Why should the hot QCD static potential be complex?

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Introduction

There is a long history of inequivalent definitions of a thermal "static potential" binding together quarks and antiquarks ...

... and recently the problem has been further "complicated" by arguing that the real-time potential should have an imaginary part as well (which, like a width, facilitates "dissociation").

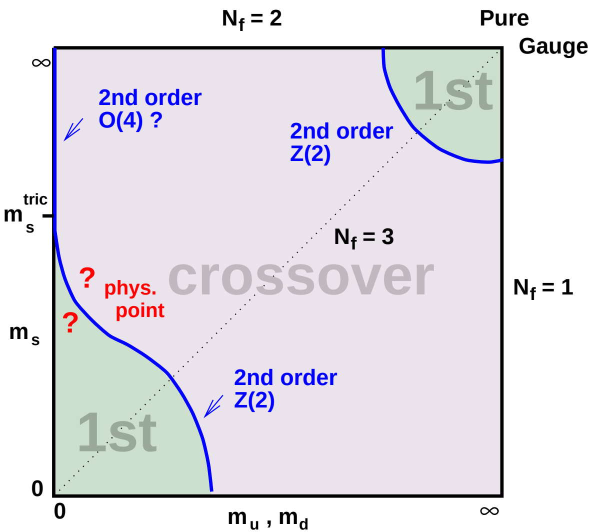
How might this be understood on a non-perturbative level?

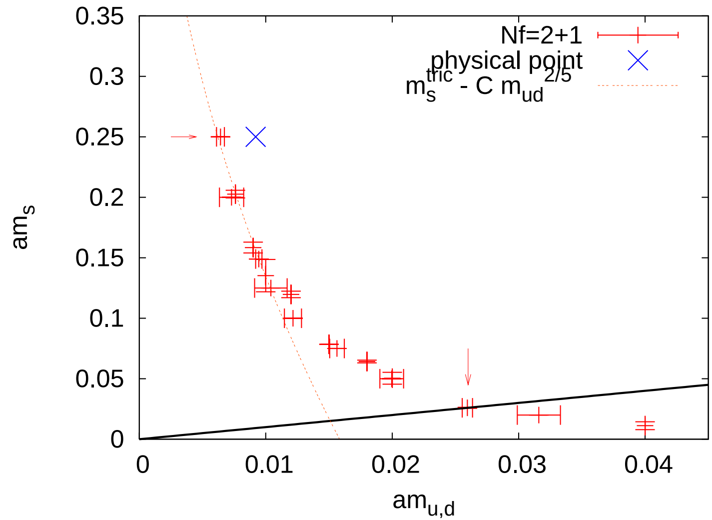
Inspired by and indebted to: A. Rothkopf, T. Hatsuda and S. Sasaki, *Proper heavy-quark potential from a spectral decomposition of the thermal Wilson loop*, 0910.2321; *Complex heavy-quark potential at finite temperature from lattice QCD*, 1108.1579.

Review and other references: M. Laine, *News on hadrons in a hot medium*, 1108.5965.

QCD has no phase transition as the temperature is increased:

[de Forcrand Philipsen hep-lat/0607017; Aoki et al hep-lat/0611014]





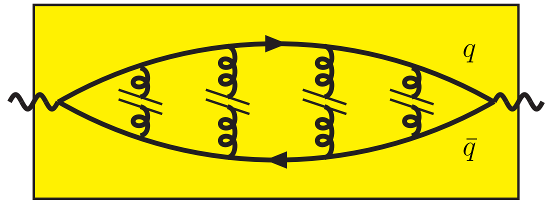
So it should be possible to understand any physics in terms of either hadrons (pions, kaons, glueballs, ...) or quarks and gluons!

⇒ "partonic" and "hadronic" pictures.

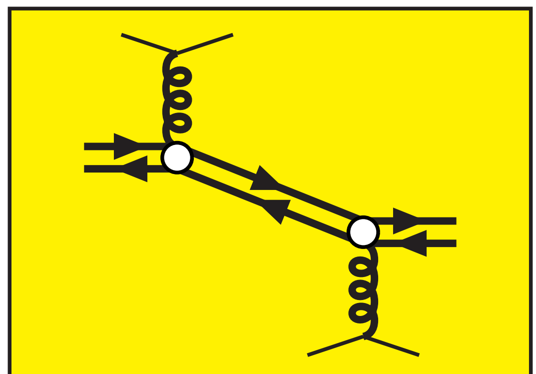
Partonic picture

Traditional view: something happens to the gluons binding the heavy quarks together.

[Here will follow this logic.]

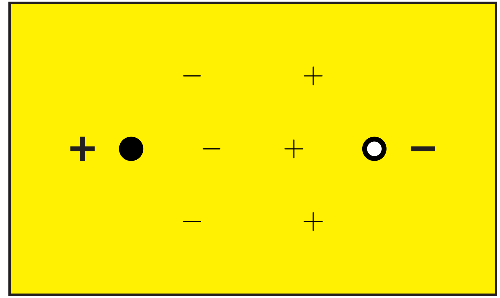


An alternative view: twist two gluons outside $\Rightarrow$ the dipole as a whole gets randomly kicked, destroying coherence.

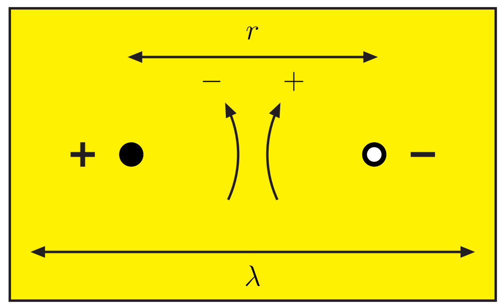


What is the "Debye screening" of gluons ($\Rightarrow \mathrm{Re}\, V_>$)?

Classical & non-relativistic: a "static" re-arrangement of free charges in the plasma.

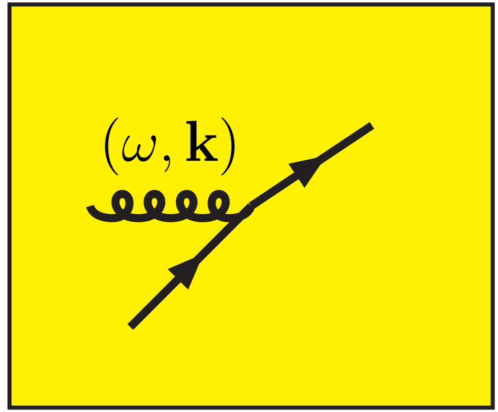


Relativistic plasma: $v \approx c$, mean free path $\lambda \overset{?}{\sim} r$; quarks and gluons cannot "hang around": it's subtle!



What is the "Landau damping" of gluons ($\Rightarrow \mathrm{Im}\, V_>$)?

If the gluons are on-shell ($\omega = k$), then $2 \to 1$ processes are not kinematically allowed.

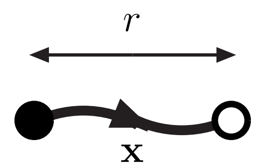


However if the gluons are off-shell and nearly static ($\omega \leq k$), like in a Coulomb field, then the process is allowed. It represents an energy transfer from "soft" to "hard" degrees of freedom; soft gauge configurations "decay".

Hadronic picture

Let us consider a gauge-invariant quark-antiquark "basis" state, with a Wilson line in between, generated by an operator $\hat{\mathcal{J}}_{\mathbf{r}}$:

$$\hat{\mathcal{J}}_{\mathbf{r}}(t,\mathbf{x})|0\rangle \quad =$$



These basis states are in general not eigenstates of the Hamiltonian (we *choose* $\hat{\mathcal{J}}_{\mathbf{r}}$ at will); however the eigenstates can be expressed as linear combinations of the basis states.

Then we define

$$C_r^{>}(t) \equiv \langle \hat{\mathcal{J}}_{\mathbf{r}}^{\dagger}(t,\mathbf{0})\hat{\mathcal{J}}_{\mathbf{r}}(0,\mathbf{0})\rangle_T \; .$$

Eigenstates of the QCD Hamiltonian, $\hat{H}$:

$|n\rangle \equiv$ states with no heavy quarks or antiquarks (i.e. glueballs, light hadrons, their scattering states).

$|n';r\rangle \equiv$ states with a quark and an antiquark, separated by r.

So the Hamiltonian operates as

$$\begin{aligned} \hat{H}\,|n\rangle &= E_n|n\rangle\ , \quad E_n = \text{“light hadron state”}\ , \\ \hat{H}|n';r\rangle &= E_{n'}(r)|n';r\rangle\ , \quad E_{n'}(r) = \text{“singlet potential”}\ , \end{aligned}$$

whereas $\hat{\mathcal{J}}_{\mathbf{r}}$ operates as

$$\hat{\mathcal{J}}_{\mathbf{r}}|n\rangle = \sum_{m'} A_{n,m'}|m';r\rangle\ , \quad A_{n,m'} \in \mathbb{C}\ .$$

Expand now $C_r^>(t)$ in the energy eigenbasis.

Key observation: for $T \ll 2M$, no states of the type $|n';r\rangle$ need to be put next to $\mathcal{Z}^{-1}\exp(-\beta\hat{H})$, because such contributions are suppressed by $\sim e^{-2M/T}$. Therefore,

$$
\begin{aligned}
C_r^>(t) &\approx \frac{1}{\mathcal{Z}}\sum_{n,n'}\langle n|e^{-\beta\hat{H}}e^{i\hat{H}t}\,\hat{\mathcal{J}}_{\mathbf{r}}^{\dagger}\,e^{-i\hat{H}t}|n';r\rangle\langle n';r|\hat{\mathcal{J}}_{\mathbf{r}}|n\rangle \\
&= \frac{1}{\mathcal{Z}}\sum_{n,n'}e^{-\beta E_n}e^{i[E_n-E_{n'}(r)]t}\,|\,\langle n';r|\hat{\mathcal{J}}_{\mathbf{r}}|n\rangle\,|^2\,.
\end{aligned}
$$

All relevant information is here!

$$C_r^>(t) \approx \frac{1}{\mathcal{Z}} \sum_{n,n'} e^{-\beta E_n} e^{-i[E_{n'}(r) - E_n]t} \, | \langle n'; r | \hat{\mathcal{J}}_{\mathbf{r}} | n \rangle |^2 \, .$$

Consider first $T \to 0$, i.e. $\beta \to \infty$.

The sum over n is saturated by the ground state $n = 0$.

The sum over n' has many states, however the excited states lead to a more rapid oscillation than the ground state.

So, the singlet potential could be identified as the smallest oscillation frequency, irrespective of the operator $\hat{\mathcal{J}}_{\mathbf{r}}$ chosen.

The motion remains periodic, or "coherent", at all times (given that the string spectrum is expected to be discrete).

$$C_r^{>}(t) \approx \frac{1}{\mathcal{Z}} \sum_{n,n'} e^{-\beta E_n} e^{-i[E_{n'}(r) - E_n]t} \, | \langle n'; r | \hat{\mathcal{J}}_{\mathbf{r}} | n \rangle |^2 \, .$$

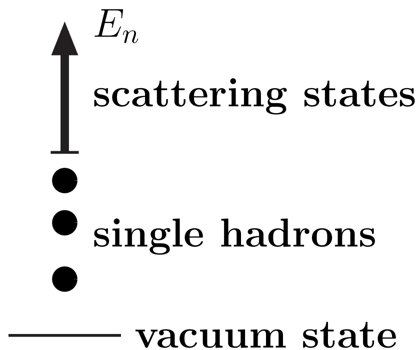
Next, let $T > 0$, i.e. $\beta < \infty$.

Then the sum over n is non-trivial: for any given n', the contributions with $n \geq 1$ make the oscillations *slower*, decreasing $E_{n'}(r) - E_0$ to $E_{n'}(r) - E_n$, where $E_n > E_0$ for $n \geq 1$.

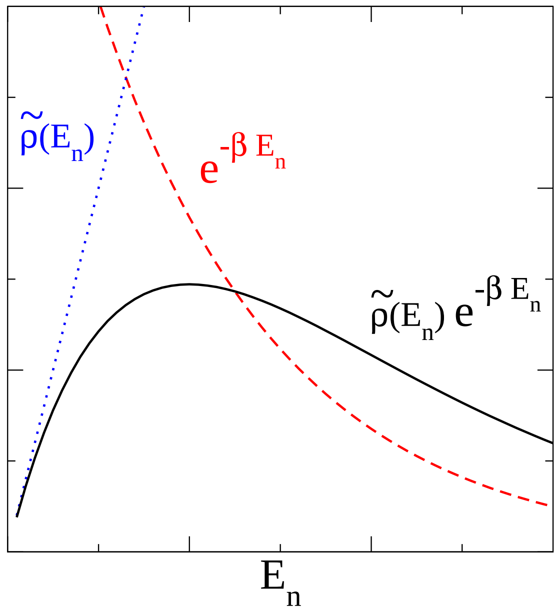
This means that effectively the energy associated with a quark-antiquark pair decreases because of the presence of states other than the vacuum one.

We could refer to this phenomenon as *Debye screening*.

But the spectrum E_n contains generically (in infinite volume) also a *continuous part*, consisting e.g. of two-glueball scattering states (or, with light dynamical fermions, two-pion states etc).



If the temperature is high enough to excite this part, then we may expect to find kind of a "resonance": the density of states, $\tilde{\rho}(E_n)$, grows rapidly with E_n, whereas $e^{-\beta E_n}$ decreases:



Model the energy dependence e.g. by a Breit-Wigner shape:

$$\sum_n e^{-\beta E_n + iE_n t} \, |\langle n'; r|\hat{\mathcal{J}}_{\mathbf{r}}|n\rangle|^2$$

$$\rightarrow \quad \int \mathrm{d}E_n \, \tilde{\rho}(E_n) e^{-\beta E_n + iE_n t} \, |\langle n'; r|\hat{\mathcal{J}}_{\mathbf{r}}|n\rangle|^2$$

$$\simeq \quad \int \mathrm{d}E_n \, e^{iE_n t} \frac{\mathcal{F}(n'; r)}{[E_n - \mathcal{E}(n'; r)]^2 + \Gamma^2(n'; r)}$$

$$\simeq \quad \frac{\pi \mathcal{F}(n'; r)}{\Gamma(n'; r)} \exp\{i\mathcal{E}(n'; r)t - \Gamma(n'; r)t\} \; .$$

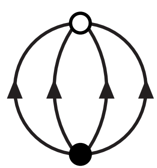
We observe that, apart from the energy shift represented by $\mathcal{E}$, the absolute value of the correlator also decreases with time.

This may generically be referred to as *decoherence*.

As the notation indicates, Debye screening and decoherence depend on n' and r, as well as on the operator $\hat{\mathcal{J}}_{\mathbf{r}}$ chosen. To undestand this let us try to depict $\langle n'; r | \hat{\mathcal{J}}_{\mathbf{r}} | n \rangle$.

Let the Wilson line in $\hat{\mathcal{J}}_{\mathbf{r}}$ be straight; so, $\hat{\mathcal{J}}_{\mathbf{r}}$ generates and $\hat{\mathcal{J}}_{\mathbf{r}}^{\dagger}$ annihilates a quark-antiquark pair connected by a flux tube.

The shape of the eigenstate $|n'; r\rangle$ is more difficult to specify; however, restricting to $n' = 0'$ and distances short enough to be in the perturbative domain, we may expect a QED-like electric dipole configuration. So:

$|n'; r\rangle =$  $,\qquad \hat{\mathcal{J}}_{\mathbf{r}}^{\dagger} |n'; r\rangle =$  .

We see that the configuration $\hat{\mathcal{J}}_{\mathbf{r}}^{\dagger}|n';r\rangle$ represents a "glue bundle"; it can conceivably have a non-zero overlap with multi-glueball states $\Rightarrow$ screening & decoherence.

(In QED itself, there are no glueballs and an overlap can exist only if the configuration represents photons energetic enough to generate an $e^{+}e^{-}$-pair.)

On the other hand, at large distances, we could expect $|n';r\rangle$ to be well-represented by a straight flux tube, and therefore almost be annihilated by $\hat{\mathcal{J}}_{\mathbf{r}}^{\dagger}$ $\Rightarrow$ little overlap to glueball states; only minor modifications with the temperature?

Recall again: $|n\rangle$, $|n';r\rangle$ are determined by $\hat{H}$, whereas $\hat{\mathcal{J}}_{\mathbf{r}}$ can be chosen at will.

$\Rightarrow$ **which operator is best for physics?**

- Whatever we choose, need to determined $\mathcal{E}(n';r)$ and $\Gamma(n';r)$ (i.e. $\mathrm{Re}\, V_>$, $\mathrm{Im}\, V_>$) *consistently from the same setup*.

- Below the melting temperature, or at the melting temperature in the weak-coupling domain, it can be shown that parametrically $rm_\mathsf{E} \ll 1$. So, $\mathbf{r} \cdot \nabla A^\mu \sim rm_\mathsf{E} A^\mu \ll A^\mu$, and the gauge potentials are essentially constants. Then the Wilson line is path independent, and the results should be universal.

Conclusions

It appears that real-time static potentials generically have both a real and an imaginary part, corresponding to screening and decoherence, respectively ...

... but that neither is unique.

It should be a good crosscheck to show that different choices lead ultimately to the same spectral function!

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