

Can Lattice QCD account for Charm Flow at PHENIX?

*Rajiv V. Gavai**

T. I. F. R., Mumbai & Universität Bielefeld

** With Debasish Banerjee, Saumen Datta & Pushan Majumdar, arXiv:1109.5738*

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Introduction

Formalism

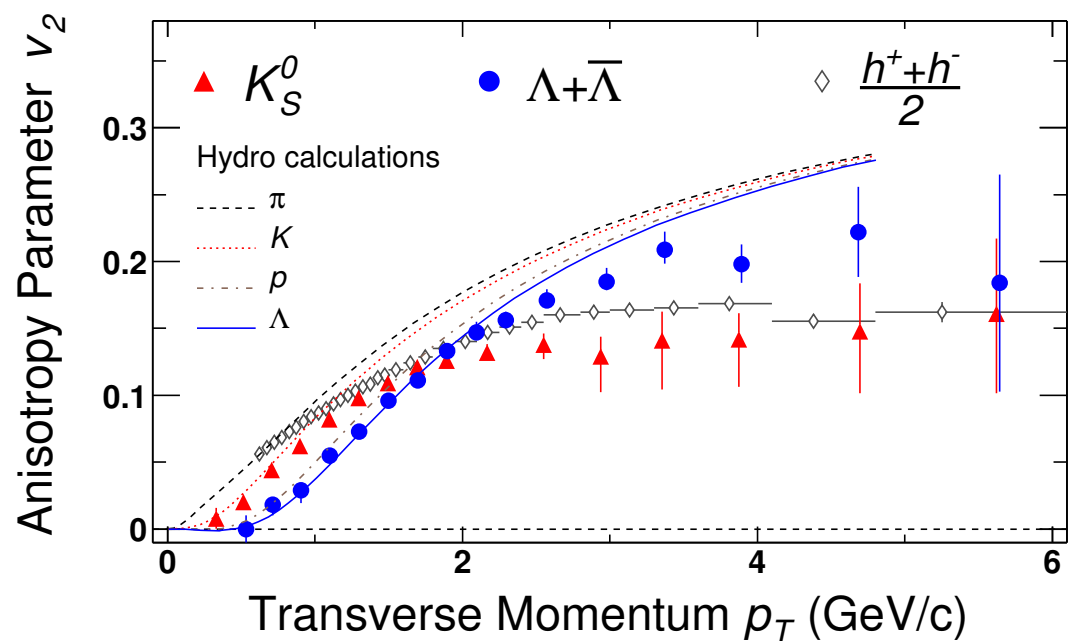
Our Lattice Results

Summary

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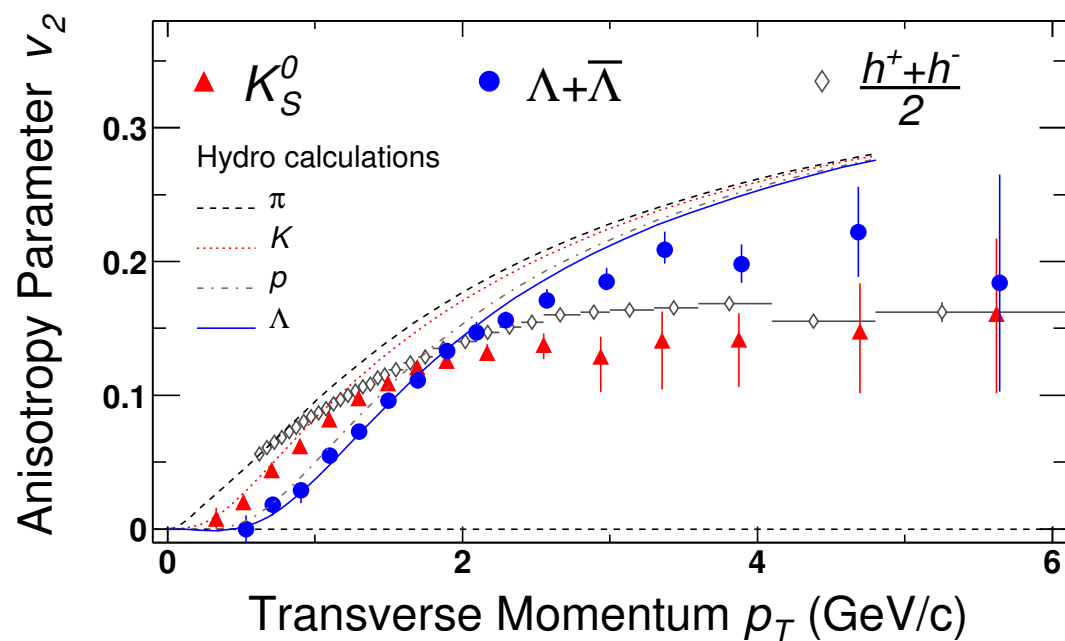
Introduction

- Exciting results from RHIC on the elliptic flow, a measure of azimuthal anisotropy.



(STAR Collaboration, Adams et al., PRL92 (2004) 052302.)

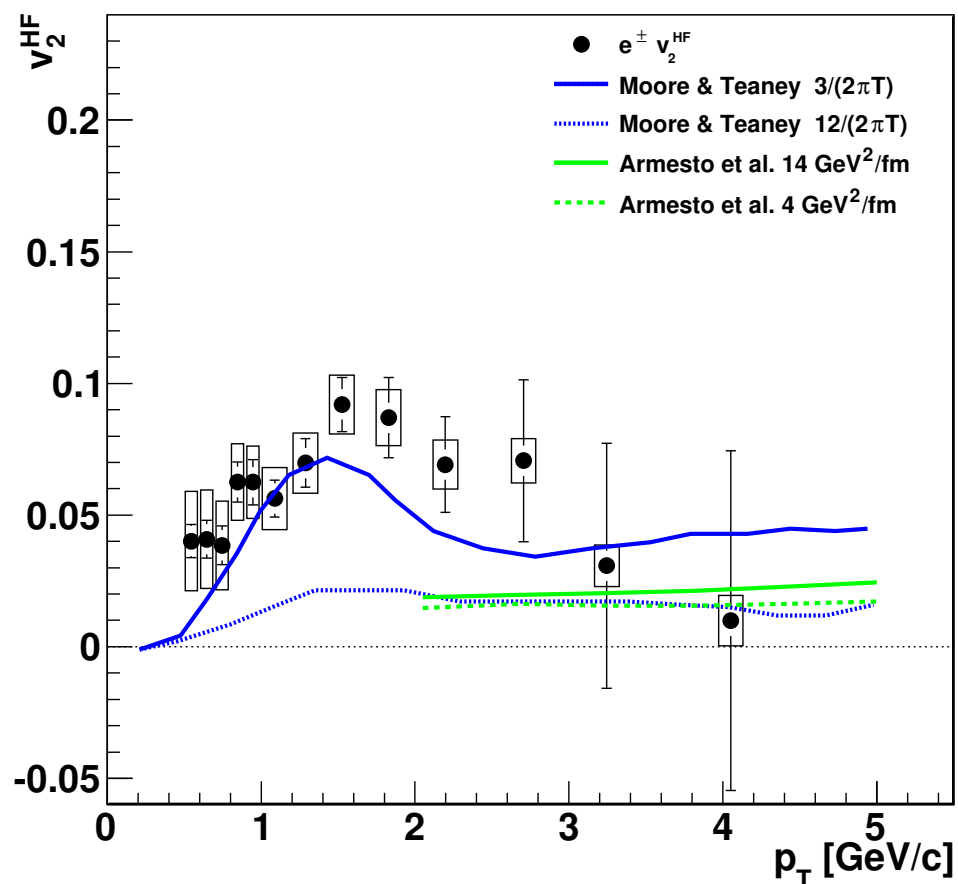
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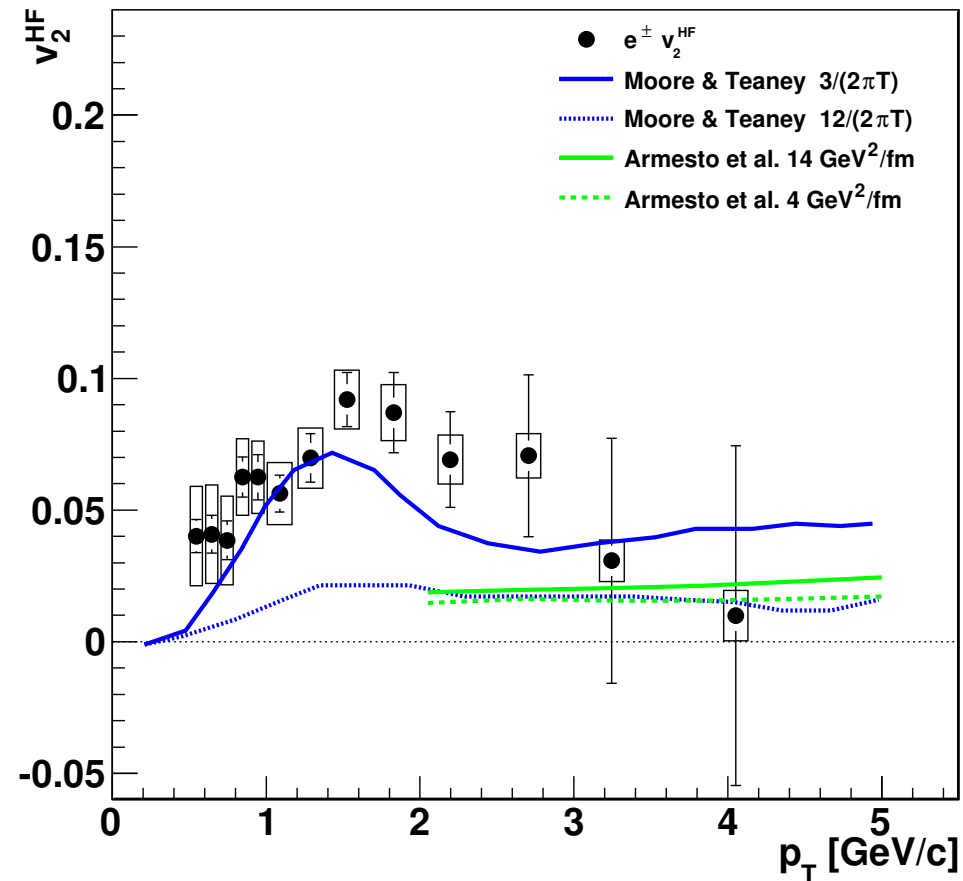
- Exciting results from RHIC on the elliptic flow, a measure of azimuthal anisotropy.
- Good agreement with ideal hydro: Suggesting early thermalization and perfect fluid and many more interesting things.

- Naively expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.

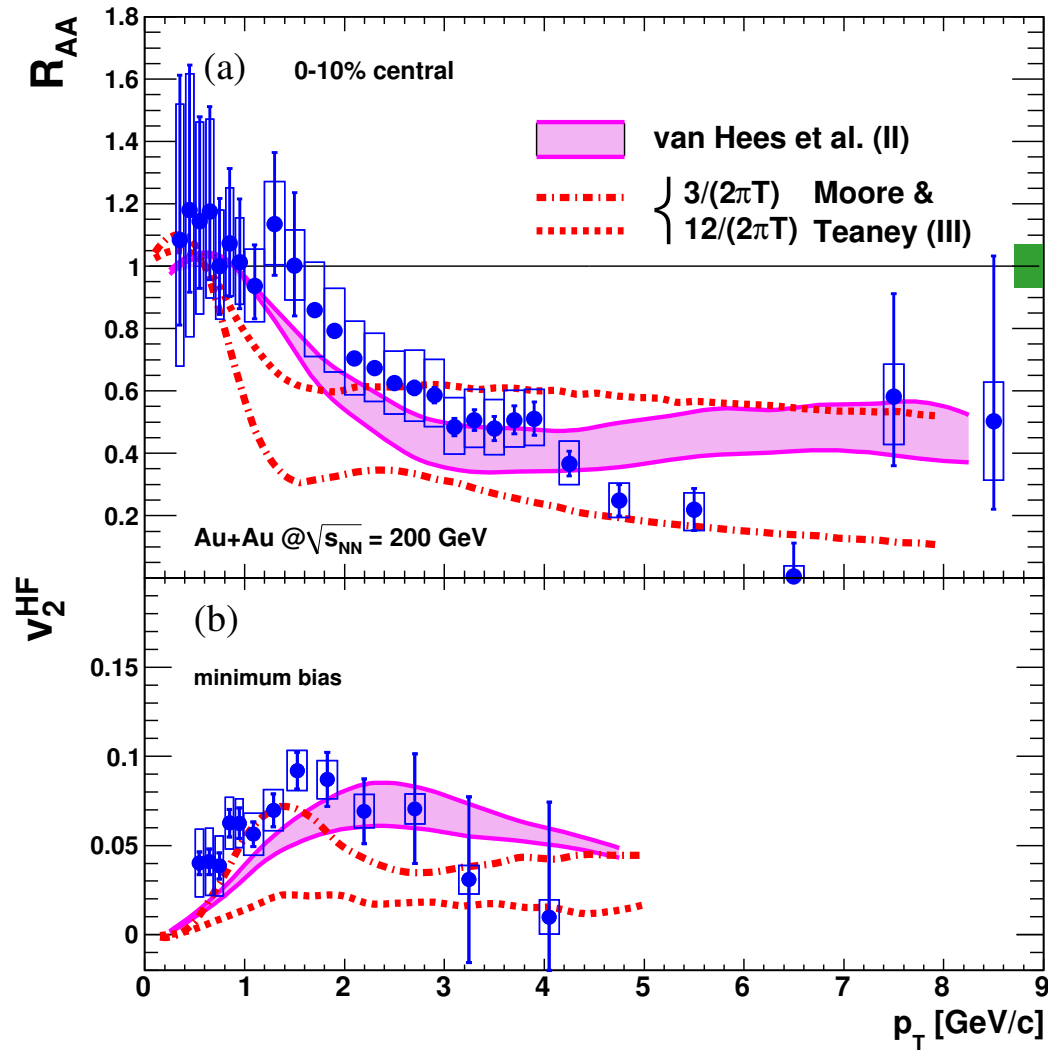


(PHENIX Collaboration, Adare et al., arXiv:1005.1627 & PRL 98 (2007) 172301.)

- Naively expect heavy quark relaxation time to be M/T times larger, leading to the expectation of small/zero flow for charm quarks.
- In models (Moore-Teaney, PRC 71, 2005), heavy quark diffusion coefficients governs its elliptic flow **and** suppression.



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- Denoting by D the heavy quark diffusion coefficient, $D = 12/2\pi T$, a 'perturbative' estimate, seems to under-predict v_2 substantially.
- **Smaller** $D \simeq 3/2\pi T$ seems required by data.
- Similar value also explains the suppression in the PHENIX R_{AA} for heavy quarks at RHIC.

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- Is it non-perturbative ? **Strong coupling models — AdS/CFT based — do lead to values in the desired range under “suitable” assumptions.**
- **Can Lattice QCD shed some light on the Charm Flow ?**

Langevin Model for Heavy Q Thermalization

- Momentum transfer from a thermal gluon is $\sim T$ at most. It takes $\sim M/T$ collisions to change momentum of the heavy Q by $\mathcal{O}(1)$.
- Its interaction with the medium can be modelled as uncorrelated momentum kicks (Moore-Teaney, PRC 71 (2005) 064904) : A Langevin Model.

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$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t) \quad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t') \quad (1)$$

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- η_D – momentum drag coefficient and 3κ is mean-squared momentum transfer per unit time.
- Diffusion constant D can be found to be $2T^2/\kappa$ with $\eta_D = \kappa/2MT$.

- Moore-Teaney also showed that an initial ($T_0 = 300$ MeV) power-law (LO pQCD) transverse momentum distribution of heavy Q finally ($T_f = 165$ MeV) approximates a thermal one in an ideal Bjorken expansion of the plasma **provided** $D \leq 3/2\pi T$.
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- Using Heavy Q Effective Theory, Caron-Huot, Laine & Moore (JHEP 0904, 053) provided a suitable definition for κ for a lattice evaluation:

$$G_E^{\text{Lat}}(\tau) = -\frac{1}{3L} \sum_{i=1}^3 \left\langle \text{Re tr} \left[U(\beta, \tau) E_i(\tau, \vec{0}) U(\tau, 0) E_i(0, \vec{0}) \right] \right\rangle.$$

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- They also suggested a suitable discrete version for Lattice QCD :

$$E_i(\vec{x}, \tau) = U_i(\vec{x}, \tau) U_4(\vec{x} + \hat{i}, \tau) - U_4(\vec{x}, \tau) U_i(\vec{x} + \hat{4}).$$

- Using this, the numerator can be written as a derivative of an extended (by spatial detour of a) Polyakov loop.

$$G_{E,\text{num}}^i(\tau) = C^i(\tau + 1) + C^i(\tau - 1) - 2C^i(\tau)$$

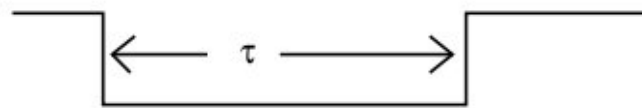
$$C^i(\tau) = \prod_{x_4=0}^{t-1} U_4(x_4) \cdot U_i(t) \cdot \prod_{x_4=t}^{t+\tau-1} U_4(x_4) \cdot U_i^\dagger(t + \tau) \cdot \prod_{x_4=t+\tau}^{\beta-1} U_4(x_4).$$

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Graphical Representation of $C(\tau)$.

Our Lattice Results

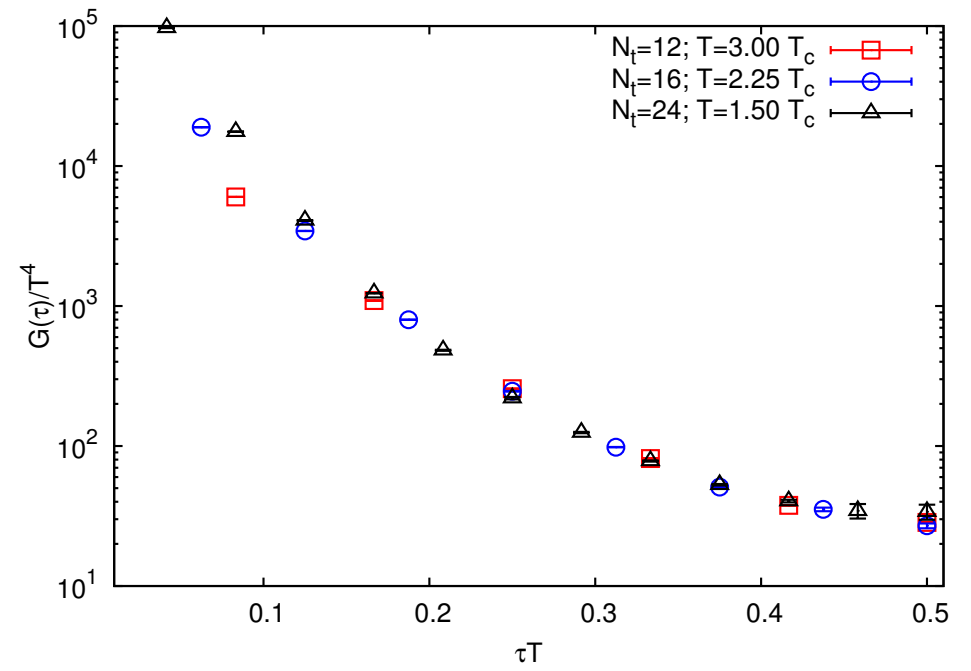
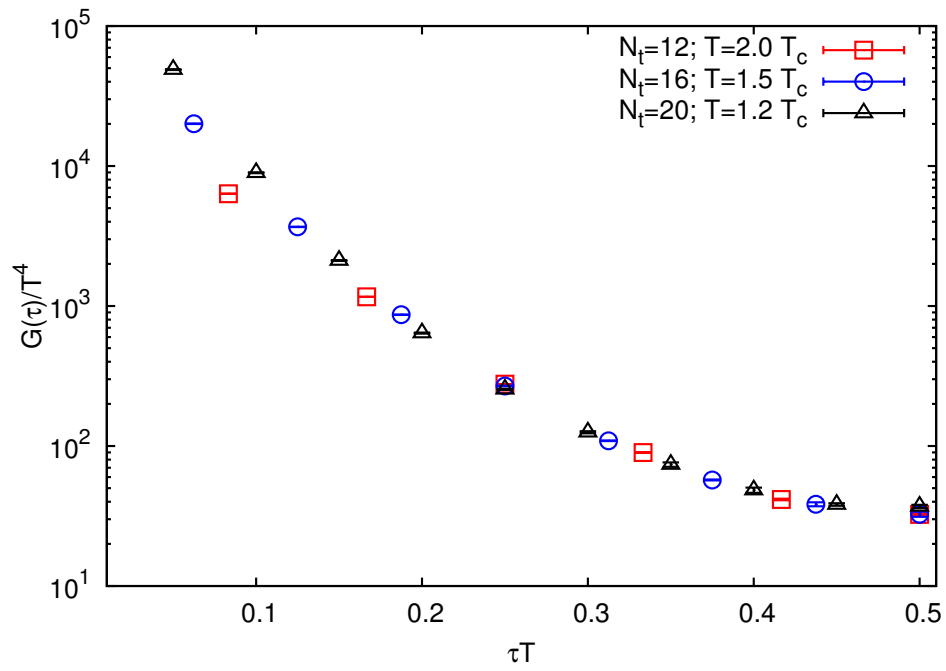
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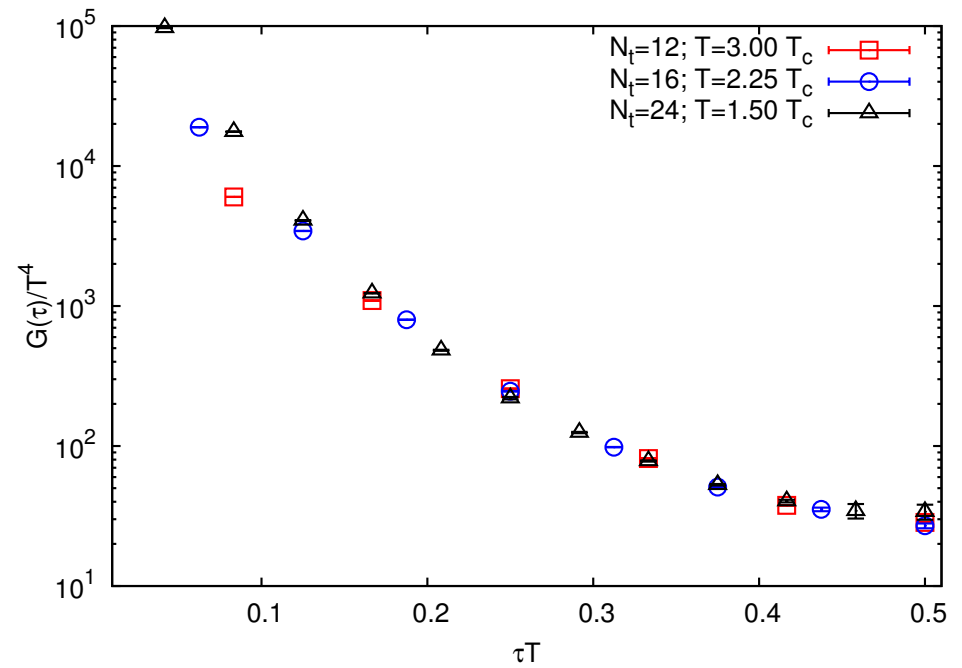
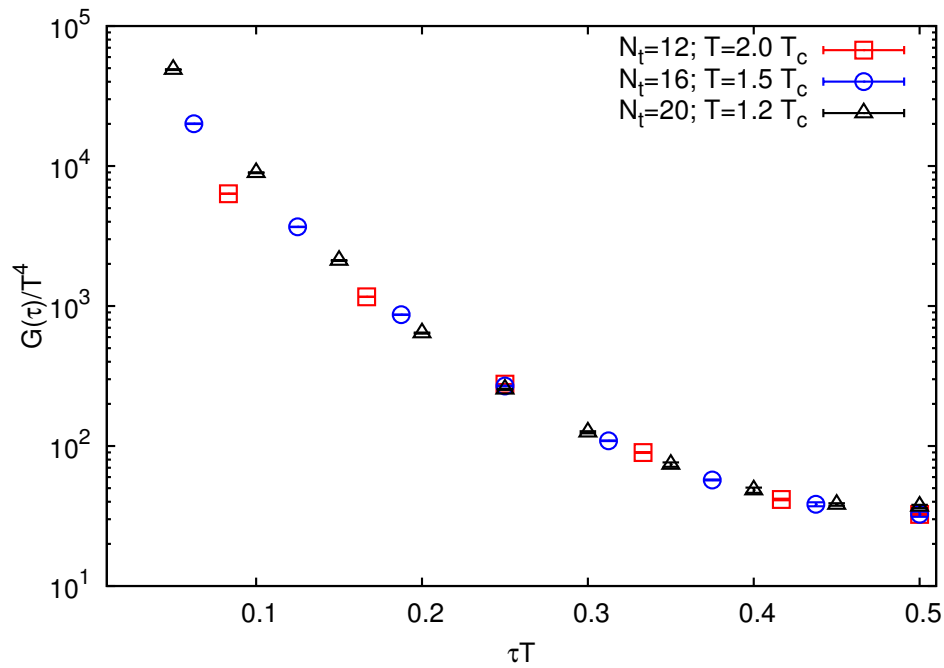
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- We attempted $N_\tau = 12, 16, 20$ and 24 . Multilevel algorithm (Lüscher-Weisz, JHEP 0109 & 0207) was suitably adopted.
- For same size error on $G(10)$, it was found ~ 2500 times more efficient: Very crucial in getting κ .

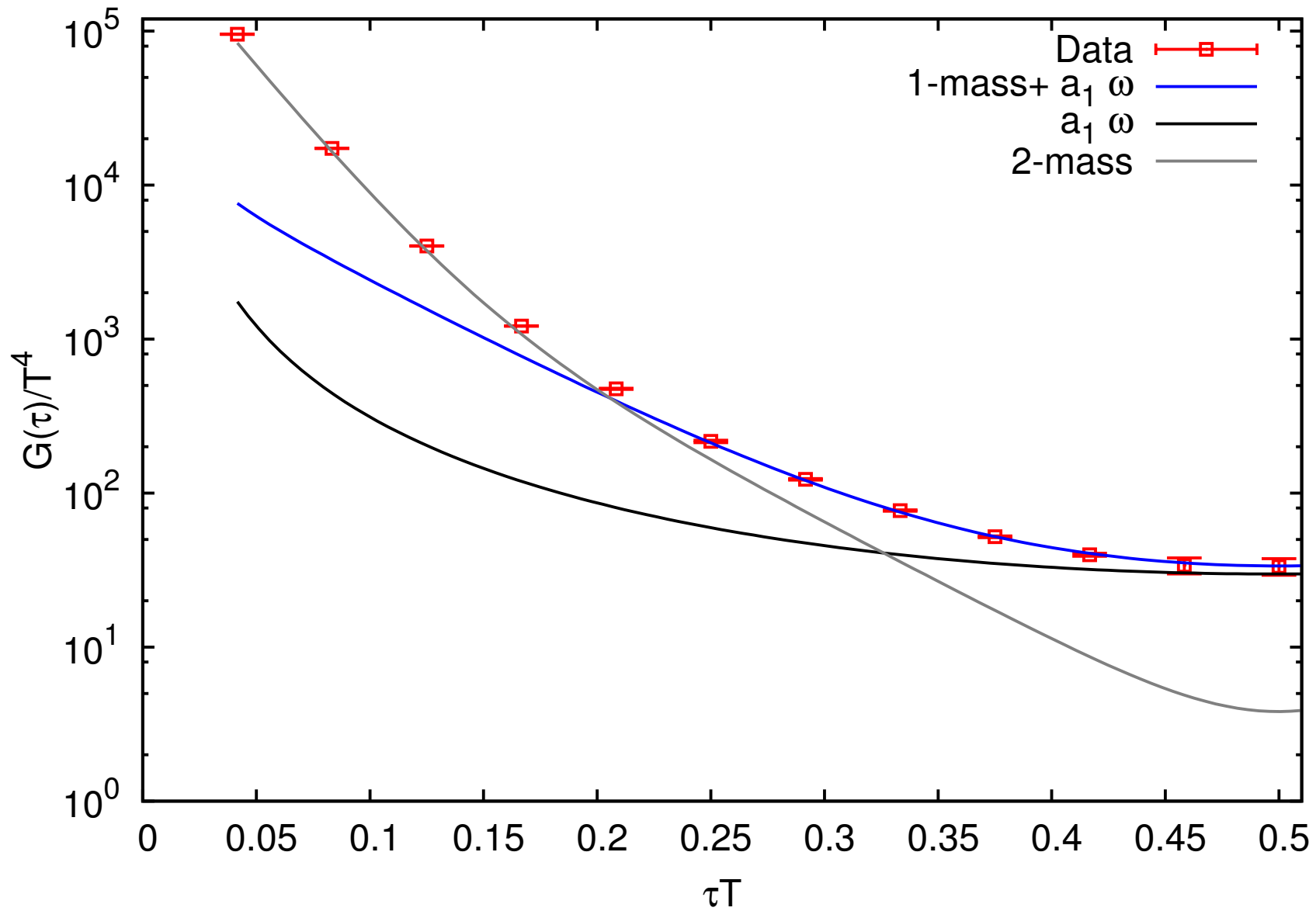
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- For same size error on $G(10)$, it was found ~ 2500 times more efficient: Very crucial in getting κ .
- Spatial volumes are such that $N_s \geq 2N_\tau$.
- Couplings were chosen suitably to make simulations at $T/T_c = 1.04, 1.09, 1.24, 1.5$ and 1.96 for the two largest N_τ .
- Typical Statistics : Few hundred Independent Configurations





- Large τ region shows scaling.
- Low τ region, on the other hand, has only lattice artifacts.

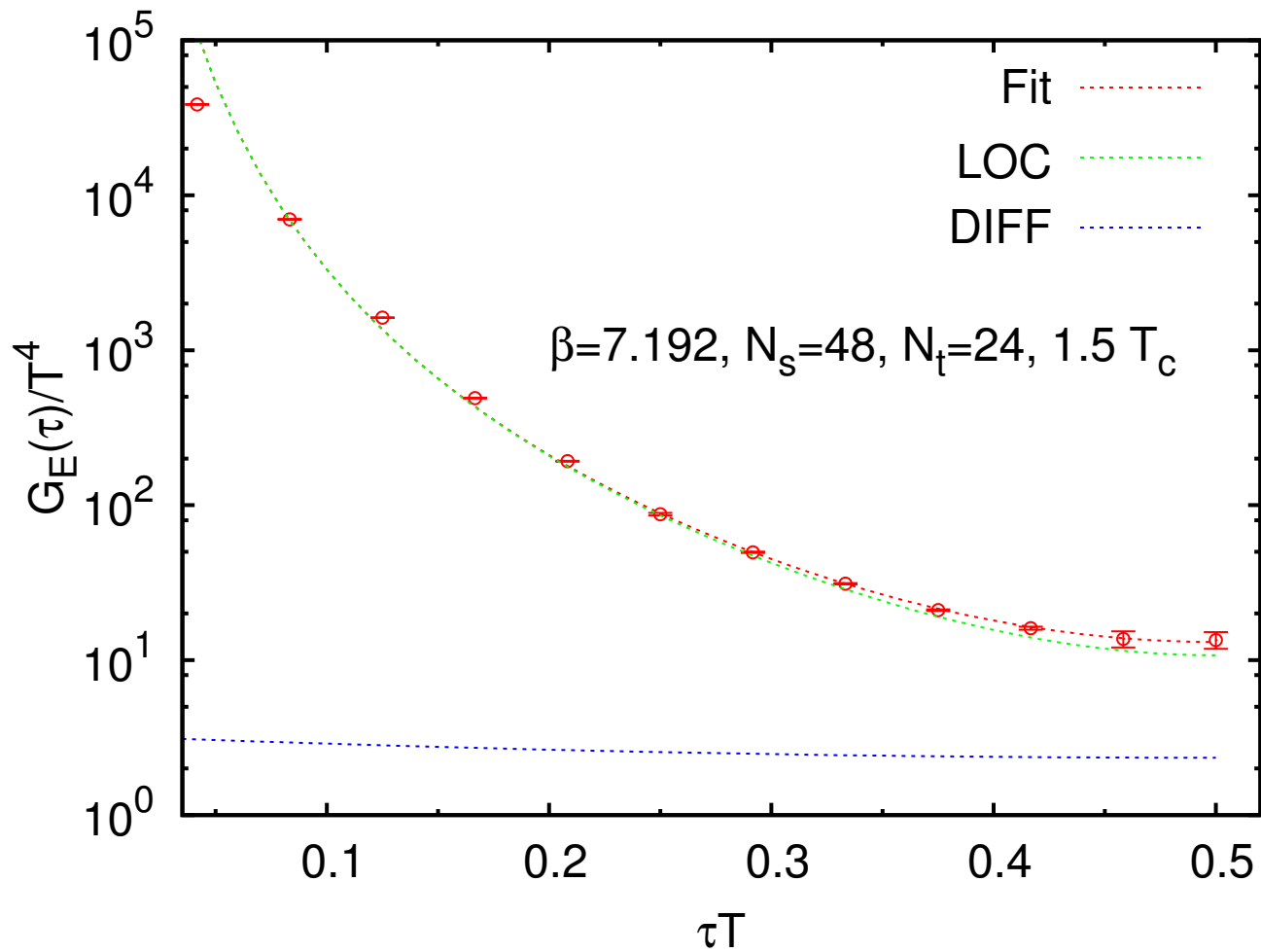


Extracting D

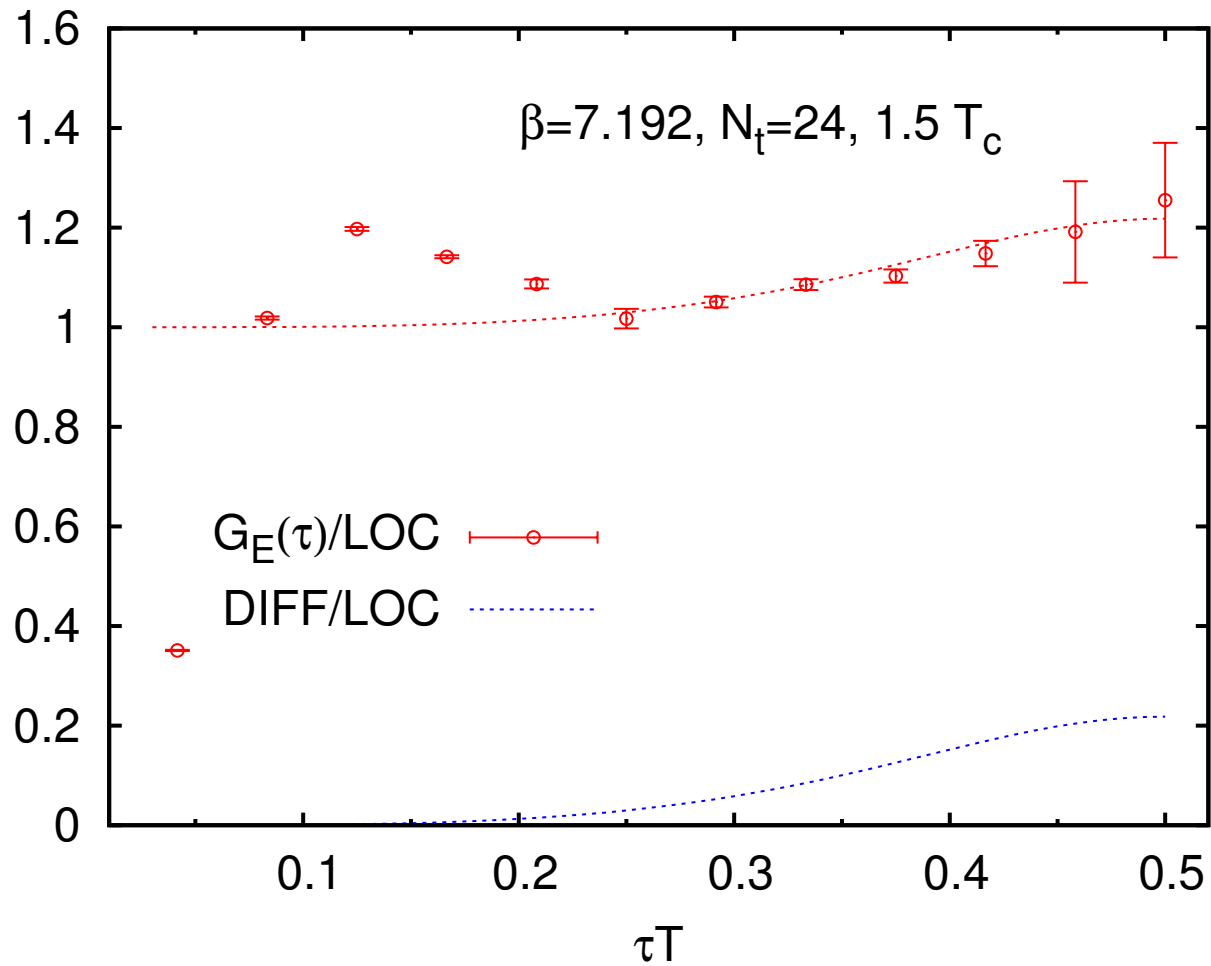
- Getting to the spectral function ρ , an ill-posed problem, has attracted a lot of attention. Many methods can be tried.
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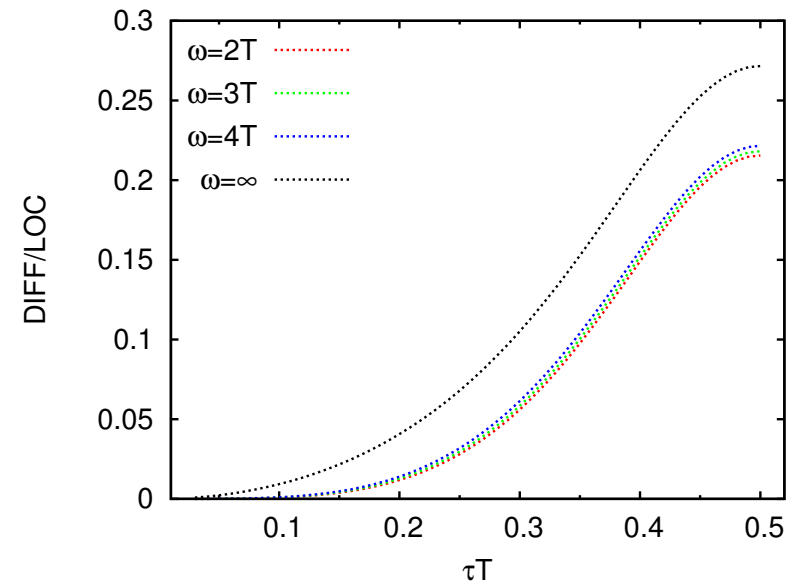
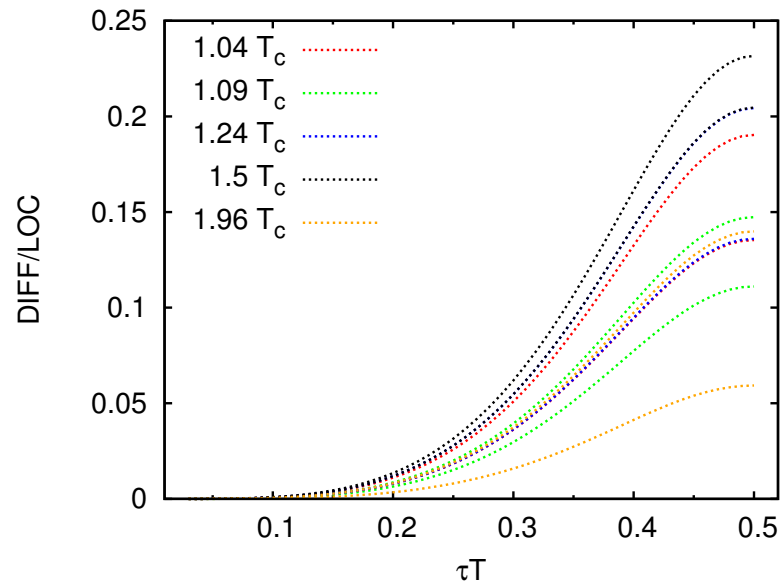
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- $\rho(\omega) = a\omega \Theta(\omega - \Lambda) + b\omega^3$
First term is the due to the expected DIFFusion constant, and the second is motivated by leading perturbation theory (LOC)
- $\Lambda = 3T$ used; varied from 2 to ∞ for systematic error.



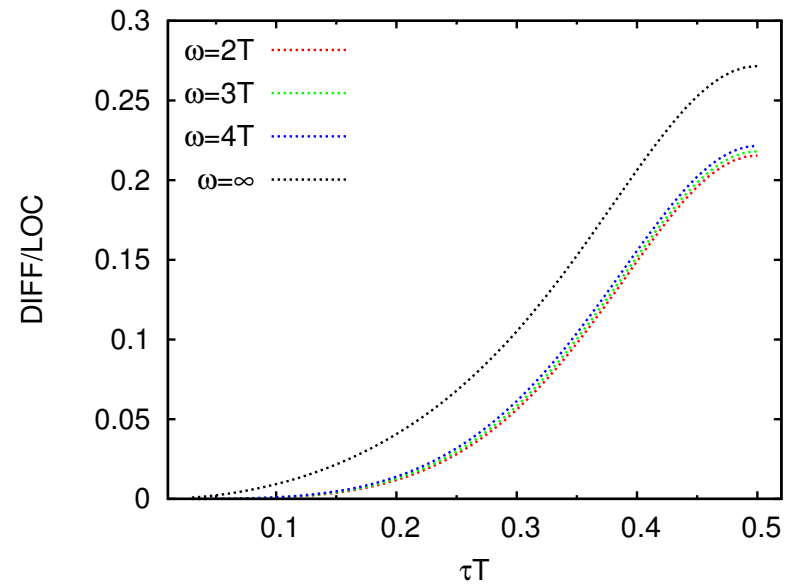
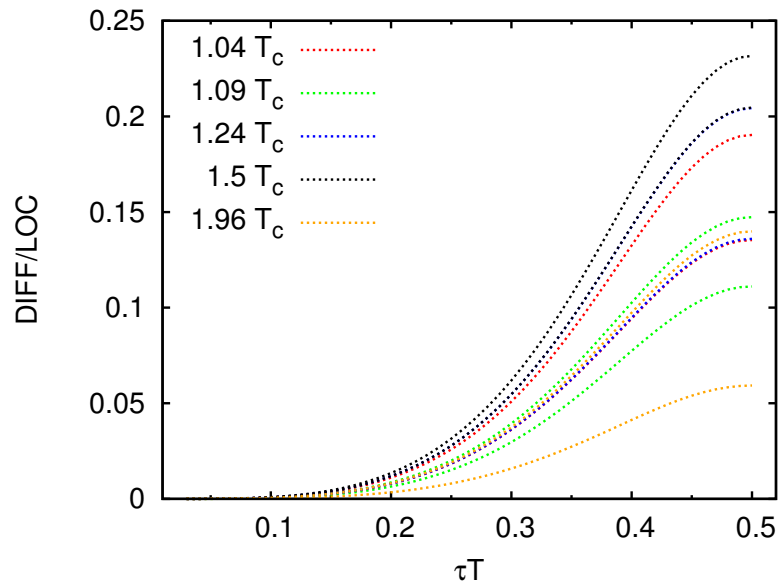
♠ Contribution of the two terms shown as DIFF and LOC.



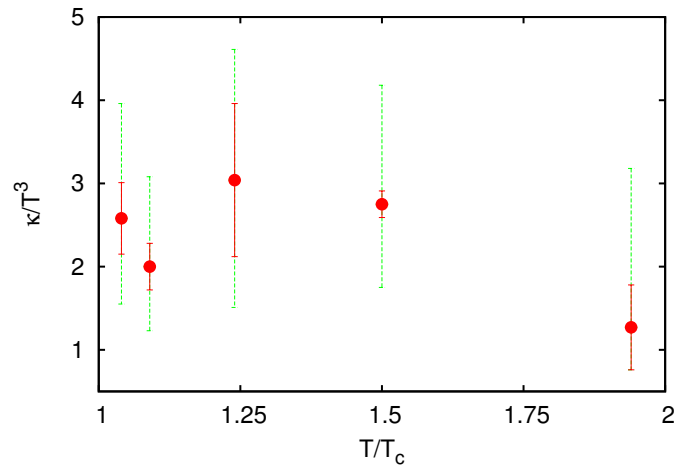
♠ Comparing the DIFF fit with the data after eliminating the LOC.



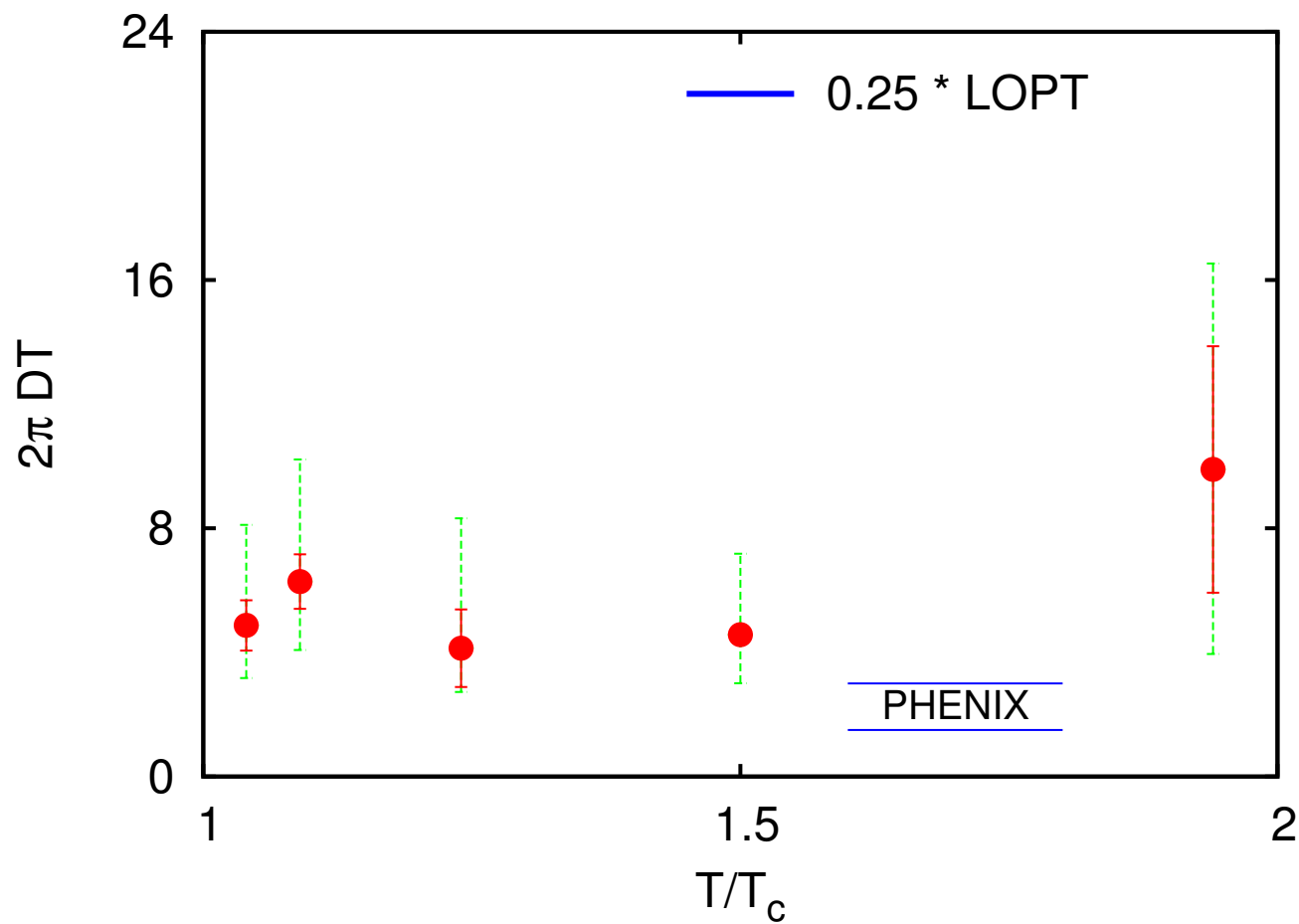
♠ Variation with the temperature and the cut-off Λ .



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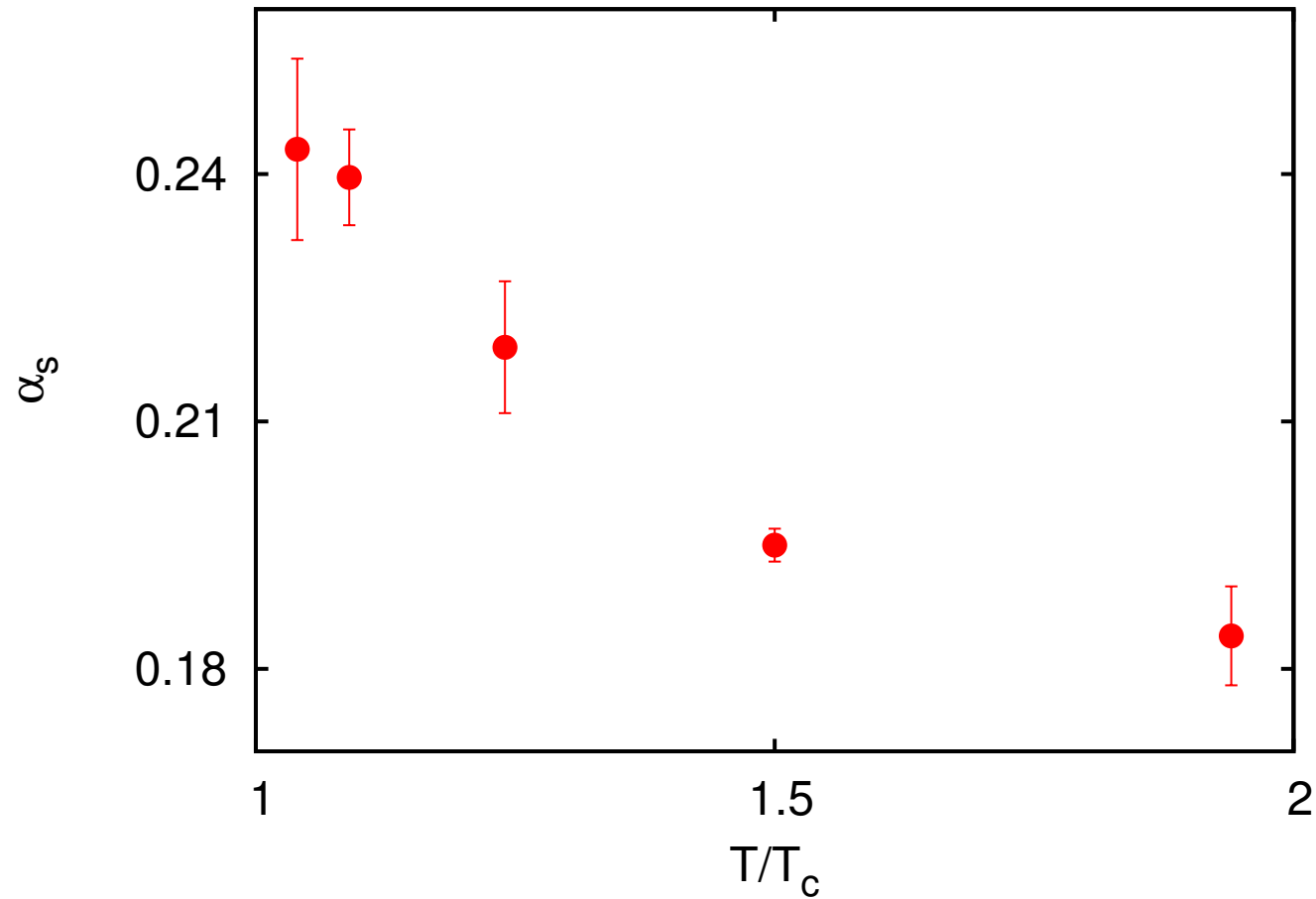


♠ Multiplying by T , obtain a quantity used by Moore-Teaney and PHENIX.



♡ In agreement with preliminary Bielefeld estimates (Ding et al. 1107.0311; Francis et al. 1109.3941).

♠ The ω^3 term comes with g^2 . Use as a scheme to define α_s non-perturbatively.



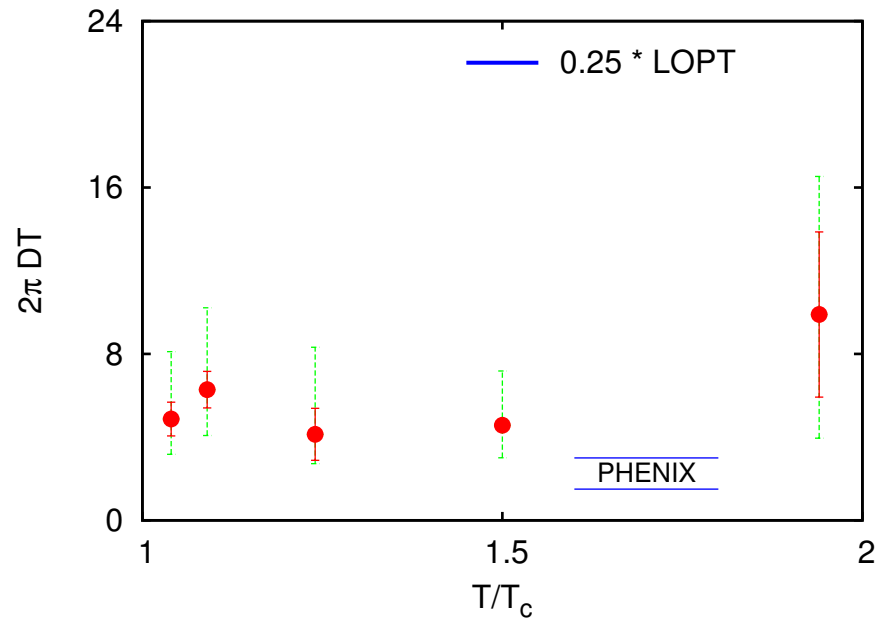
♡ In agreement with other similar estimates (Ding et al. PRD 83 (2011) 034504).

Summary

- We have obtained the diffusion constant D as a function of T/T_c in quenched QCD in the temperature range of interest to RHIC and LHC.
- Our results for DT are almost constant in the range studied.

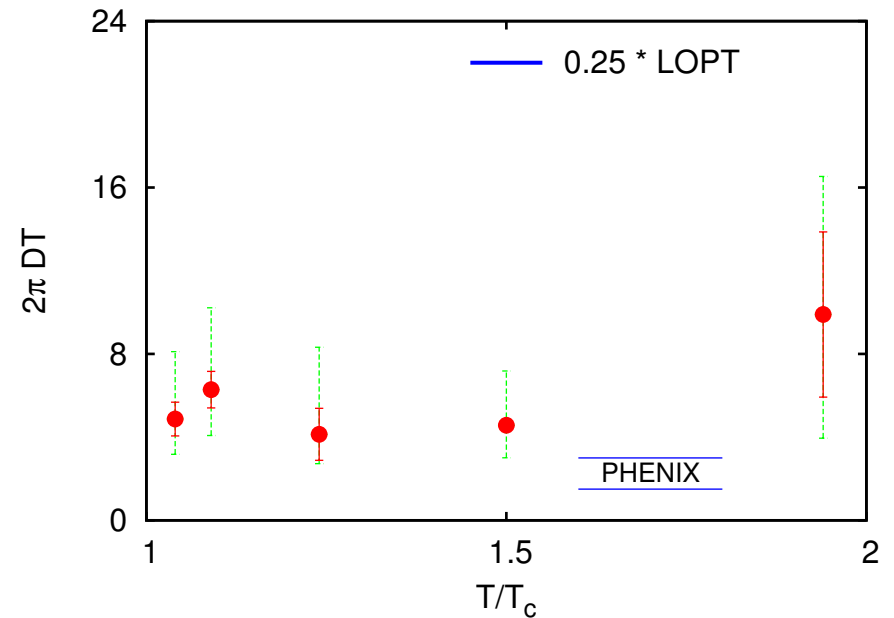
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It would be interesting to see if DT vs. T/T_c exhibits similar flavour independence as the pressure.