The Hypothesis of Superluminal Neutrinos: comparing OPERA with other Data

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Abstract: The OPERA Collaboration reported evidence for muonic neutrinos traveling slightly faster than light in vacuum. While waiting further checks from the experimental community, here we aim at exploring some theoretical consequences of the hypothesis that muonic neutrinos are superluminal, considering in particular the tachyonic and the Coleman-Glashow cases. We show that a tachyonic interpretation is not only hardly reconciled with OPERA data on energy dependence, but that it clashes with neutrino production from pion and with neutrino oscillations. A Coleman-Glashow superluminal neutrino beam could instead be safely produced from pions and describe the OPERA data; it could also be easily reconciled with SN1987a data, but then it would be very problematic to account for neutrino oscillations.

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I. INTRODUCTION

Recently the OPERA Collaboration [1] reported an early arrival time for CNGS muon neutrinos with respect to the one expected assuming neutrinos to travel at the speed of light in vacuum c. The relative difference of the velocity of the muon neutrinos v with respect to light quoted by OPERA is:

$$\frac{v-c}{c} = (2.48 \pm 0.28(stat) \pm 0.30(sys)) \times 10^{-5} \ , \tag{1}$$

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for a mean energy of the neutrino beam of 17 GeV.

Similar hints, but with much less significance, were also reported for muon neutrino beams produced at Fermilab. Dealing with energies peaked at 3 GeV, the MINOS Collaboration [2] found in 2007 that $(v-c)/c = (5.1 \pm 3.9) \times 10^{-5}$. In 1979, a bound on the relative velocity of the muon with respect to muon neutrinos (with energies from 30 to 200 GeV) was also extracted: $|v - v_\mu|/v_\mu \leq 4 \times 10^{-5}$ [3].

While urging the experimental community to further check and debate on these results, in particular the most recent ones, it is worth to explore which theoretical consequences would follow from the hypothesis that *the muon neutrino is a superluminal particle.*

This clearly requires a deep modification of the Standard Model (SM) of particle physics, that assumes particles to be *subluminal.* The energy and momentum of a subluminal particle of mass m and velocity $\vec{v}$ are: $E = mc^2/\sqrt{1-\frac{v^2}{c^2}}$, $\vec{p} = m\vec{v}/\sqrt{1-\frac{v^2}{c^2}}$. They are related by the dispersion relation $E^2 = p^2c^2 + m^2c^4$, where $p = |\vec{p}|$, and the deviation from the speed of light is:

$$\frac{c-v}{c} = \frac{c}{c+v}\left(\frac{mc^2}{E}\right)^2 . \tag{2}$$

Here we focus on two possibilities to account for a superluminal particle: the tachyon [5] and the Coleman-Glashow particle [6, 7].

The usual expressions for energy and momentum can be extended to the region $v > c$ provided we substitute in the numerator $m \to i\tilde{m}$, where $\tilde{m}$ is a real number. For such a particle, the energy and momentum are thus $E = \tilde{m}c^2/\sqrt{\frac{v^2}{c^2}-1}$, $\vec{p} = \tilde{m}\vec{v}/\sqrt{\frac{v^2}{c^2}-1}$, and satisfy the dispersion relation

$$E^2 = p^2c^2 - \tilde{m}^2c^4 . \tag{3}$$

A tachyonic particle of mass $\tilde{m}c^2$ and energy E then travels faster than c by an amount

$$\frac{v-c}{c} = \frac{c}{c+v}\left(\frac{\tilde{m}c^2}{E}\right)^2 . \tag{4}$$

The deviation from the speed of light for a tachyon is thus opposite with respect to the one of a subluminal particle with the same mass, eq. (2). In both cases, for energies much larger than the mass, the particle speed v approaches c. We display in fig. 1 the relative speed of the tachyon with respect to light as a function of the tachyon mass and for selected energy values.

The proposal that the neutrino could be a tachyon dates back to 1985 [4]. In the light of the recent results, it is worth to consider this hypothesis as a possible explanation of the OPERA data. As we are going to discuss in section II, this interpretation is very problematic for various reasons: not only the deviation from c would depend on energy, but it would not even be possible for a pion to produce a tachyon with the mass in the range required to fit the OPERA data.

Another proposal to account for a superluminal particle has been suggested by Coleman and Glashow (CG) [6, 7]. The idea is that the i-th particle has, in addition to its own mass m_i, its own maximum attainable velocity c_i, and obeys the standard dispersion relation:

$$E_i^2 = p_i^2 c_i^2 + m_i^2 c_i^4 \ . \tag{5}$$

The CG muon neutrino can indeed account for the OPERA data without any trouble associated with its production from pion, as we are going to discuss in section III. To explain the observation of neutrinos associated in time with SN1987a, it is however necessary to introduce another neutrino with speed practically equal to c. This brings severe problems to neutrino oscillations, so that even the CG muon neutrino appears not to be a fully satisfactory explanation.

We draw our conclusions is section IV.

II. PROBLEMS OF A TACHYONIC INTERPRETATION

A. Energy independence of the early arrival times

If the neutrinos produced at CERN are tachyons with mass $\tilde{m}$, after having travelled a distance $L \approx 730$ km, their associated early arrival time is $\delta t = \frac{L}{c}\frac{v-c}{c}$, with $\frac{L}{c} \approx 2.4$ ms. Consider two tachyonic neutrino beams of energy E_1 and E_2, with $E_1 \leq E_2$ for definiteness. As follows from eq.(4), the ratio of their early arrival times δt_1 and δt_2 has a simple energy scaling:

$$\frac{\delta t_1}{\delta t_2} = \left(\frac{E_2}{E_1}\right)^2 \ . \tag{6}$$

The early arrival time of a tachyon neutrino beam is indeed smaller the larger is its energy. In particular, for $E_2 \approx 3E_1$, one expects $\delta t_2 \approx \delta t_1/9$. The difference of the arrival times is thus negative: $\delta t_2 - \delta t_1 \approx -\delta t_1$.

Now, the OPERA Collaborations considers two sample neutrino beams with mean energy equal to $\bar{E}_1 = 13.9$ GeV and $\bar{E}_2 = 42.9$ GeV respectively [1]. The ratio of these energies is indeed close to 3. However, the experimental values of the associated early arrival times are respectively $\delta t_1 = (53.1 \pm 18.8(stat) \pm 7.4(sys))$ ns and $\delta t_2 = (67.1 \pm$

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[1] If neutrinos were tachyons, the energy reconstruction of the OPERA Collaboration should be revisited. However, for tachyonic masses smaller than GeV, such effect is negligible for the sake of the present considerations.

18.2($stat$)±7.4(sys)) ns. These data display no evidence of an energy dependence. OPERA quotes a value $\delta t_2 - \delta t_1 = (14.0 \pm 26.2)$ ns for the difference of the arrival times $\delta t_2 - \delta t_1$. Far from being close to $-\delta t_1$ as expected for a tachyon, the latter value is even slightly positive, although consistent with zero.

This simple argument disfavors the tachyon explanation of the OPERA data. The same conclusions were drawn in refs. [8, 9] (appeared when this paper was completed), carrying out a detailed numerical analysis and including in the fit not only the recent OPERA data but also the Fermilab data, which do not display any energy dependence too.

One could however still question this conclusion, since the energies $\bar{E}_1 = 13.9$ GeV and $\bar{E}_2 = 42.9$ GeV quoted by OPERA are mean ones and if we consider the 3σ range associated to $\delta t_2 - \delta t_1$ we find the interval $[-65, 93]$ ns.

B. Tachyon mass range from OPERA

Suppose that we close an eye on the energy dependence and we stick to the interpretation of the OPERA early arrival time in terms of a tachyonic muon neutrino. As we are going to discuss, arguments based solely on kinematics allow to obtain an indication for the value of the tachyonic muon neutrino mass.

In terms of the muon neutrino energy E and the muon neutrino velocity v, the tachyonic muon neutrino mass $\tilde{m}$ is simply given by (see eq.(4)):

$$\tilde{m}c^2 \approx \sqrt{2\frac{v-c}{c}}E \ . \tag{7}$$

Since OPERA deals with neutrinos with mean energy $\langle E \rangle = 17$ GeV and observes $(v - c)/c = (2.48 \pm 0.28 \pm 0.30) \times 10^{-5}$, the corresponding tachyonic mass value is $\tilde{m}c^2 = (110 - 130)$ MeV at 1σ, and $(85 - 146)$ MeV at 3σ (statistical and systematic errors are summed in quadrature)[2] . This is also graphically shown in fig.1.

This range of values is close to the muon mass, $m_\mu c^2 \approx 105$ MeV, and it would be fascinating to postulate that the muon neutrino mass is just the charged muon one, upon a rotation of $\pi/2$ in the complex plane. However, these conjectures are going to be severely challenged by other experimental data.

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[2] Clearly this value is just a rough estimate, due to the significant energy spread of the neutrino beam, but we said that we ignore this as it would also cause an energy dependence of the early arrival time.

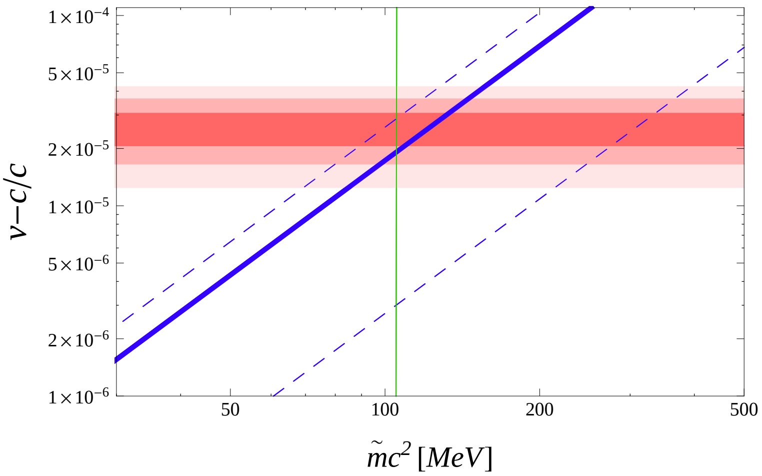

FIG. 1: Relative speed of the tachyon with respect to light as a function of the tachyon mass and for selected values of the its energy: from left to right $E = 13.9, 17.0, 42.9$ GeV. The shaded regions display the OPERA measurement, with $1, 2, 3\sigma$ error bands (statistical and systematic errors are summed in quadrature). The vertical line marks the value of the muon mass, $m_\mu c^2 \approx 105$ MeV.

C. Production from pion

Indeed, first of all we must wonder whether a tachyonic muon neutrino with such a large mass could ever be produced in pion decay: $\pi \to \mu\nu_\mu$. The OPERA muon neutrinos are in fact obtained from pions decaying in flight in a 1 km long vacuum tunnel[3].

Let us focus on the kinematics of pion decay at rest. Clearly, momentum conservation requires the muon and the neutrino to be produced back to back and with the same momentum p. Energy conservation requires in addition:

$$m_\pi c^2 = \sqrt{(pc)^2 + (m_\mu c^2)^2} + \sqrt{(pc)^2 - (\tilde{m}c^2)^2}\ , \tag{8}$$

where m_π, m_μ stand for the pion and muon masses respectively. As $\tilde{m}$ reaches its maximum allowed value when the muon neutrino has null energy ($p = \tilde{m}c$), one derives an upper bound $\tilde{m} \leq \sqrt{m_\pi^2 - m_\mu^2} \approx 92$ MeV$/c^2$. This limit is marginally compatible with the tachyonic neutrino mass range derived before from OPERA results, see fig.1.

To better support this conclusion, in fig. 2 we display the μ and ν_μ energies and momenta as a function of the muon neutrino mass, assuming the latter to be a tachyon (T) or a standard subluminal particle, also called bradyon (B). Deviations from the well

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[3] A proton beam of 400 GeV$/c$ from SPS hits a graphite target producing pions which are focused by two magnetic horns and directed towards the tunnel.

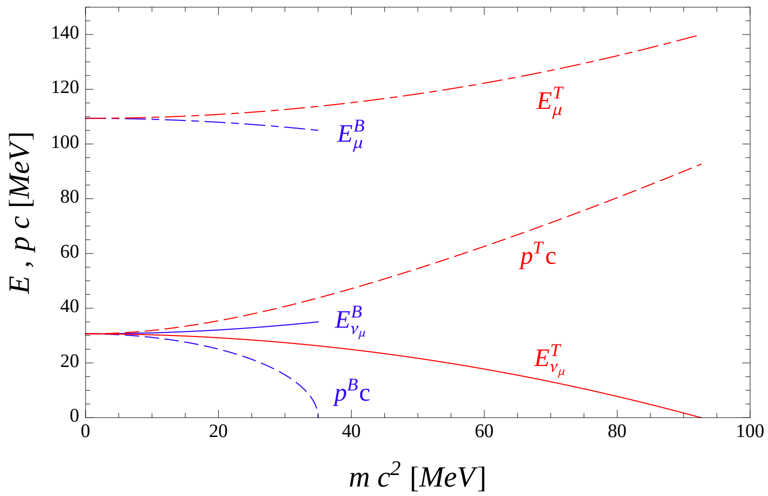

FIG. 2: Kinematics for $\pi \to \mu\nu_\mu$ assuming that the muon neutrino is a tachyon (T) or a bradyon (B). The muon neutrino energy E_{ν_μ} (solid), the muon energy E_μ (dot-dashed) and the associated momentum (dashed) are shown as a function of the muon neutrino mass.

tested values of the muon energy and momentum allow only a small tachyonic mass, say smaller than about 10 MeV/c^2. When inserted in eq. (4) and keeping the muon neutrino energy $E = 17$ GeV, the bound $\tilde{m} \leq 10$ MeV/c^2 would imply $(v-c)/c \leq 1.7 \times 10^{-7}$, which corresponds to an early arrival time at OPERA of $\delta t \leq 0.6$ ns. This is at least two orders of magnitude below the observed value.

We can rephrase all this also in another way: To end up with $(v-c)/c \approx 2 \times 10^{-5}$ while keeping $\tilde{m} = 10$ MeV/c^2, a muon neutrino beam energy $E \approx 1.4$ GeV would have been necessary. The latter value seems to be definitely too small with respect to the reconstructed muon neutrino energy. We conclude that *the tachyonic explanation of the early arrival times of muon neutrinos at OPERA is ruled out.*

We also note that a kinematical analysis of muon decay ($\mu \to e\bar{\nu}_e\nu_\mu$) using the tachyonic mass range suggested by OPERA would produce serious difficulties. For simplicity, consider that ν_μ has a tachyonic mass $\tilde{m}$ while e and $\bar{\nu}_e$ are massless. Then, in the corner of the phase space where $E_{\nu_\mu} = 0$ (and consequently $p_{\nu_\mu} = \tilde{m}c$), the electron energy can be as high as $E_e \leq m_\mu c^2/2(1 + \tilde{m}/m_\mu)$ (intuitively, e and $\bar{\nu}_e$ have to balance the large momentum of the tachyon, and they have a lot of energy to do so, since the tachyon energy is zero). Even more significantly, one can see that for every value of the allowed energy range for the tachyon, $0 \leq E_{\nu_\mu} \leq m_\mu c^2/2(1 - \tilde{m}^2/m_\mu^2)$, the maximum value of E_e is larger than $m_\mu c^2/2(1 + \tilde{m}^2/m_\mu^2)$. In conclusion, values of $\tilde{m} \geq 10$ MeV/c^2 would be immediately detectable in the electron spectrum, whose endpoint would be much larger that $m_\mu/2$. Once again, this argument rules out the hypothesis that the OPERA muon neutrino is a tachyon.

D. Supernovae SN1987a requires an $\lesssim$eV electron-neutrino

Let suppose that we close another eye on the problems associated with the production of a beam of tachyonic muon neutrinos with 100 MeV mass and persevere on this road. Can we agree with the SN1987a data? As we are going to discuss this is possible.

The SN1987a is $L = 1.68 \times 10^5$ ly far from the Earth and exploded releasing a huge neutrino signal, with typical energies $10 - 20$ MeV, which allowed the first direct detection of astrophysical neutrinos. All neutrino flavors were emitted but Kamiokande-II, IMB and Baksan were designed to detect mainly electron anti-neutrinos. The signal lasted about 10 s and the photons also arrived within a few hours. See for instance ref. [10] for a recent review and a list of references.

The advance (delay) of a tachyonic (bradyonic) anti-neutrino with respect to light is $\delta T = T|v - c|/c$, where $T = L/c$ is the time associated to the photon trip from SN1987a to the Earth[4]. The fact that photons and electron anti-neutrinos arrived without few hours implies that $\delta T/T = |v - c|/c \sim 10^{-9}$, which in turn translates into an upper bound for the (bradyonic or tachyonic) mass of the electron anti-neutrino of about 1 keV.

Electron anti-neutrinos arrived with a time spread $\Delta T \lesssim 10$ s, as indicated by observations. This poses a much tighter limit on their (tachyonic or bradyonic) mass $m_{\bar{\nu}_e}$ than the one just discussed. The time spread $\Delta T = |T_2 - T_1|$ of two neutrinos with energies E_1 and E_2 (with $E_1 \leq E_2$) is

$$\frac{\Delta T}{T} = \frac{m^2_{\bar{\nu}_e}}{2E_1^2}\left(1 - \frac{E_1^2}{E_2^2}\right) . \tag{9}$$

For the numerical values mentioned above, one obtains $m_{\bar{\nu}_e} \lesssim 40$ eV$/c^2$. This limit, applies however to the electron anti-neutrino[5].

The SN emits all neutrino flavors. Let suppose that it emits also a 100 MeV tachyonic muon neutrino: its advance with respect to light would be of about $\delta t \approx 4yr$, but with an enormous spread as can it can be realized by considering eq.(9) in the case of a particle with mass bigger than its energy. These neutrinos would have certainly escaped detection.

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[4] The SN exploded when the Earth was in the quaternary period and on such timescales effects due to the expansion of the universe can be neglected.

[5] We recall that an even stronger bound on the mass of a tachyonic electron neutrino follows from tritium beta decay experiments [11], which set a limit of few eV$/c^2$.

E. Oscillations: game over

According to the picture emerged so far, the ratio between the tachyonic mass of the muon neutrino suggested by OPERA and the mass of the electron anti-neutrino suggested by SN1987a would be as large as 10^5. In principle, the formalism of neutrino oscillation in the tachyonic case is the same as for an ordinary neutrino [12], but it appears difficult to come to pact with the robust informations coming from neutrino oscillation experiments. Indeed, these experiments put stringent bounds on the difference of neutrino masses squared: $|\Delta m^2_{32}| \approx 2.4 \times 10^{-3}$ eV^2 and $\Delta m^2_{21} \approx 7.6 \times 10^{-5}$ eV^2 [13]. Even though the analysis of the experimental data should be redone since the fluxes at the production in the Sun for electron anti-neutrinos and in the atmosphere for muon neutrinos would change, it seems hopeless to find an agreement with experimental data on oscillations.

III. MUON NEUTRINO Á LA COLEMAN-GLASHOW

As an alternative scenario, consider now two CG neutrino mass eigenstates with masses m_1 and m_2 not larger than $\mathcal{O}(eV)/c^2$, with different limit speeds c_1 and c_2 [6, 7]. One may infer $|c_1 - c|/c \lesssim 10^{-9}$ from SN1987a as discussed in the previous section, while $(c_2 - c)/c \approx 2 \times 10^{-5}$ as suggested by OPERA.

Let us suppose that ν_2 has a significant mixing with the muon neutrino of the OPERA beam, and that ν_1 mixes significantly with the electron neutrino. We now revisit for this CG superluminal neutrino model the same issues discussed for the tachyon.

First of all, the early arrival time of the muon neutrino beam is energy independent, since now c_2 is a constant (already chosen to reproduce the results from OPERA) and we assumed $m_2 c_2^2 \leq \mathcal{O}(eV)$; with these assumptions the standard kinematics used for event reconstruction at OPERA need not be modified.

Similarly, there are no problems for the production of such CG muon neutrino from pion decay.

Also in the case of SN1987a, a beam of CG ν_2 would pose no problem, because it would have simply arrived about 4 yr in advance with respect to the photons and the other ν_1's. Most probably it would have escaped detection since the detectors had a lower sensitivity to muon neutrinos (moreover Kamiokande-II started taking data only in 1985). At variance with the tachyon case, it is important to remark that the ν_2 beam does not spread out in time but all the neutrinos arrive within a few seconds, because we assumed their mass to be smaller than eV$/c_2^2$.

The real problem for CG neutrinos is again due to neutrino oscillations, as can be shown by using the formalism of refs.[6, 7]. The two CG neutrino eigenstates travel at different speeds and this affects the neutrino oscillation probability similarly to a difference in mass:

$$P(\nu_\ell \to \nu_\ell) = 1 - \sin^2 2\theta \, \sin^2 \left(\frac{R}{\hbar \bar{c}} (\frac{(m_2^2 - m_1^2)\bar{c}^4}{4E} + \frac{\delta c}{\bar{c}} \frac{E}{2}) \right) \;, \tag{10}$$

where θ is the mixing angle, R is the distance from source to detector, $\bar{c} = (c_1 + c_2)/2$, $\delta c = c_2 - c_1$ and E is the neutrino energy, typically in the range of a few MeV for reactor and solar experiments. For numerical estimates, it is perfectly safe to replace $\bar{c}$ with c. Oscillation experiments (see for instance [14]) indicate a value for $m_2^2 - m_1^2 \approx 10^{-4}$ eV$^2/c^4$. This translates in a sensitivity to $\delta c/c$ of about 10^{-18}, much smaller than what would be needed to explain the OPERA data. This can be seen as follows: the experimentally tolerated oscillation frequency is the one of the the first term of the $\sin^2$ argument in eq.(10) with $(m_2^2 - m_1^2)c^4 \sim 10^{-4}$ eV^2 and $E \sim$ MeV. A comparable frequency would result from the second term of the argument of $\sin^2$ only if $\delta c/c \sim 10^{-18}$. Also due to the different energy dependence of these two terms, it seems unlikely that a cancellation might be at work for a much larger value of $\delta c/c \sim 10^{-5}$ in the energy range probed by oscillation experiments. A similar analysis was done in refs.[6, 7], suggesting an even tighter limit $\delta c/c \sim 6 \times 10^{-22}$.

In conclusion, also CG superluminal neutrinos seem not to provide a fully satisfactory explanation of the OPERA results.

IV. CONCLUSIONS

The evidence for muonic neutrinos traveling slightly faster than light in vacuum as reported by the OPERA Collaboration, motivated us to explore two possible interpretations of the data: the hypothesis that the muon neutrino is a tachyon or that it is a Coleman-Glashow neutrino.

We demonstrated that the tachyonic interpretation is hardly reconciled with the energy independence of the OPERA data, as shown also by [8, 9]. The real problem that we point out here is that it would be impossible to produce a 100 MeV tachyon from pion decay. The data associated with SN1987a can be interpreted by assuming an eV electron anti-neutrino. This picture however clashes with what is known concerning neutrino oscillations.

A Coleman-Glashow superluminal neutrino beam could instead be safely produced from pions and describe the OPERA data; it could also be easily reconciled with SN1987a data. On the other hand, also in this case it would be not possible to reconcile the model with neutrino oscillations.

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