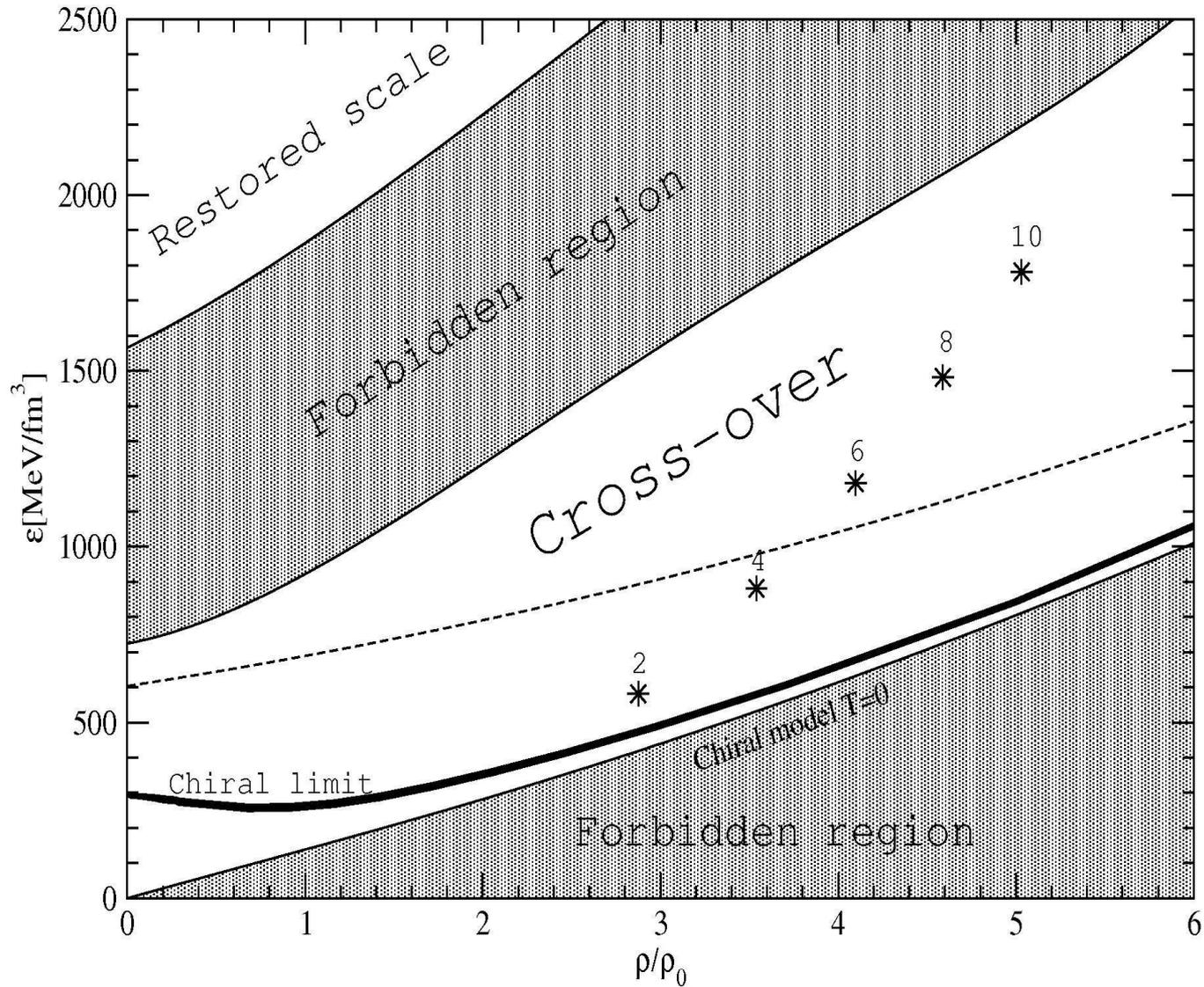


# Chiral transition in the $\rho$ - $\epsilon$ plane

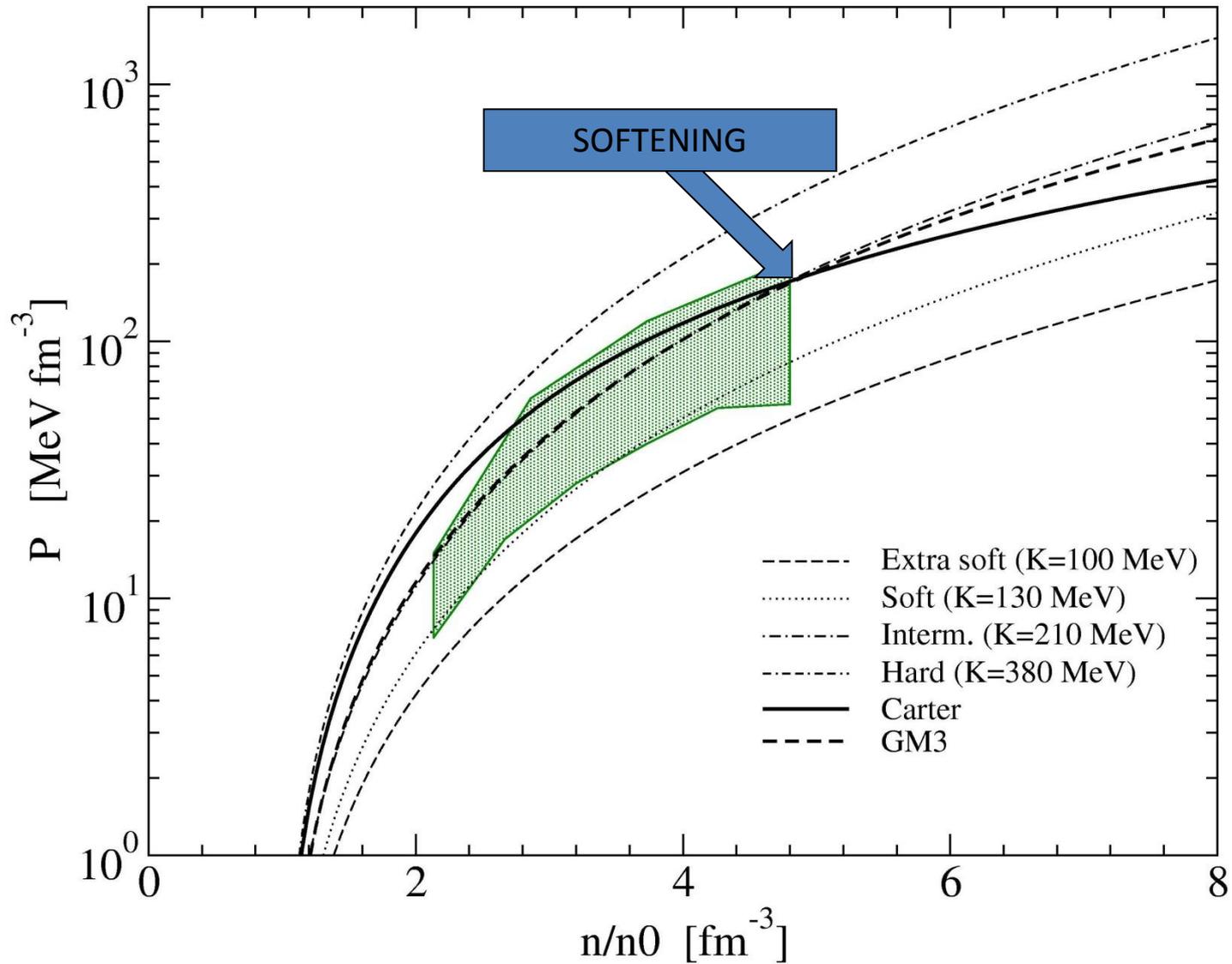
chiral limit  $m_\pi = 0$



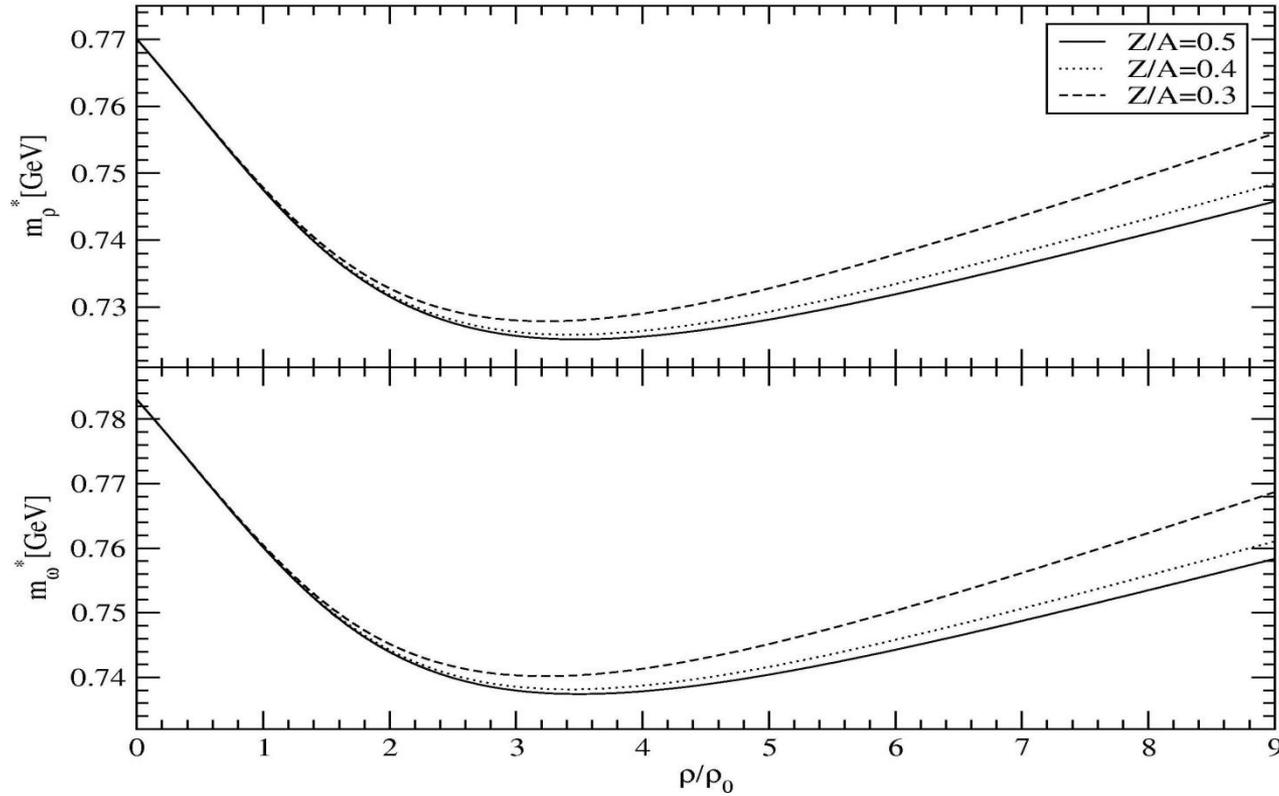
# Chiral symmetry broken case ( $m_\pi = 138$ MeV)



# EOS Symmetric nuclear matter at T=0



# Vector Meson effective masses



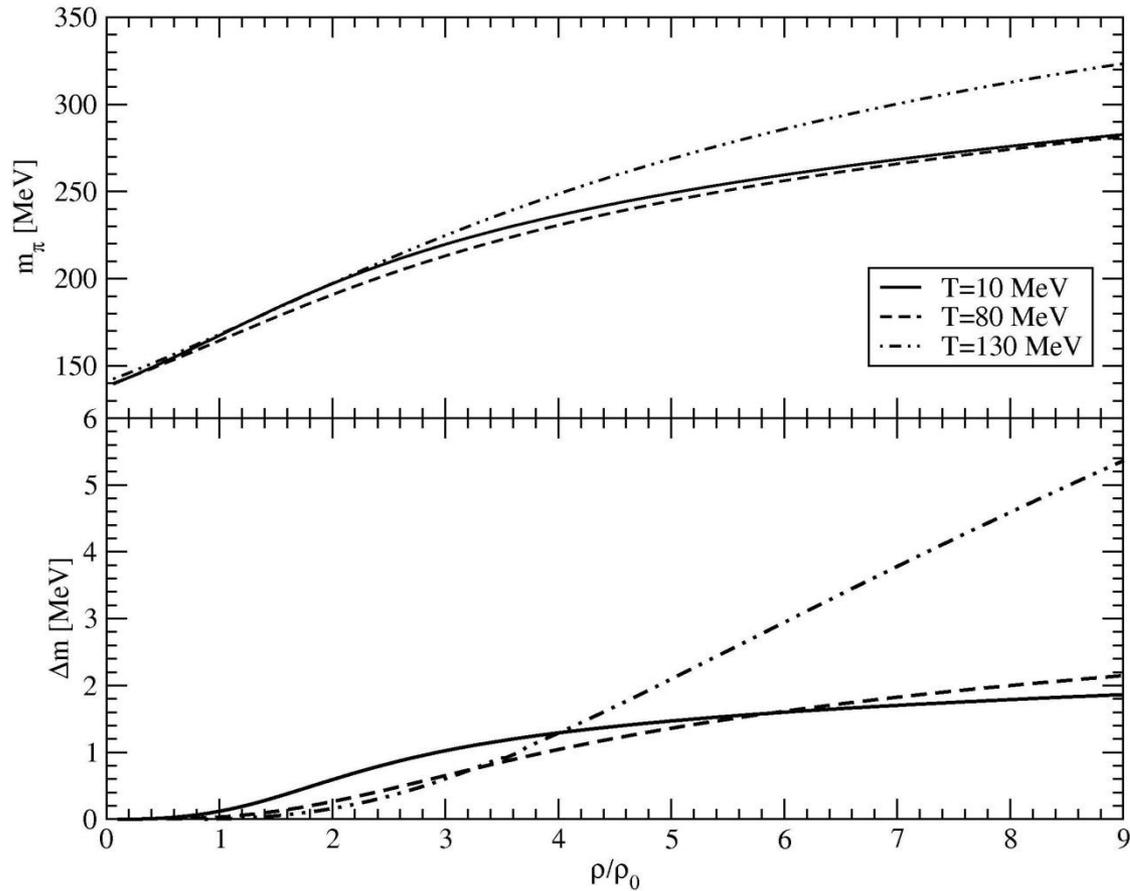
## Experimental indications

Metag, J. Phys. G34, S397 (2007)

Table 1. Medium modifications of light vector mesons reported by different collaborations.

	KEK	Jlab	CBELSA/TAPS	CERES	NA60
$\rho$	$\frac{\Delta m}{m} = -9\%$ no	$\Delta m \approx 0$ some broadening		broadening favoured over density dependent mass shift	$\Delta m \approx 0$ strong broadening
$\omega$	broadening		$\frac{\Delta m}{m} \approx -14\%$ $\frac{\Gamma_\rho(\omega)}{\Gamma_\omega} \approx 10$		
$\Phi$	$\frac{\Delta m}{m} = -3.4\%$ $\frac{\Gamma_\rho(\Phi)}{\Gamma_\Phi} = 3.6$				

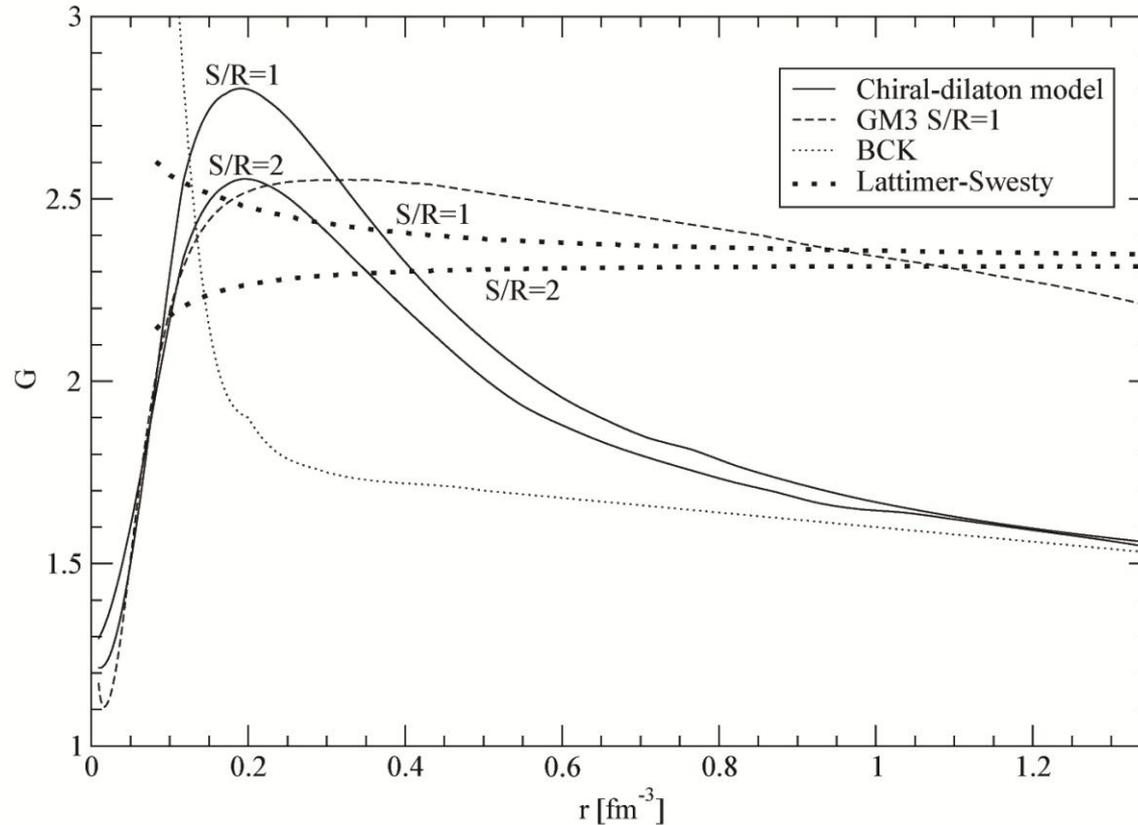
# Effective pion mass



Still preliminary experiments on deeply bound 1s and 2p states in pionic atoms indicate an increase of about 30 MeV at saturation density.

Friedman and Gal, Phys .Lett. B432, 235 (1998)

## Adiabatic index in pre-supernova matter



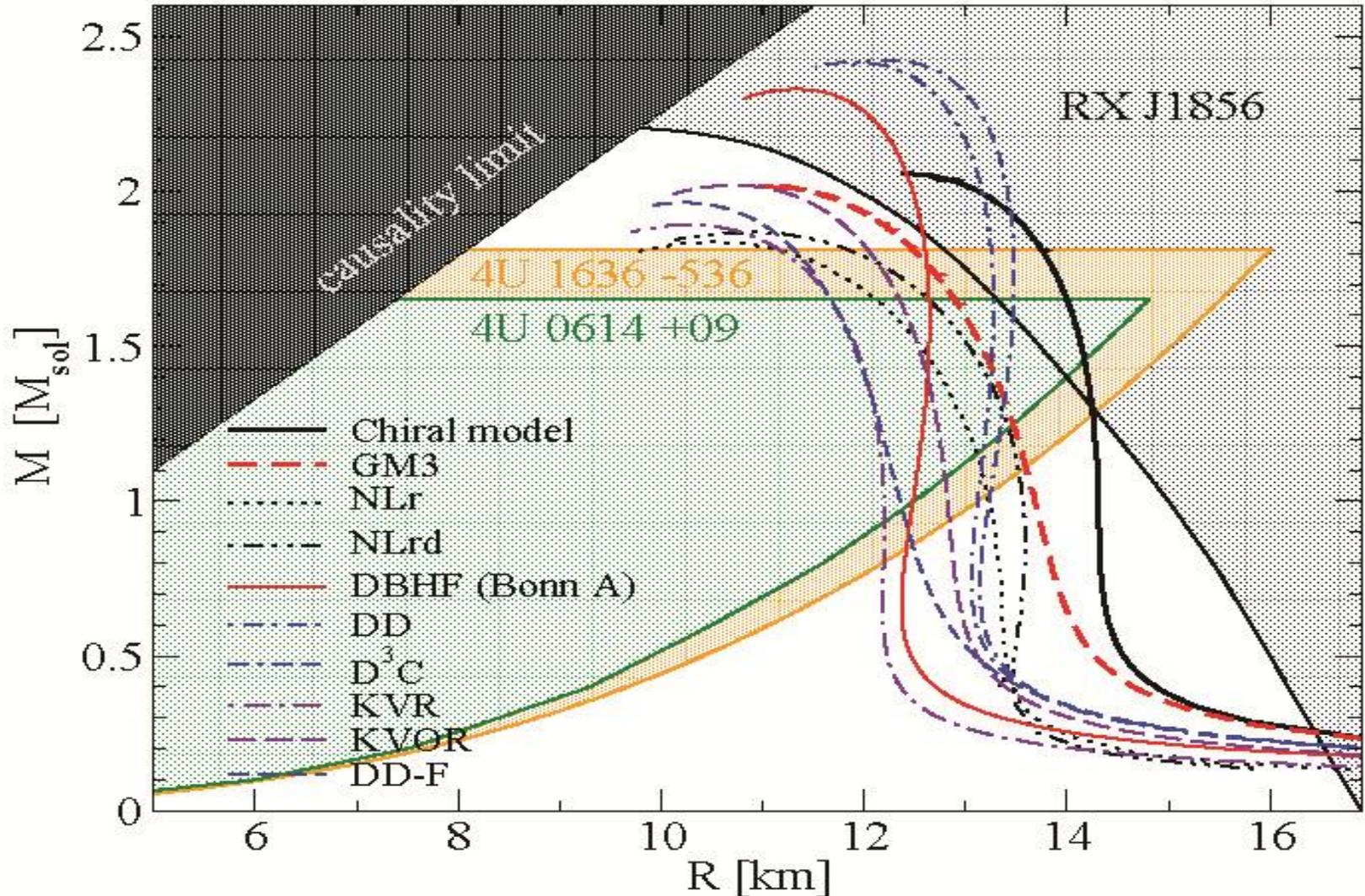
**Chiral symmetry restoration dramatically reduces the adiabatic index,**

**but at too high densities to allow a prompt explosion.**

**Interesting effects could be observed in neutron stars merging.**

# Mass-radius relation

from Klahn et al. 2006, adapted



# From nucleons to quarks

**Main idea:** to use as a quark Lagrangian the same used as a nucleon lagrangian, but **intepreting now the fermions as quarks.**

The nucleons should now emerge as **chiral solitons** from the dynamics of the quarks.

Points to be checked:

- Do solitonic solutions exist at all?
- Are those solitons able to provide a reasonable description of nucleons?
- Can a lattice of solitons be built? Can it describe nuclear matter saturation?

It has been done before using the  $\sigma$ -model, but:

- At nucleons' level the  $\sigma$ -model does not provide a good description for nuclear matter, Lee-Wick phase is the ground state already at low densities
- At quarks' level the  $\sigma$ -model does not allow to describe nuclear saturation, solutions disappear at low densities

# Chiral-dilaton solitons

- Non-topological chiral solitons
- Baryon number provided by the quarks
- First discussed by Kahana-Ripka-Soni 1984 using a non-linear  $\sigma$ -model
- Broniowski and Banerjee 1986 include vector mesons in the linear  $\sigma$ -model soliton
- Mean field solutions based on the so-called hedgehog ansatz,  $\mathbf{G} = \mathbf{J} + \mathbf{I} = \mathbf{0}$
- Projection on spin and isospin in order to describe a nucleon
- **New ingredient: log potential** for dilaton and chiral fields

**MAIN IDEA:** use the same nucleon Lagrangian, but now introducing **quarks** degrees of freedom → fermionic fields are **quarks**

### Simple version without vector mesons

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - g_\pi(\sigma + i\pi \cdot \tau \gamma_5)) \psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \cdot \partial^\mu \pi) - V(\phi_0, \sigma, \pi)$$

Keeping the dilaton frozen at its vacuum value  $\phi_0$ , the potential reads:

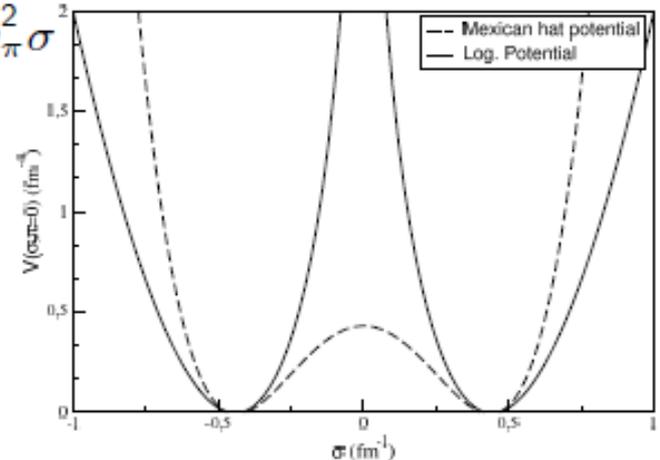
$$V(\sigma, \pi) = \lambda_1^2(\sigma^2 + \pi^2) - \lambda_2^2 \ln(\sigma^2 + \pi^2) - \sigma_0 m_\pi^2 \sigma$$

$$\lambda_1^2 = \frac{1}{4}(m_\sigma^2 + m_\pi^2)$$

$$\lambda_2^2 = \frac{\sigma_0^2}{4}(m_\sigma^2 - m_\pi^2)$$

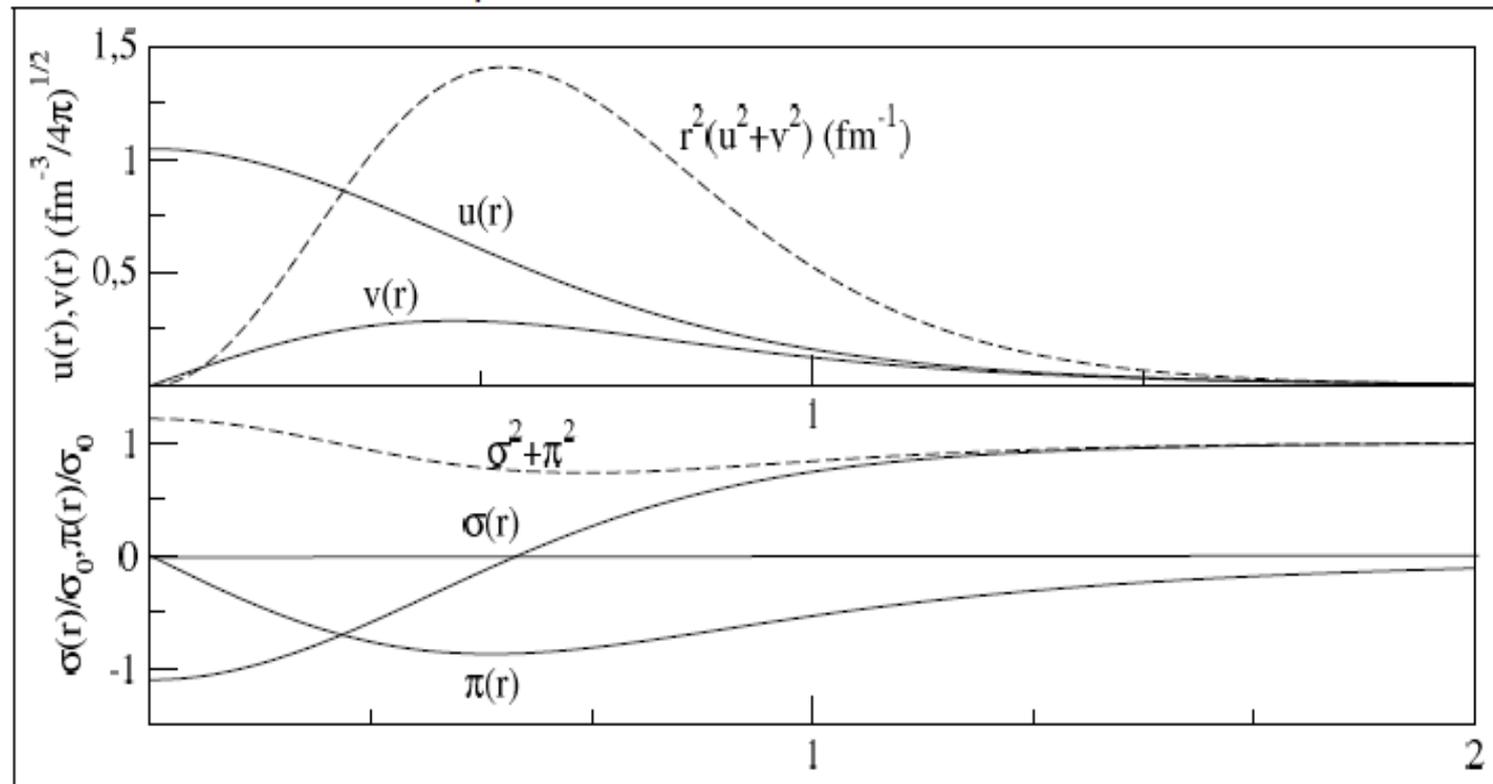
$$\sigma_0 = f_\pi = 93 \text{ MeV}, m_\sigma = 550 \text{ MeV},$$

$$m_\pi = 139.6 \text{ MeV}, g = 5$$



# Single soliton in vacuum

Fields in vacuum for the present model:



# Single soliton in vacuum

Quantity	Log. Model	$\sigma$ Model	Exp.
$E_{1/2}$ (MeV)	1075.4	1002.1	
$M_N$ (MeV)	959.6	894.1	938
$E_{3/2}$ (MeV)	1140.5	1075.2	
$M_\Delta$ (MeV)	1032.	975.4	1232
$\langle r_E^2 \rangle_p$ (fm <sup>2</sup> )	0.82	0.92	0.74
$\langle r_E^2 \rangle_n$ (fm <sup>2</sup> )	-0.03	-0.01	-0.12
$\langle r_M^2 \rangle_p$ (fm <sup>2</sup> )	0.82	0.87	0.74
$\langle r_M^2 \rangle_n$ (fm <sup>2</sup> )	0.86	0.9	0.77
$\mu_p$ ( $\mu_N$ )	3.	3.24	2.79
$\mu_n$ ( $\mu_N$ )	-2.6	-2.5	-1.91
$g_a$	1.52	1.1	1.26
$\langle N_\pi \rangle_J$	1.6, ( $J = 1/2$ )	1.2, ( $J = 1/2$ )	/
	2., ( $J = 3/2$ )	1.6, ( $J = 3/2$ )	

The values of most of the observables in CDM turn out to be slightly closer to the experimental ones than in the  $\sigma$  model

# Wigner-Seitz approximation to nuclear matter

- Approximating nuclear matter by a lattice of nucleons  $\rightarrow$  we consider the meson fields configuration centered at each lattice point, generating a **periodic potential** in which the quarks move
- **Wigner-Seitz approximation**: replace the cubic lattice by a spherical symmetric one  $\rightarrow$  each soliton sits on a spherical cell of radius  $R$  with specific boundary conditions on the surface of the sphere

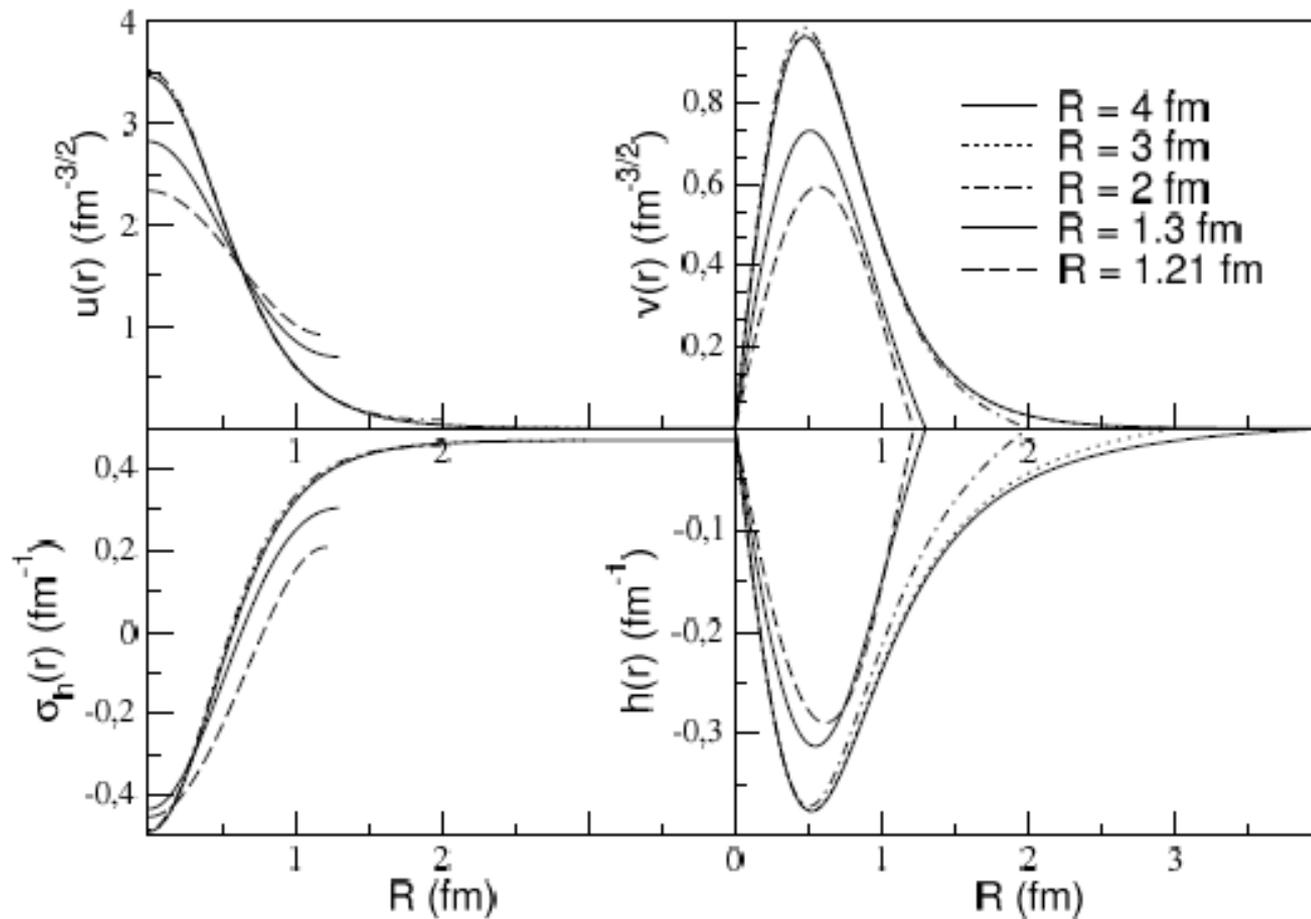
The Hamiltonian for a periodic system must obey **Bloch's theorem**, so the quark spinor must be of the form:

$$\psi_{\mathbf{k}}(r) = e^{i\mathbf{k}\cdot r} \Phi_{\mathbf{k}}(r), \quad (\mathbf{k} = 0 \text{ for the ground state})$$

The **bottom of the band** is defined as the state satisfying the following periodic boundary conditions, dictated by symmetry arguments (parity):

$$\begin{aligned} v(R) &= h(R) = 0, \\ u'(R) &= \sigma'_h(R) = 0. \end{aligned}$$

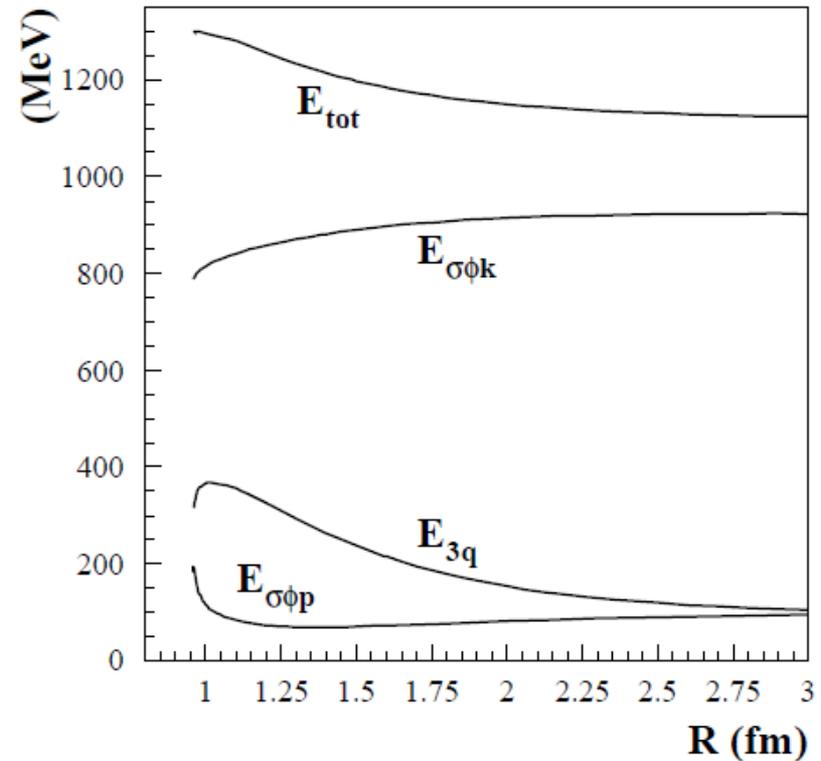
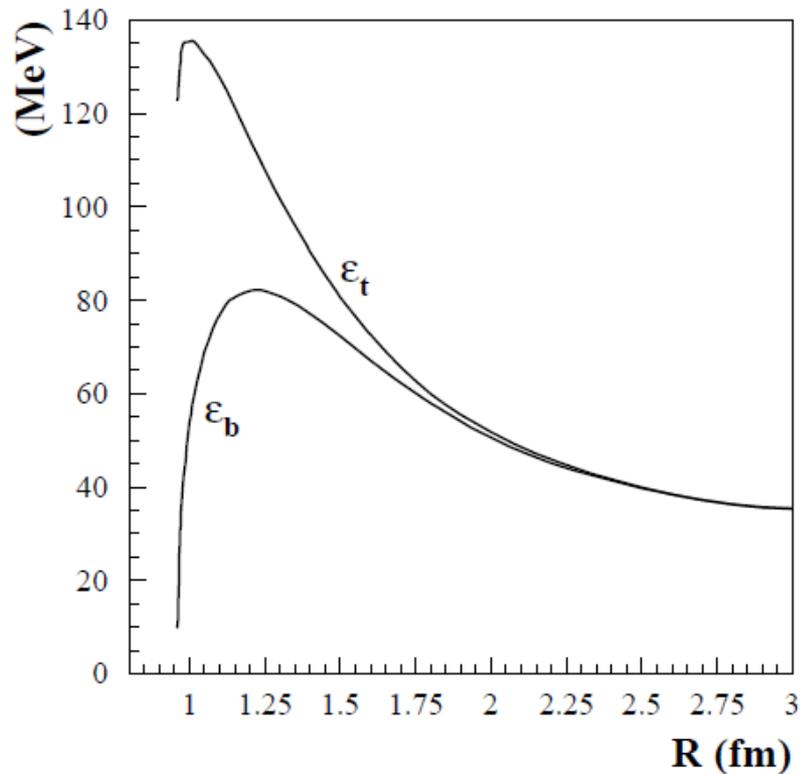
# Fields at finite density



# Results in the linear $\sigma$ -model

## Weber and McGovern 1997

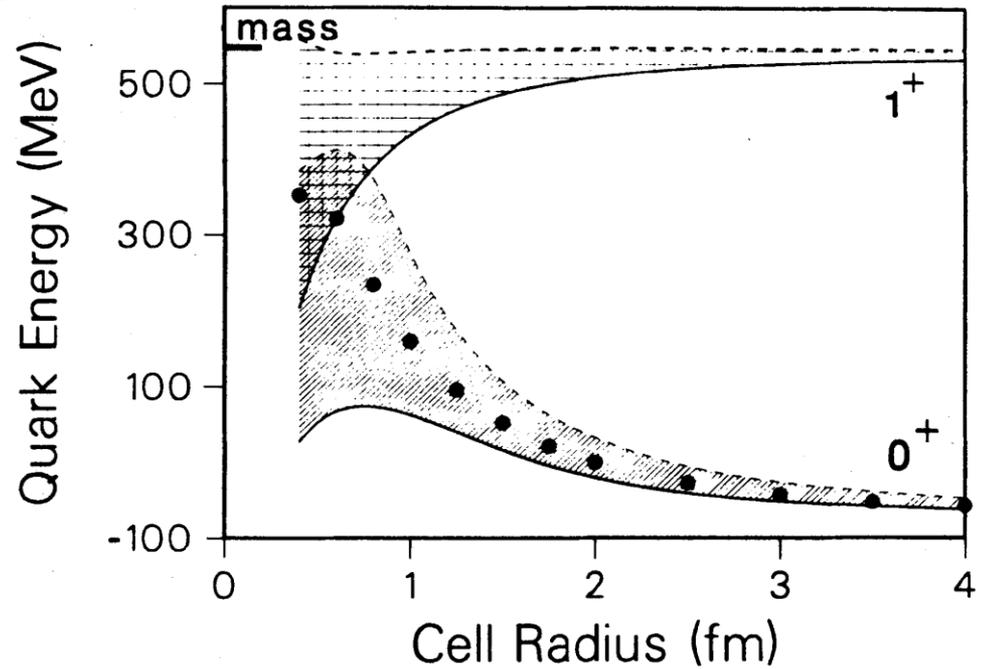
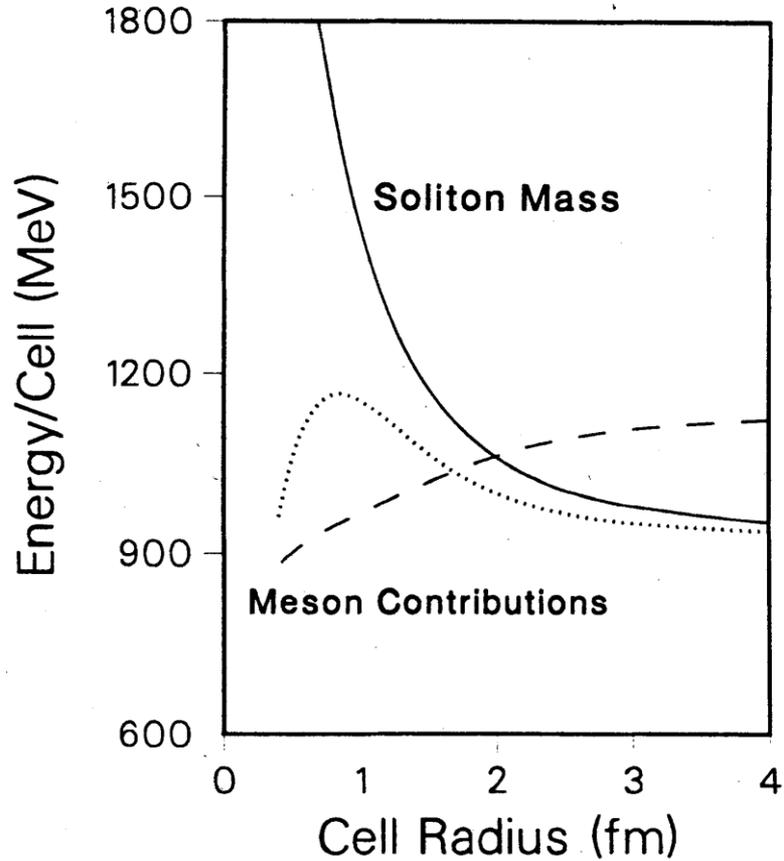
**No saturation. No solutions at moderate densities.**



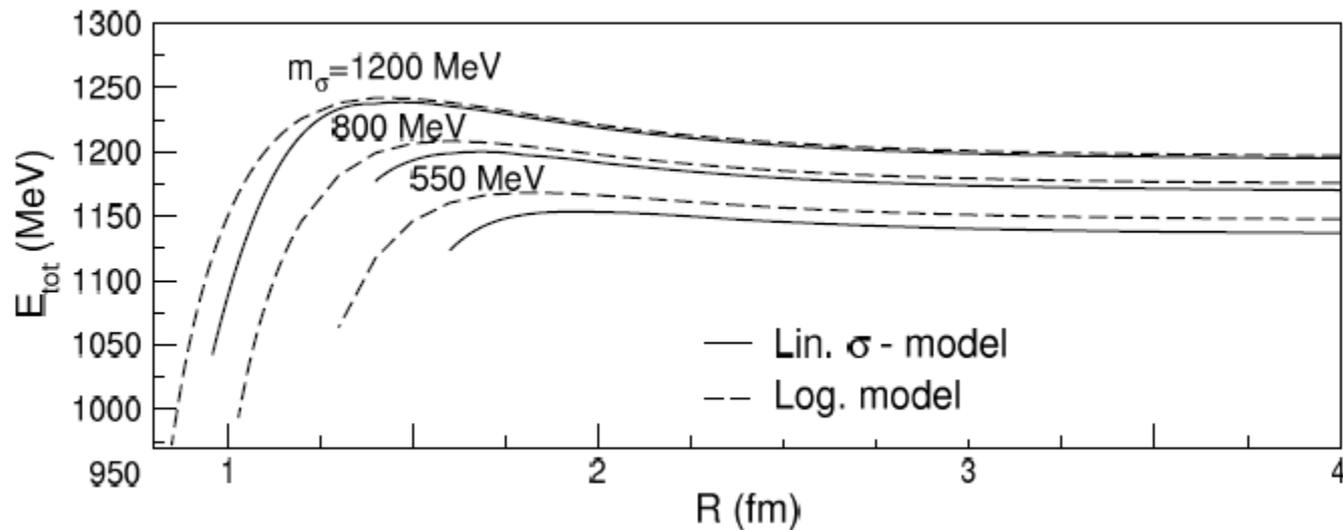
Here  $m_\sigma = 1200$  MeV

# Results in the **non-linear $\sigma$ -model** Hahn and Glendenning 1987

**No saturation**



# Stability at large densities



**At each given  $m_\sigma$  the log. model is more stable than the  $\sigma$ -model**

$$R = 1.0 \text{ fm} \rightarrow \rho = 0.25 \text{ fm}^{-3} = 1.5 \rho_0$$

$$R = 0.8 \text{ fm} \rightarrow \rho = 0.5 \text{ fm}^{-3} = 3 \rho_0$$

## How to define a band

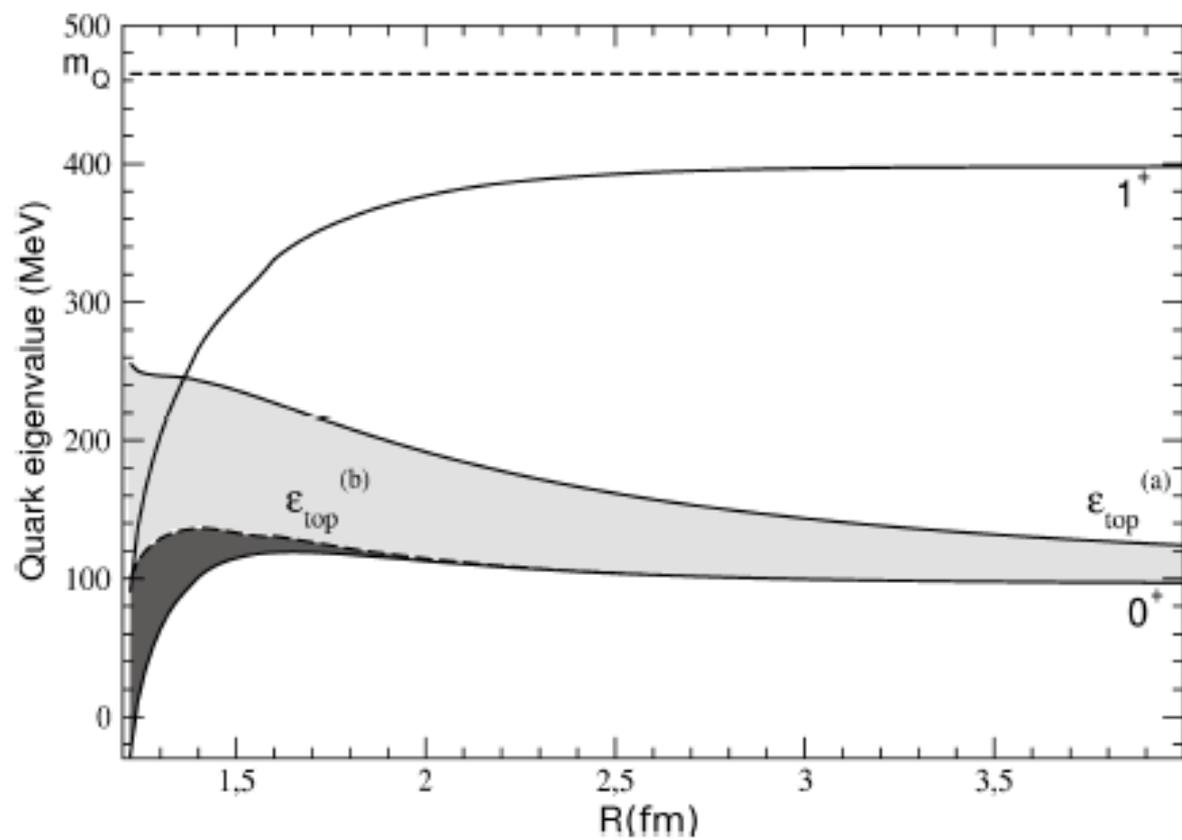
In our work we use two different methods to estimate the band width:

- A (rather crude) approximation to the width of a band can be obtained by using (Glendenning, Banerjee PRC 34(1986)):

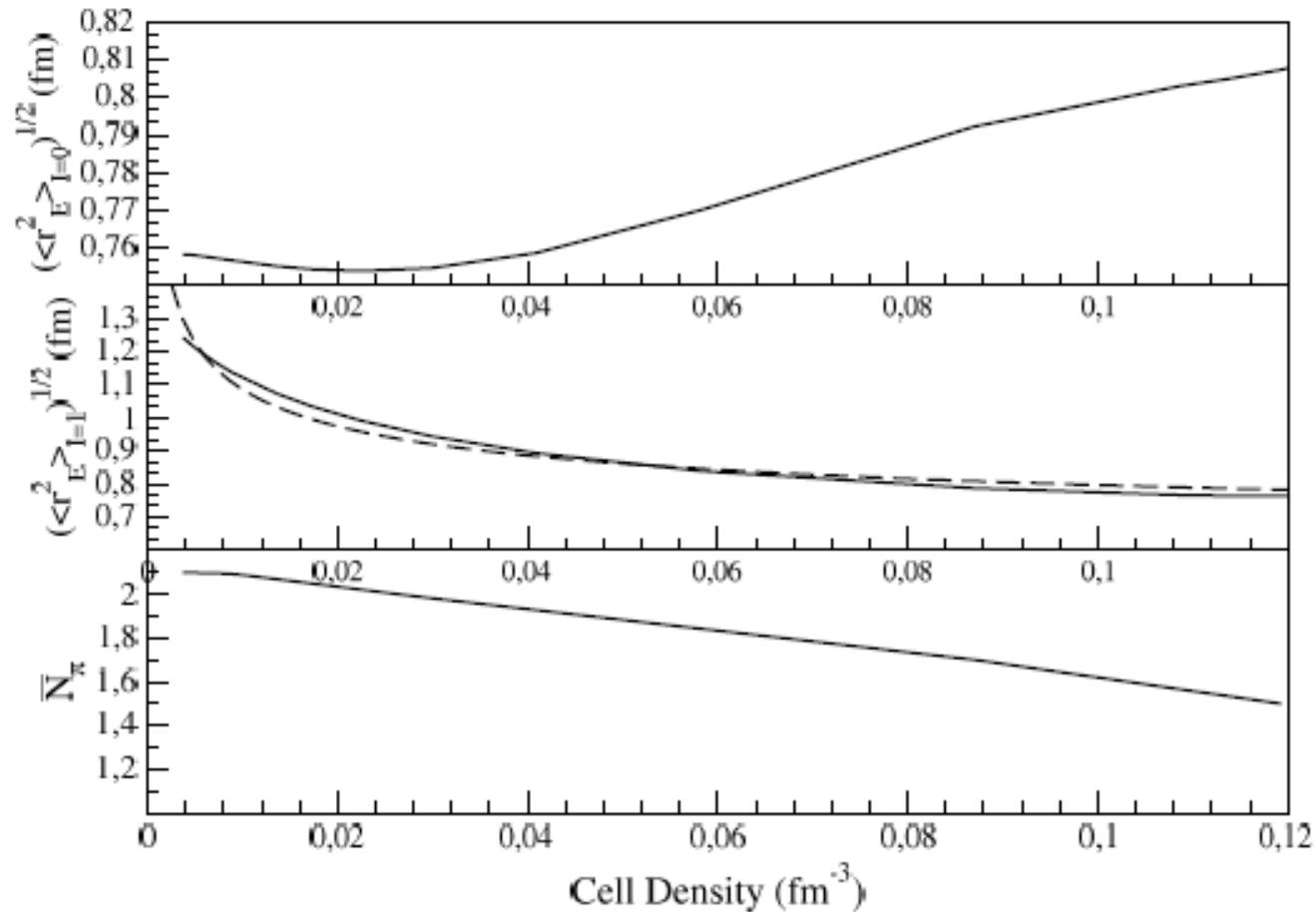
$$\begin{aligned}\Delta &= \sqrt{\epsilon_0^2 + \left(\frac{\pi}{2R}\right)^2} - |\epsilon_0|, \\ \epsilon_{top} &= \epsilon_0 + \Delta.\end{aligned}$$

- An alternative approximation is obtained by imposing that the upper Dirac component vanishes at the boundary (U.Weber, J.A.McGovern, PRC 57 (1998)):

$$u(R) = 0$$



# Modification of nucleon properties at finite density and using MFA



# Including vector mesons

$$\begin{aligned}\mathcal{L} = & \bar{\psi} \left( i\gamma^\mu \partial_\mu - g_\pi (\sigma + i\pi \cdot \boldsymbol{\tau} \gamma_5) + g_\rho \gamma^\mu \frac{\boldsymbol{\tau}}{2} \cdot (\boldsymbol{\rho}_\mu + \gamma_5 \mathbf{A}_\mu) - g_\omega \gamma^\mu \omega_\mu \right) \psi \\ & + \frac{1}{2} (D_\mu \sigma D^\mu \sigma + D_\mu \pi \cdot D^\mu \pi) - \frac{1}{4} (\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu} + \mathbf{A}_{\mu\nu} \cdot \mathbf{A}^{\mu\nu} + \omega_{\mu\nu} \omega^{\mu\nu}) \\ & + \frac{1}{2} m_\rho^2 (\boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu + \mathbf{A}_\mu \cdot \mathbf{A}^\mu) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - V(\phi, \sigma, \pi)\end{aligned}$$

# Field equations and hedgehog

$$[i\gamma^\mu \partial_\mu - g_\pi(\sigma + i\pi \cdot \tau \gamma_5) + g_\rho \gamma^\mu \frac{\tau}{2} \cdot (\rho_\mu + \gamma_5 \mathbf{A}_\mu) - \frac{g_\omega}{3} \gamma^\mu \omega_\mu] \psi = 0,$$

$$\partial_\mu D^\mu \sigma = -g_\rho \mathbf{A}_\mu \cdot \boldsymbol{\pi} - g \bar{\psi} \psi - \frac{\partial V}{\partial \sigma},$$

$$\partial_\mu D^\mu \boldsymbol{\pi} = g_\rho (-\boldsymbol{\rho}_\mu \times D^\mu \boldsymbol{\pi} + \mathbf{A}_\mu D^\mu \sigma) - ig \bar{\psi} \boldsymbol{\tau} \gamma_5 \psi - \frac{\partial V}{\partial \boldsymbol{\pi}},$$

$$-\partial^\mu \rho_{\mu\nu} = g_\rho v_\nu + m_\rho^2 \rho_\nu,$$

$$-\partial^\mu \mathbf{A}_{\mu\nu} = g_\rho \mathbf{a}_\nu + m_\rho^2 \mathbf{A}_\nu,$$

$$-\partial^\mu \omega_{\mu\nu} = -\frac{1}{3} g_\omega \bar{\psi} \gamma_\nu \psi + m_\omega^2 \omega_\nu.$$

Here  $v_\nu$  and  $\mathbf{a}_\nu$  are the vector and the axial-vector currents:

$$v_\nu = \boldsymbol{\rho}_\mu \times \rho_{\mu\nu} + \mathbf{A}_\mu \times \mathbf{A}_{\mu\nu} + \boldsymbol{\pi} \times D^\nu \boldsymbol{\pi} + \bar{\psi} \gamma_\nu \frac{\boldsymbol{\tau}}{2} \psi,$$

$$\mathbf{a}_\nu = \boldsymbol{\rho}_\mu \times \mathbf{A}_{\mu\nu} + \mathbf{A}_\mu \times \rho_{\mu\nu} + \boldsymbol{\pi} \times D^\nu \sigma - \sigma D_\nu \boldsymbol{\pi} + \bar{\psi} \gamma_5 \gamma_\nu \frac{\boldsymbol{\tau}}{2} \psi$$

$$\psi = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} u(r) \\ iv(r)\boldsymbol{\sigma} \cdot \hat{\mathbf{r}} \end{pmatrix} \frac{1}{\sqrt{2}} (|u \downarrow\rangle - |d \downarrow\rangle)$$

$$\langle \hat{\sigma} \rangle = \boldsymbol{\sigma}(r),$$

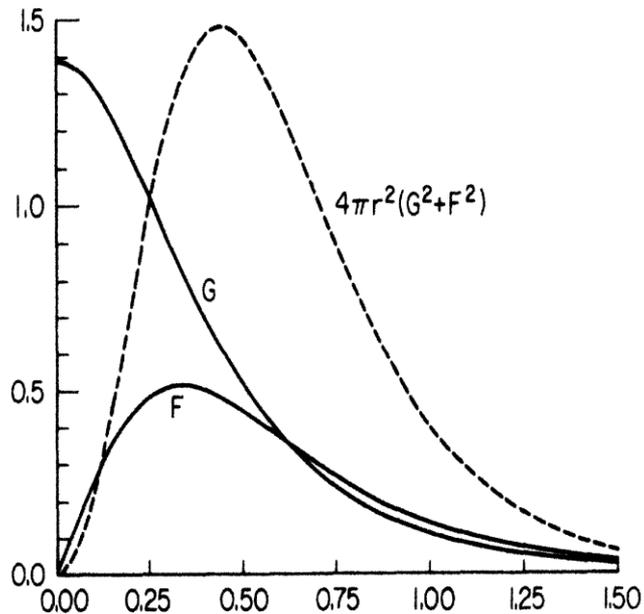
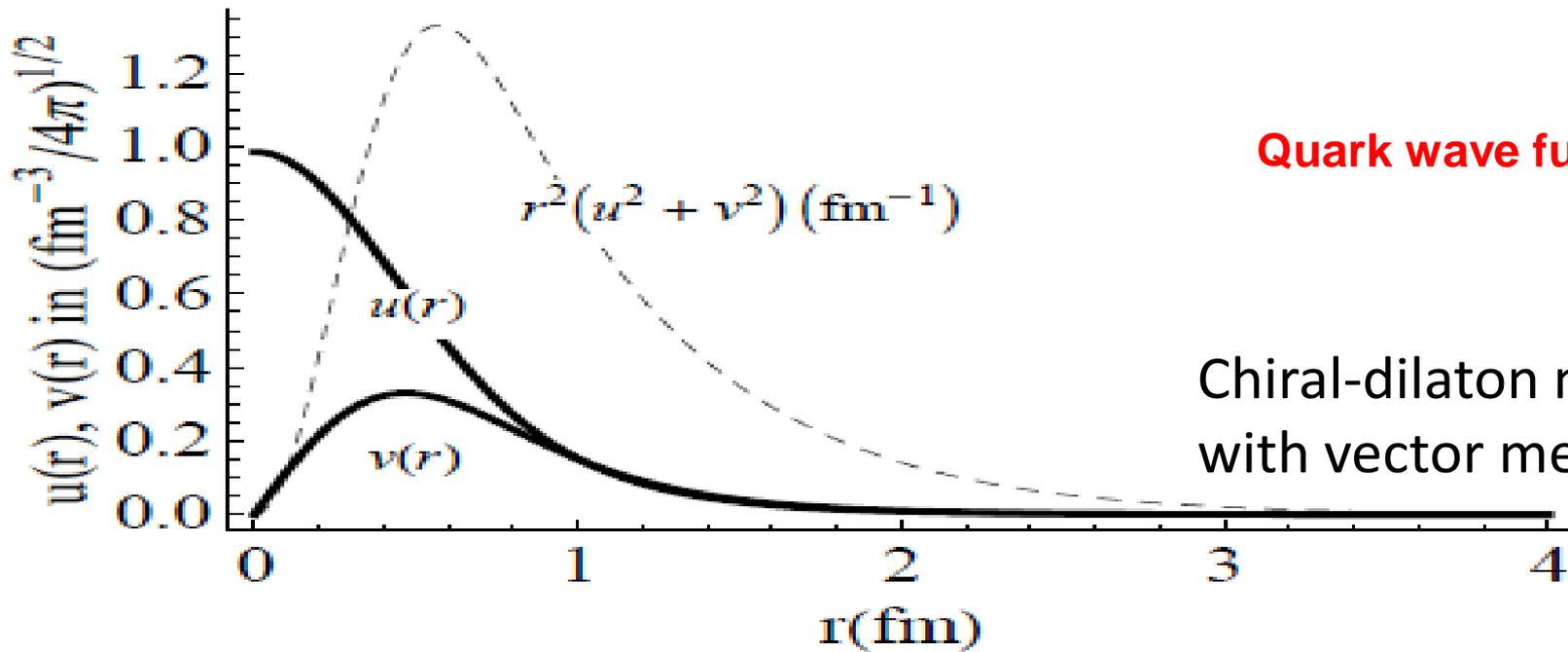
$$\langle \hat{\boldsymbol{\pi}}_a \rangle = r_a \boldsymbol{\pi}(r),$$

$$\langle \hat{\omega}_0 \rangle = \omega(r),$$

$$\langle \hat{\boldsymbol{\rho}}_a^i \rangle = -\epsilon^{ika} r^k \rho(r),$$

$$\langle \hat{\mathbf{A}}_a^i \rangle = A_S(r) \delta_{ia} + A_T(r) (\hat{r}_i \hat{r}_a - \frac{1}{3} \delta_{ia})$$

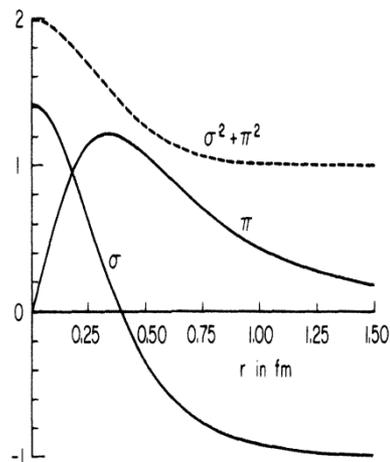
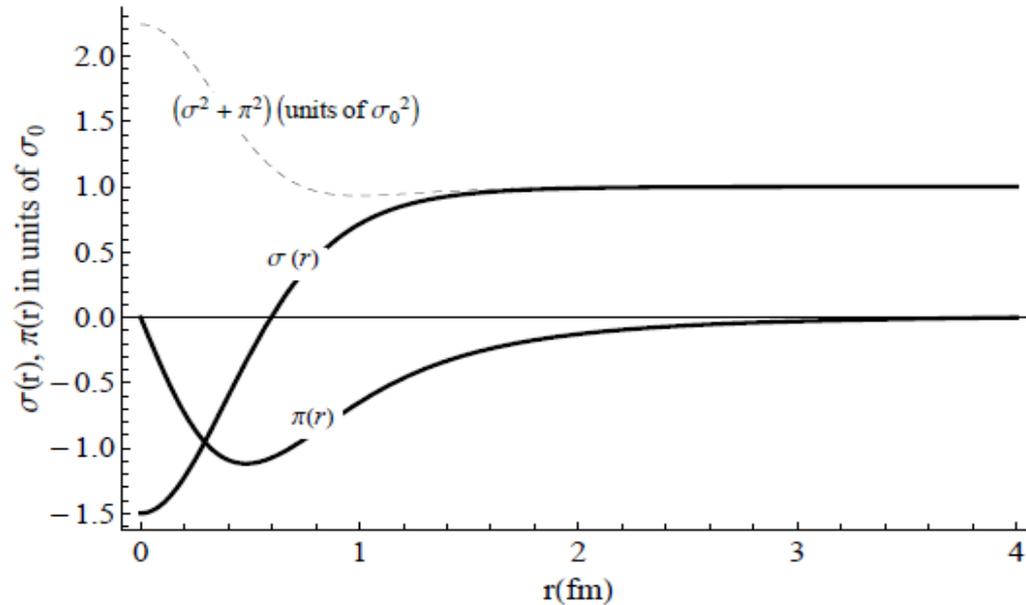
Hedgehog ansatz



Linear  $\sigma$ -model with vector mesons  
 Broniowski and Banerjee PRD34 (1986) 849

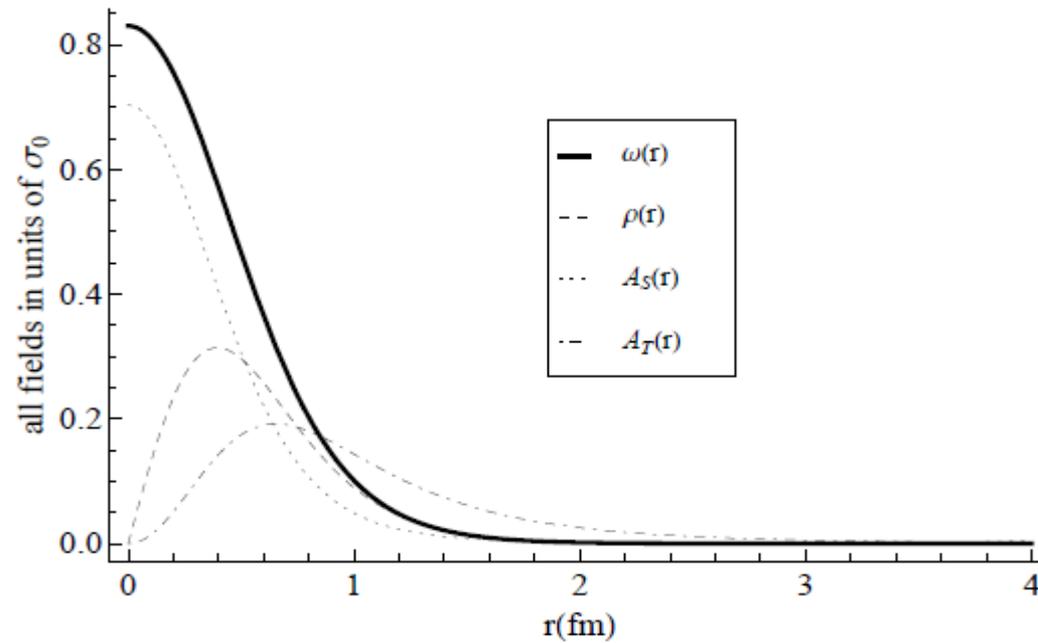
## Chiral fields

Chiral-dilaton model  
with vector mesons

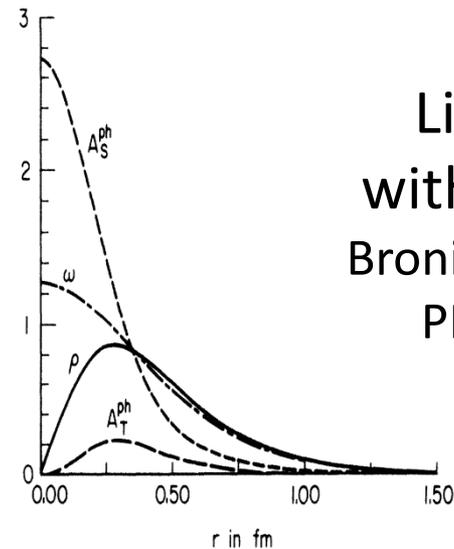
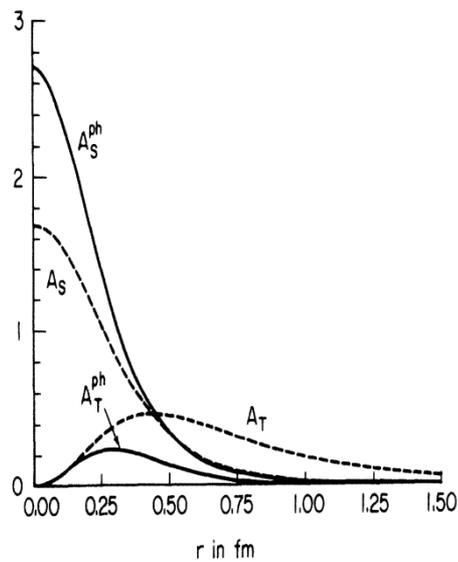


Linear  $\sigma$ -model with vector mesons  
Broniowski and Banerjee PRD34 (1986) 849

## Vector mesons

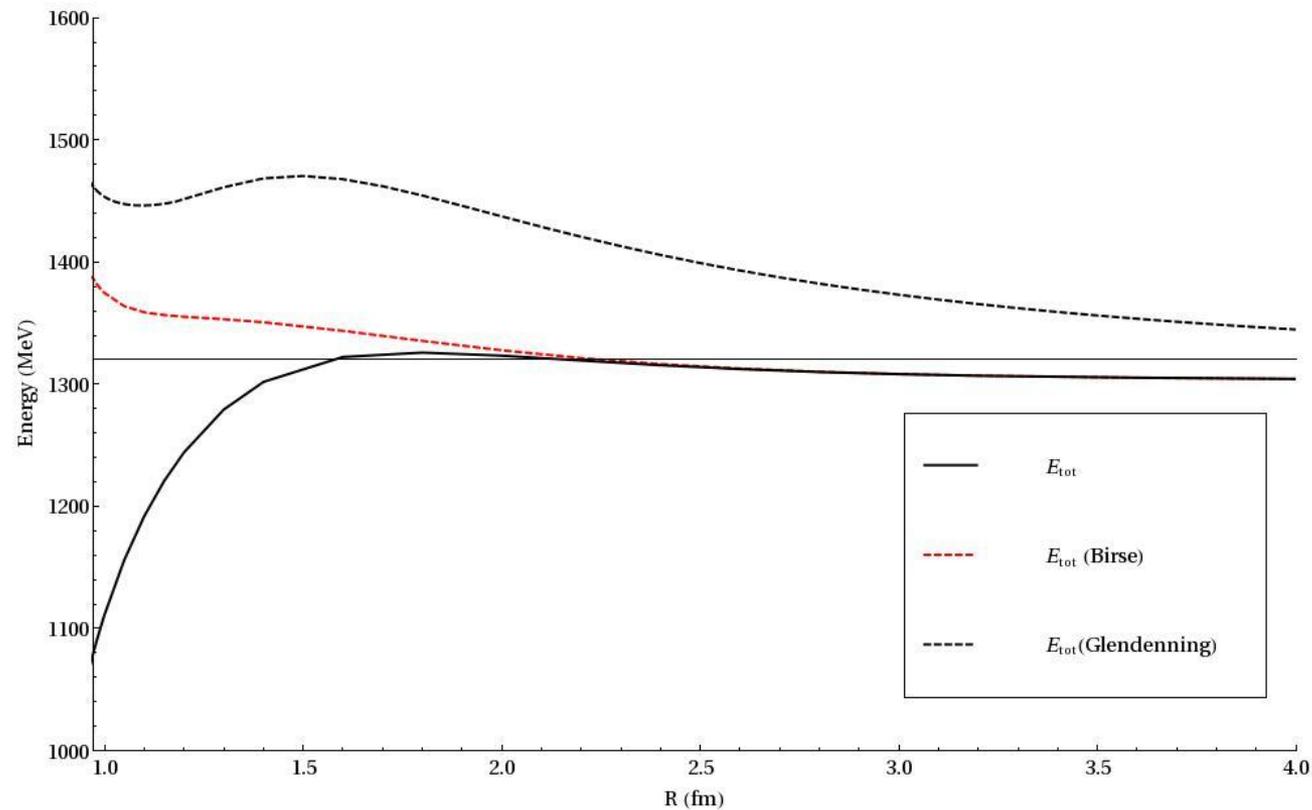


Chiral-dilaton model  
with vector mesons



Linear  $\sigma$ -model  
with vector mesons  
Broniowski and Banerjee  
PRD34 (1986) 849

# Preliminary results with vector mesons at finite density



# Conclusions and outlook

- A chiral-dilaton model can be used to describe nuclear matter (and nuclei). It provides a description of the phase space in the  $\rho$ -T plane not too different from the McLerran-Pisarski scenario.
- The same chiral-dilaton model can be used to describe the quark dynamics. Nucleons emerge as chiral solitons.
- A Wigner-Seitz lattice can be built and soliton remains stable at larger densities than in  $\sigma$ -model
- Saturation is maybe possible, *via* the interplay between vector mesons and chiral fields (*but, has  $G=0$  matter to be saturating?*)
- The extension to finite temperature is very difficult but also extremely interesting: dynamics of the dilaton field, critical end-point...
- We are still **very far** from being able to describe nuclei starting from quark dynamics!