Spectral functions in potential models and lattice QCD

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Can ground state charmonium and excited bottomonia exist deep in the deconfined phase ?

NO !

Lattice calculations of the spatial charmonium correlation functions
Karsch, Laermann, Mukherjee, P.P, work in progress

Potential model with complex potential
Miao, Mócsy, P.P., arXiv:1012.4433

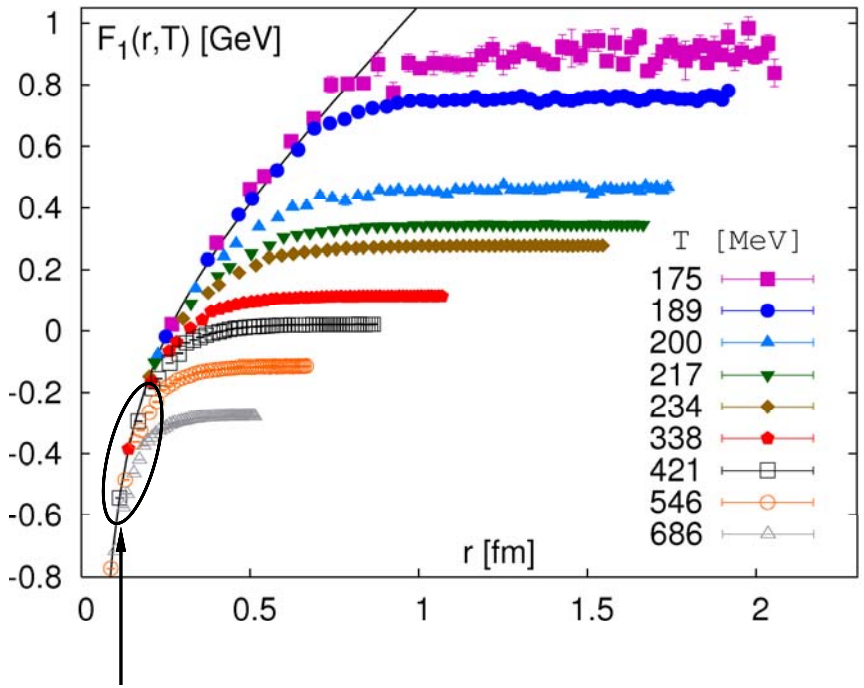
Comments on charmonium spectral functions from MEM

Quarkonia in Deconfined Matter, Acitrezza, September 27-30, 2011

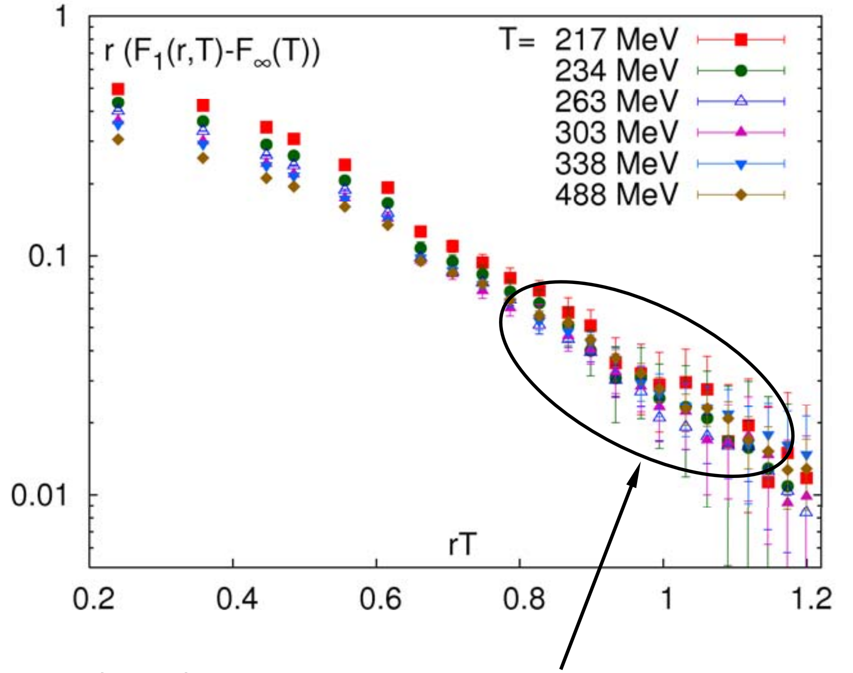
Color screening in lattice QCD

p4 action, $(2+1)-$flavor QCD, $16^3 \times 4$ lattices, $m_\pi \simeq 220$ MeV

P.P., JPG 37 (10) 094009 ; arXiv:1009.5935



$F_1(r,T)$ T-independent at short distances



$F_1(r,T)$ scales with T and is exponentially screened for $r > 0.8/T$

Significant temperature dependence of the static quark anti-quark free energy for $r \simeq 0.3 - 0.5$ fm.

⇩

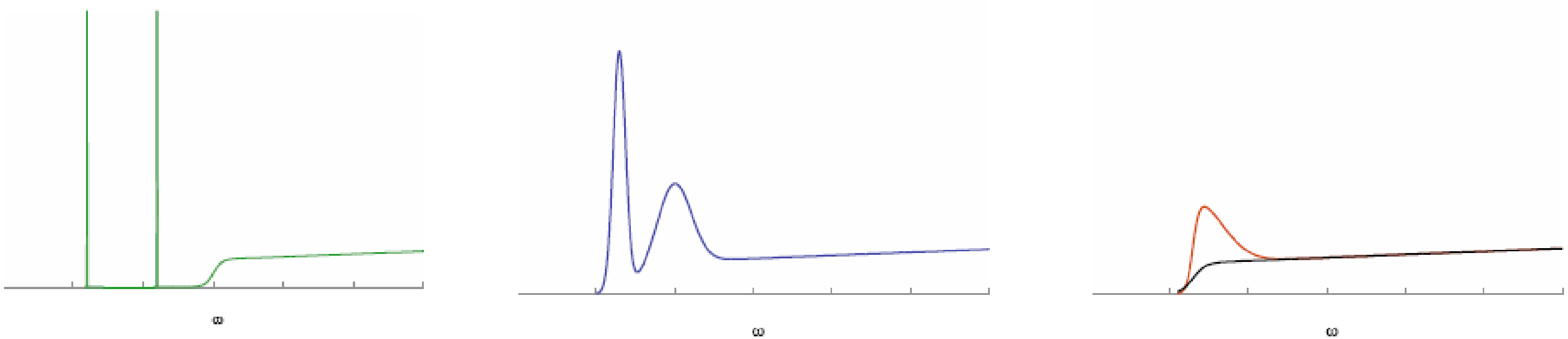
charmonium melting @ RHIC   Digal, P.P., Satz, PRD 64 (01) 094015

Quarkonium spectral functions

In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions

$$\sigma(\omega,p,T)=\frac{1}{2\pi}\,\mathrm{Im}\int_{-\infty}^{\infty}dt e^{i\omega t}\int d^3x e^{ipx}\langle[J(x,t),J(x,0)]\rangle_T$$

Melting is see as progressive broadening and disappearance of the bound state peaks



Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice

$$G(\tau,p,T)=\int d^3x e^{ipx}\langle J(x,-i\tau),J(x,0)\rangle_T$$

$$G(\tau,p,T)=\int_0^{\infty}d\omega\sigma(\omega,p,T)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))} \xrightarrow{\text{MEM}} \sigma(\omega,p,T)$$

1S charmonium survives to $1.6T_c$??

Umeda et al, EPJ C39S1 (05) 9, Asakawa, Hatsuda, PRL 92 (2004) 01200, Datta, et al, PRD 69 (04) 094507, ...

Charmonium correlators at T>0

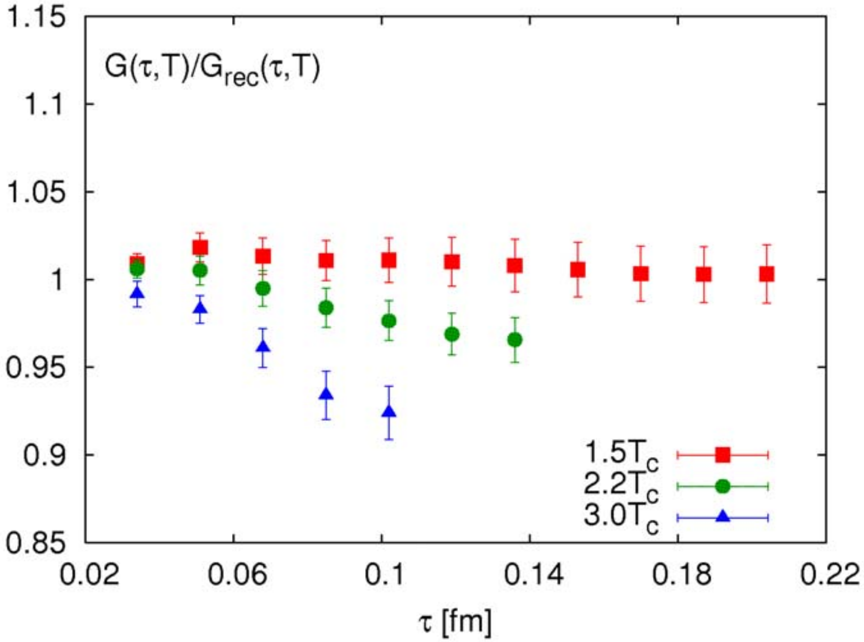
temperature dependene of $G(\tau,T)$

$$\boxed{G(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}}$$

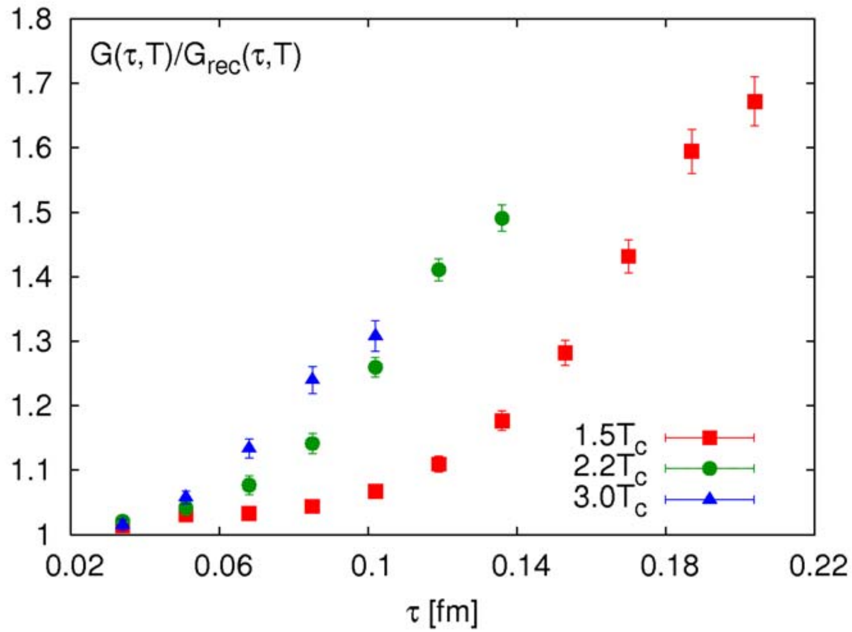
If there is no *T*-dependence in the spectral function, $G(\tau,T)/G_{rec}(\tau,T)=1$

$$G_{rec}(\tau,T)=\int_0^\infty d\omega\sigma(\omega,T=0)\frac{\cosh(\omega\cdot(\tau-\frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Pseudo-scalar $\Leftrightarrow 1S$



Scalar $\Leftrightarrow 1P$

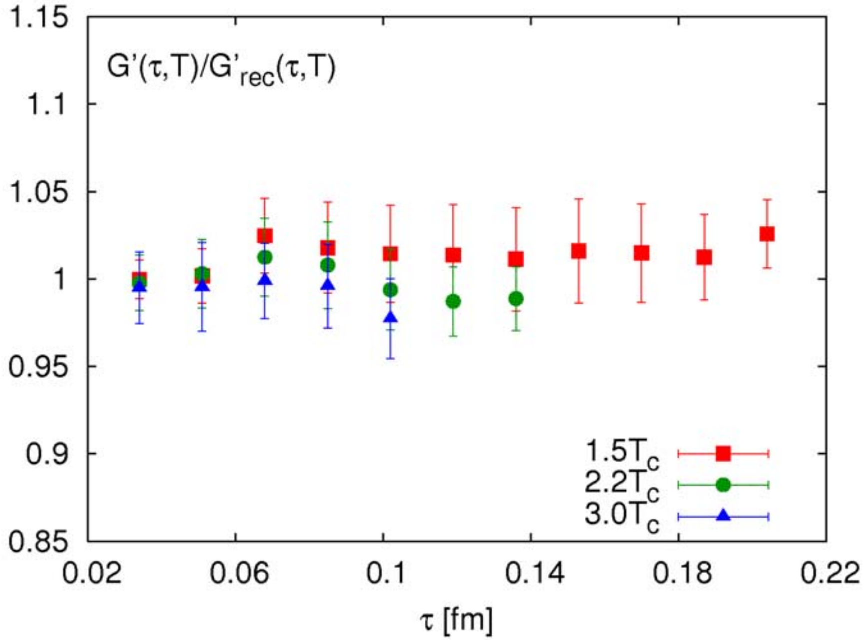


Datta, Karsch, P.P., Wetzorke, PRD 69 (04) 094507

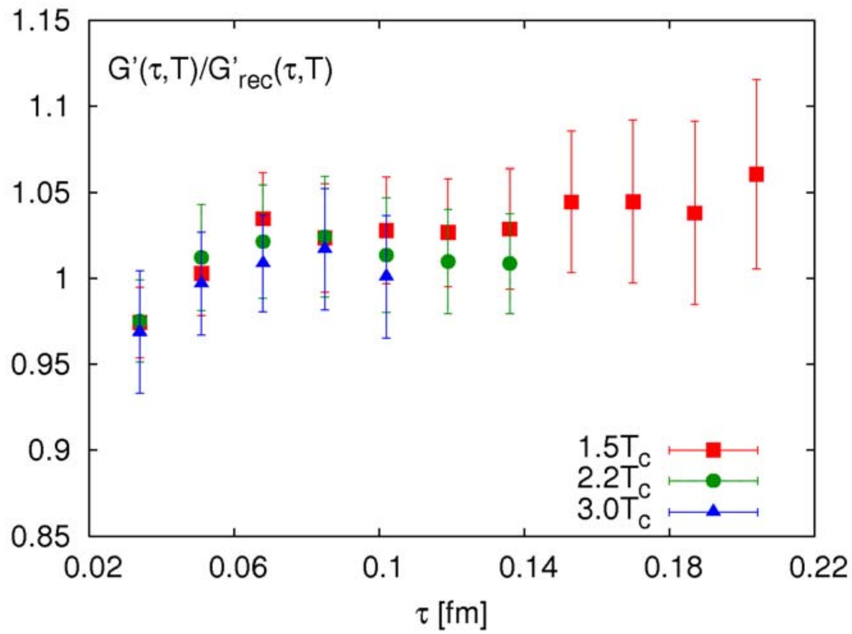
Charmonium correlators at $T>0$

zero mode contribution is not present in the time derivative of the correlator
Umeda, PRD 75 (2007) 094502

Pseudo-scalar $\Leftrightarrow 1S$



Scalar $\Leftrightarrow 1P$



P.P., EPJC 62 (09) 85

the derivative of the scalar correlators does not change up to $3T_c$, all the T-dependence was due to zero mode

⇓

either the 1P state (χ_c) with binding energy of 300MeV can survive in the medium with $\varepsilon=100$GeV/fm^3

or temporal quarkonium correlators are not very sensitive to the changes in the spectral functions due to the limited $\tau_{max}=1/(2\,T)$

Spatial charmonium correlators

Spatial correlation functions can be calculated for arbitrarily large separations $z \to \infty$

$$G(z,T) = \int_0^{1/T} d\tau \int dxdy \langle J(\mathbf{x}, -i\tau), J(\mathbf{x}, 0)\rangle_T, \quad G(z \to \infty, T) \simeq Ae^{-m_{scr}(T)z}$$

but related to the same spectral functions $G(z,T) = \int_{-\infty}^{\infty} e^{ipz} \int_0^{\infty} d\omega \frac{\sigma(\omega,p,T)}{\omega}$

Low T limit :

$$\sigma(\omega,p,T) \simeq A_{mes}\delta(\omega^2 - p^2 - M_{mes}^2)$$

$$A_{mes} \sim |\psi(0)|^2 \to m_{scr}(T) = M_{mes}$$

$$G(z,T) \simeq |\psi(0)|^2 e^{-M_{mes}(T)z}$$

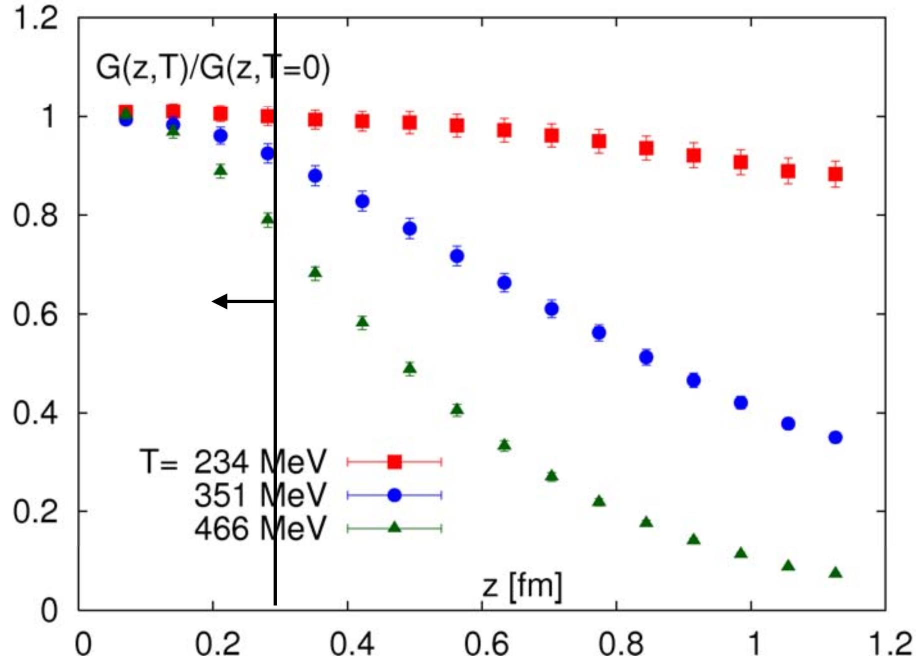
High T limit :

$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

p4 action, dynamical *(2+1)-f* *32*^{3}x*8* and *32*^{3}x*12* lattices

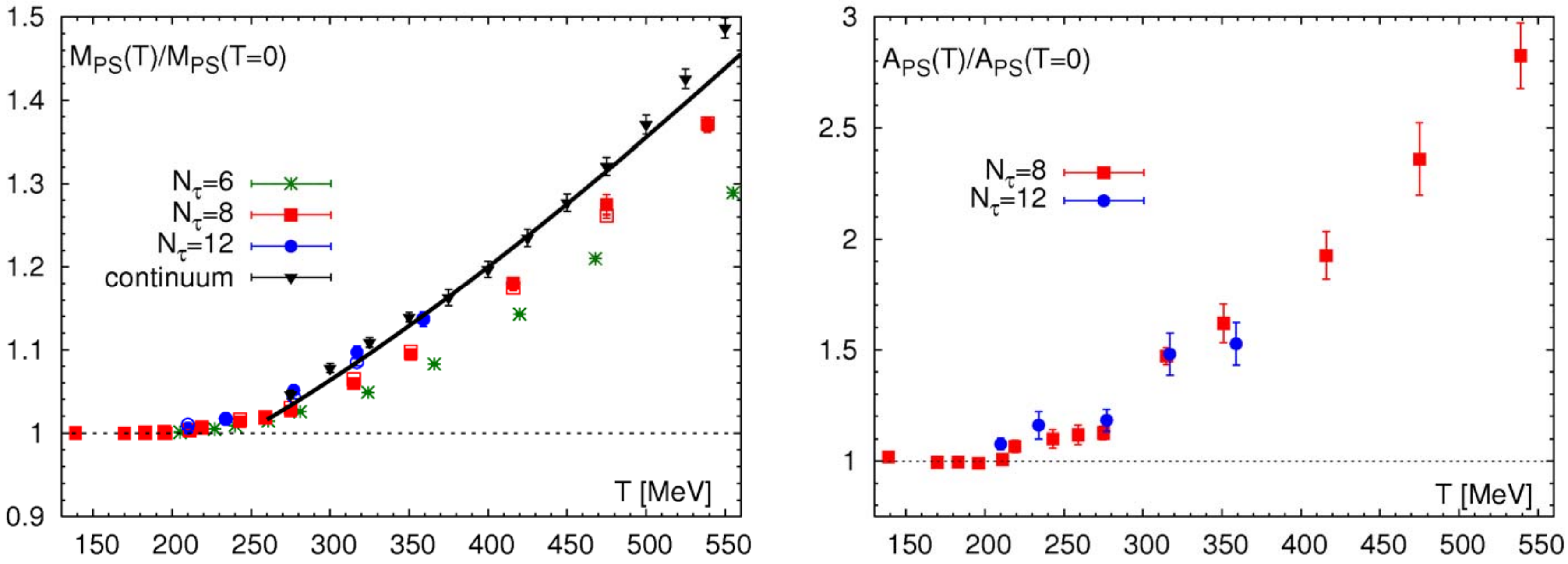
Significant temperature dependence already for *T=234* MeV, large *T*-dependence in the deconfined phase

For small separations *(z T<1/2)* significant *T*-dependence is seen



Spatial charmonium correlators at large distances

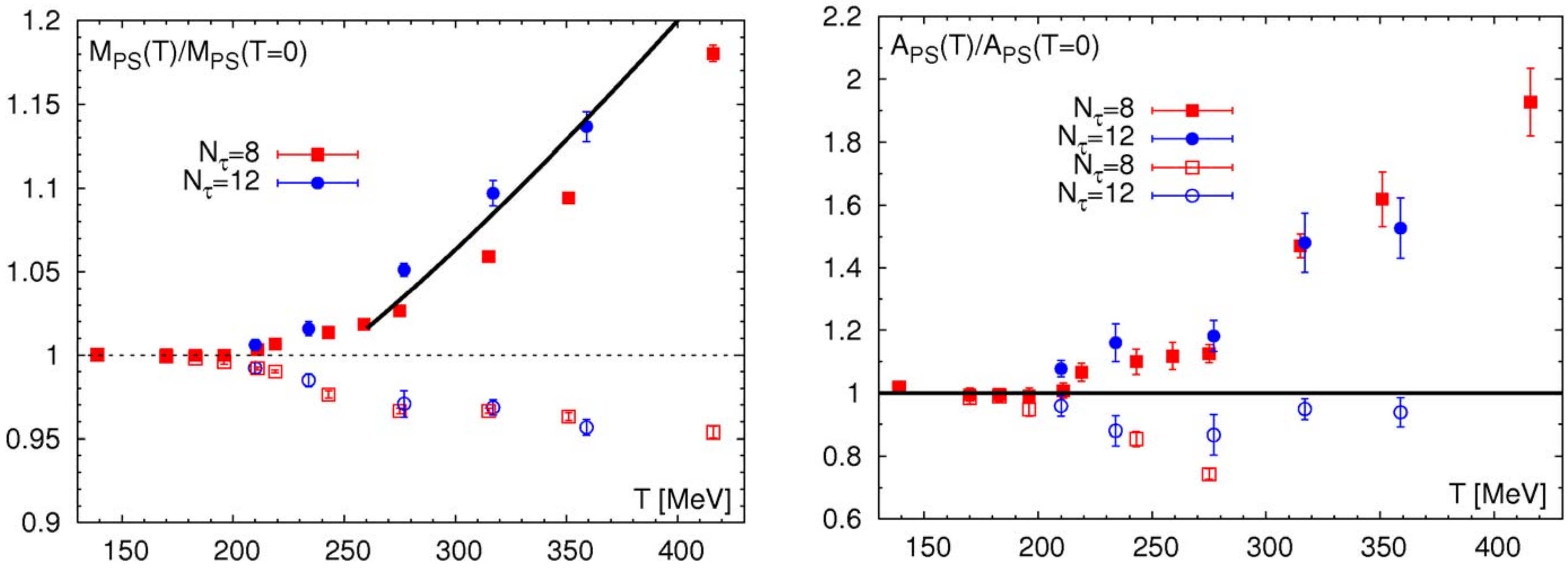
pseudo-scalar channel => 1S state , point sources: filled; wall sources: open



- no T-dependence in the screening masses and amplitudes (wave functions) for ***T<200* MeV**
- moderate T-dependence for ***200<T<275* MeV** => medium modification of the ground state
- Strong T-dependence of the screening masses and amplitudes for ***T>300* MeV**, compatible with free quark behavior assuming ***m_c=1.28* GeV** => dissolution of 1S charmonium !

Dependence of the correlators on boundary conditions

For compact bound states there is no dependence on the temporal boundary conditions in the correlators (quark and anti-quark cannot pick up the thermal momentun)



- no dependence on the boundary conditions for ***T<200* MeV**
- moderate dependence on the boundary conditions for ***200* MeV<*T<275* MeV**
- strong dependence of the screening masses and amplitudes for ***T>300* MeV**

=> dissolution of 1S charmonium !

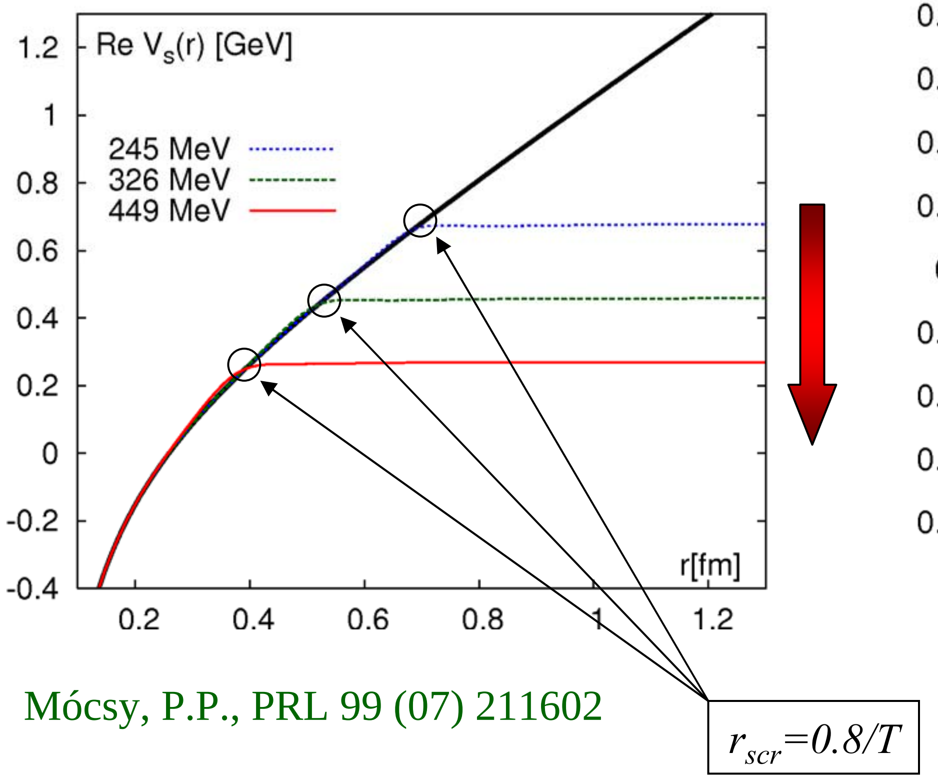
pNRQCD beyond weak coupling and potential models

Above deconfinement the binding energy is reduced and eventually $E_{bind} \sim m v^2$ is the smallest scale in the problem (zero binding) $2\pi T,\ m_D,\ \Lambda_{QCD} >> m v^2$ => most of medium effects can be described by a T-dependent potential

Determine the potential by non-perturbative matching to static quark anti-quark potential calculated on the lattice

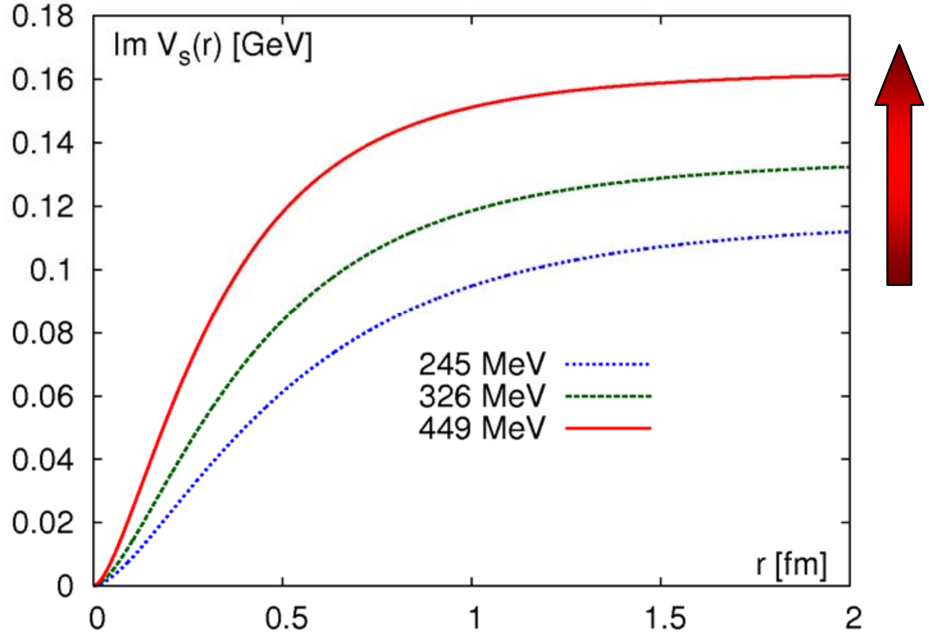
Caveat : it is difficult to extract static quark anti-quark energies from lattice correlators => constrain Re$V_s(r)$ by lattice QCD data on the singlet free energy, take Im$V_s(r)$ from pQCD calculations

"Maximal" value for the real part



Mócsy, P.P., PRL 99 (07) 211602

Minimal (perturbative) value for imaginary part



Laine et al, JHEP0703 (07) 054,
Beraudo, arXiv:0812.1130

Lattice QCD based potential model

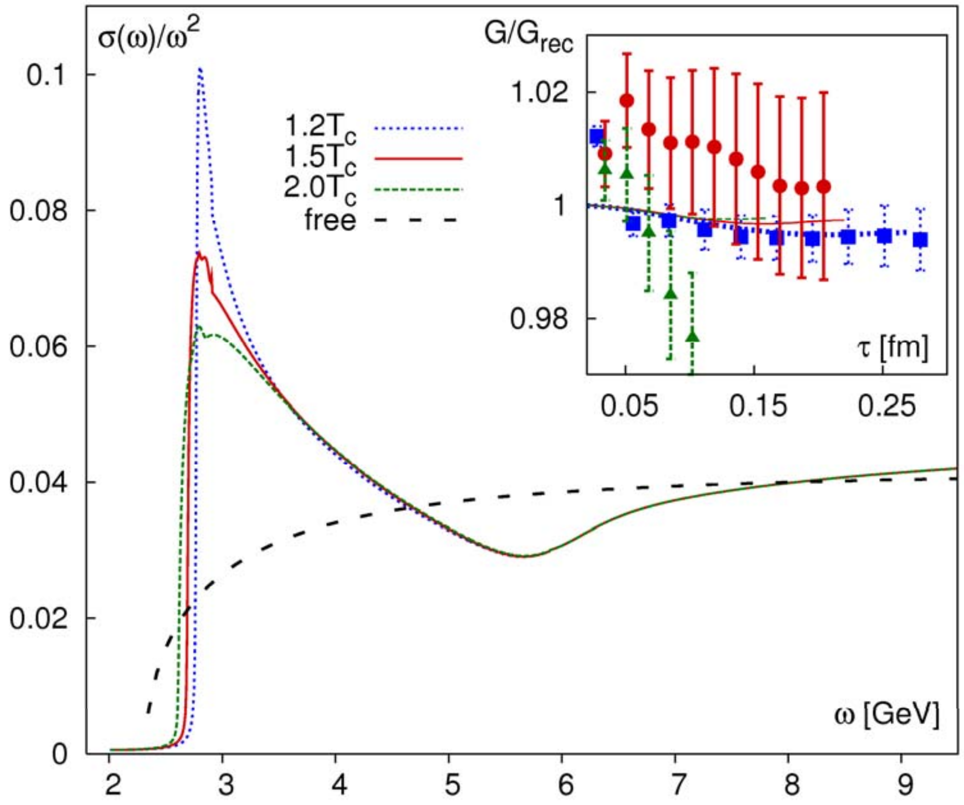
If the octet-singlet interactions due to ultra-soft gluons are neglected :

$$\left[i\partial_0 - \frac{-\nabla^2}{m} - V_s(r,T)\right] S(r,t) = 0 \quad \Rightarrow \quad \sigma(\omega, T)$$

potential model is not a model but the tree level approximation of corresponding EFT that can be systematically improved

Test the approach vs. LQCD : quenched approximation, $F_1(r,T) < \mathrm{Re}V_s(r,T) < U_1(r,T)$, $\mathrm{Im}V(r,T) \approx 0$

Mócsy, P.P., PRL 99 (07) 211602, PRD77 (08) 014501, EPJC ST 155 (08) 101



- resonance-like structures disappear already by $1.2T_c$
- strong threshold enhancement above free case

  => indication of correlations
- height of bump in lattice and model are similar
- The correlators do not change significantly despite the melting of the bound states => it is difficult to distinguish bound state from threshold enhancement in lattice QCD

The role of the imaginary part for charmonium

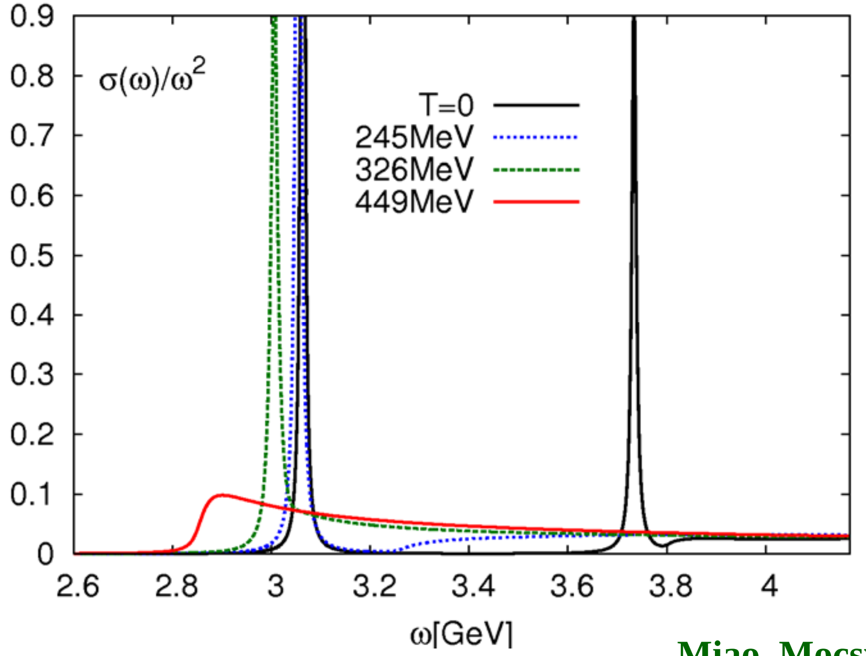
Take the upper limit for the real part of the potential allowed by lattice calculations

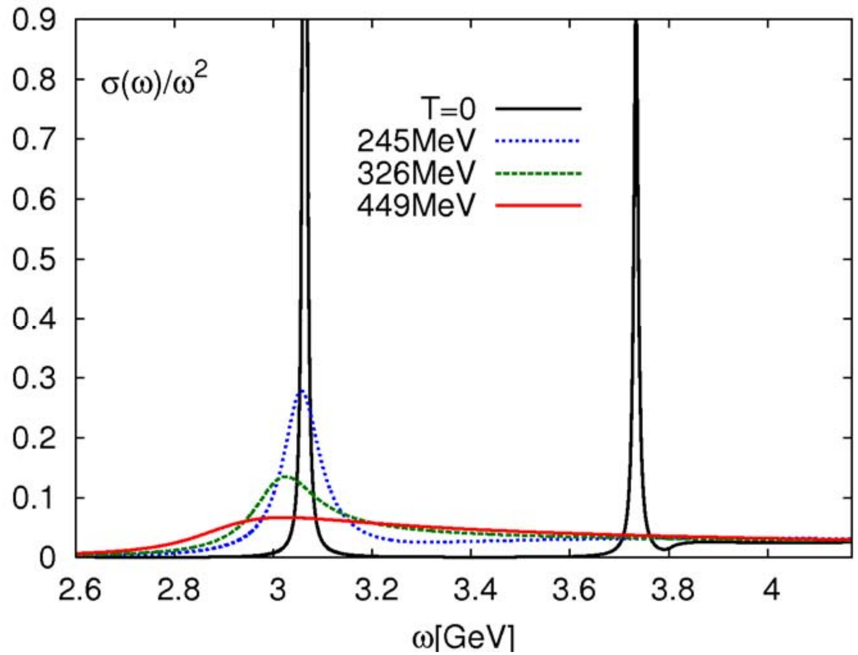
Mócsy, P.P., PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r) = 0$:
1S state survives for $T = 330$ MeV

imaginary part of $V_s(r)$ is included :
all states dissolves for $T > 240$ MeV





Miao, Mocsy, P.P., arXiv:1012.4433

no charmonium state could survive for $T > 240$ MeV
this is consistent with our earlier analysis of **Mócsy, P.P., PRL 99 (07) 211602** ($T_{dec} \sim 204 MeV$)
as well as with **Riek and Rapp, arXiv:1012.0019 [nucl-th]**

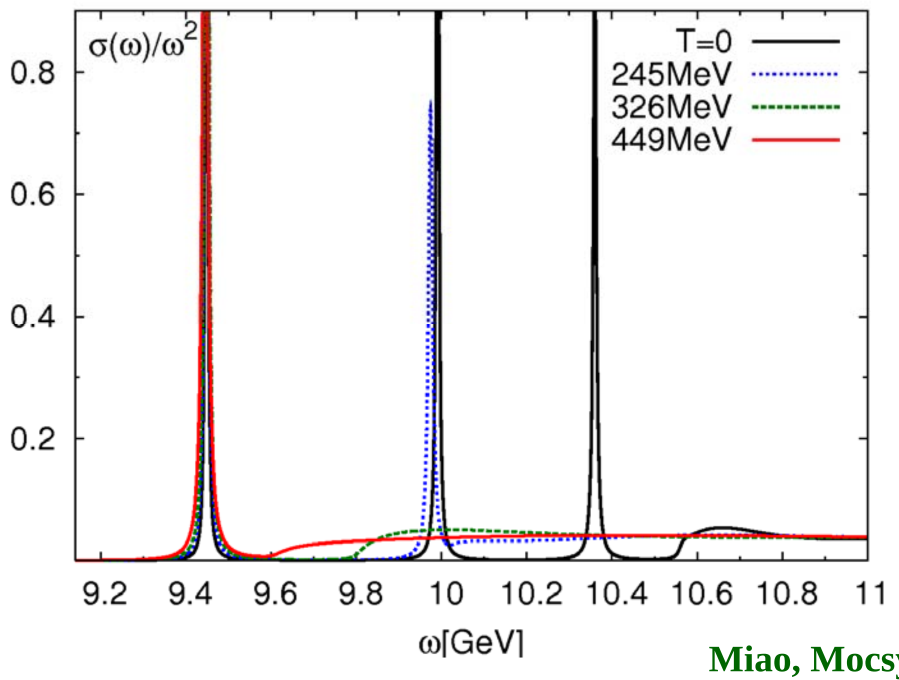
The role of the imaginary part for bottomonium

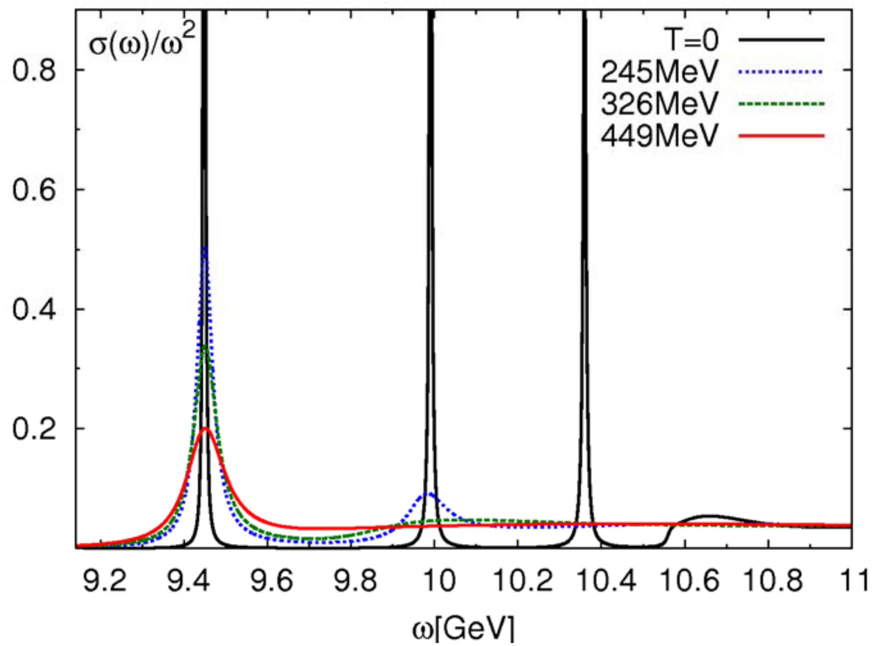
Take the upper limit for the real part of the potential allowed by lattice calculations
Mócsy, P.P., PRL 99 (07) 211602,

Take the perturbative imaginary part
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

Im $V_s(r) = 0$:
2S state survives for $T > 245$ MeV
1S state could survive for $T>450$ MeV

with imaginary part:
2S state dissolves for $T>245$ MeV
1S states dissolves for $T>450$ MeV





Miao, Mocsy, P.P., arXiv:1012.4433

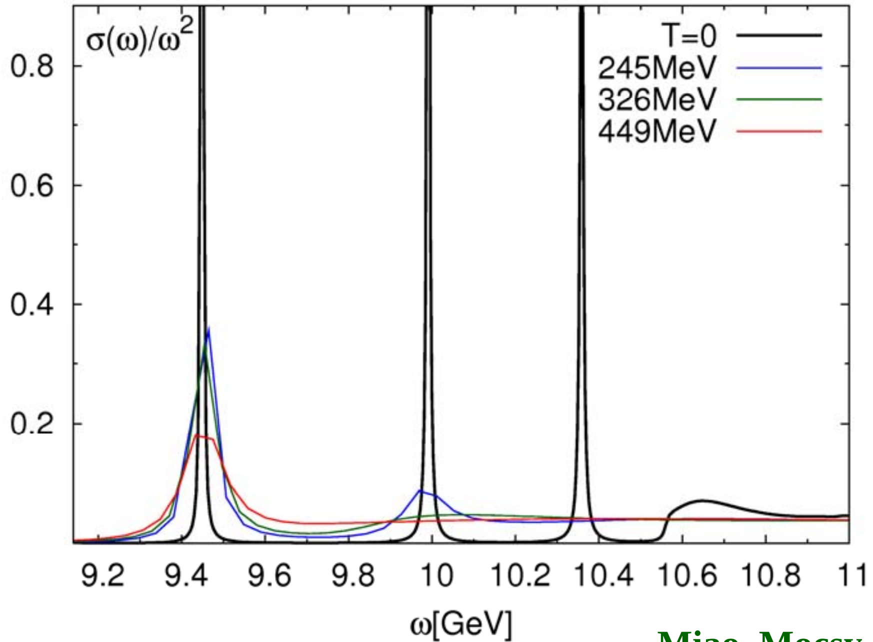
Excited bottomonium states melt for $T \approx 250$ MeV ; 1S state melts for $T \approx 450$ MeV
this is consistent with our earlier analysis of **Mócsy, P.P., PRL 99 (07) 211602** $(T_{dec} \sim 204 MeV)$
as well as with **Riek and Rapp, arXiv:1012.0019 [nucl-th]**

Sensitivity of the spectral functions to real part ofthe potential

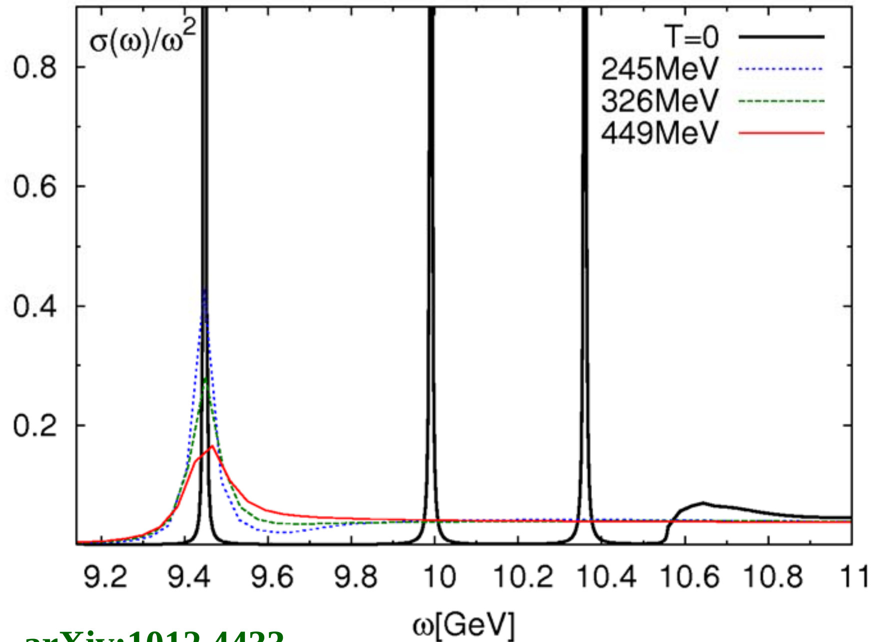
Constraints : $F_1(r,T) < \mathrm{Re}V_s(r,T) < U_1(r,T)$

- If the potential is chosen to be close to the free energy charmonium states dissolve for $T \approx 250$ MeV even if the imaginary part is neglected
- For 1S bottomonium melting does not happen for any choice of the real part

Maximally binding real part



Minimally binding real part



Miao, Mocsy, P.P., arXiv:1012.4433

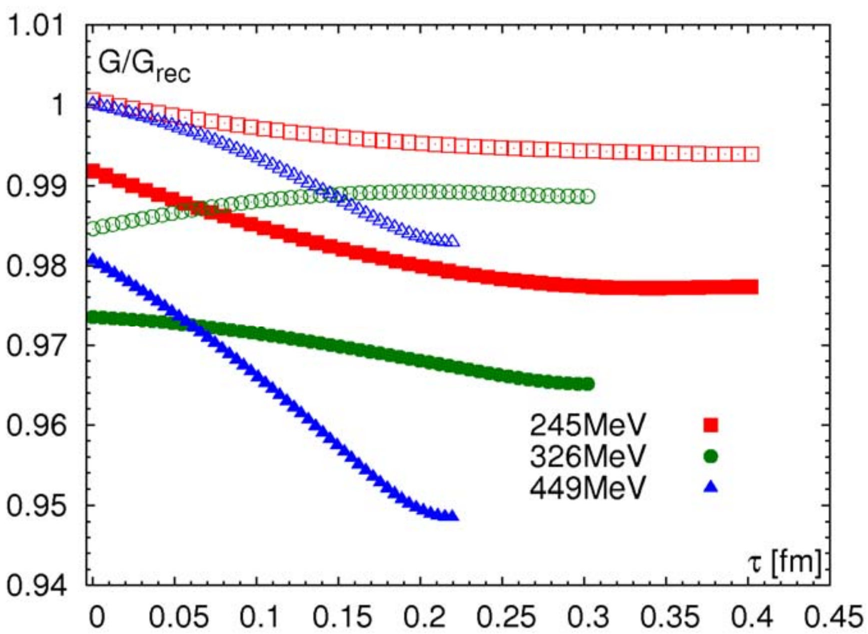
⇨ the shape of the bottomonium spectral functions is not very sensitive to the choice of the real part

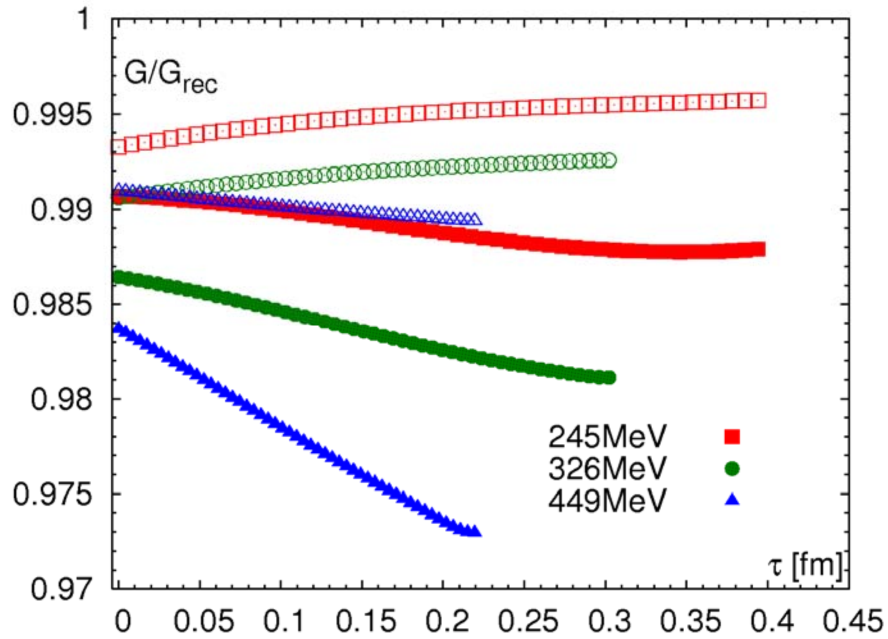
From spectral functions to Euclidean correlators

Charmonium

Bottomonium

open symbols : imaginary part =0 ; filled symbols imaginary part is included



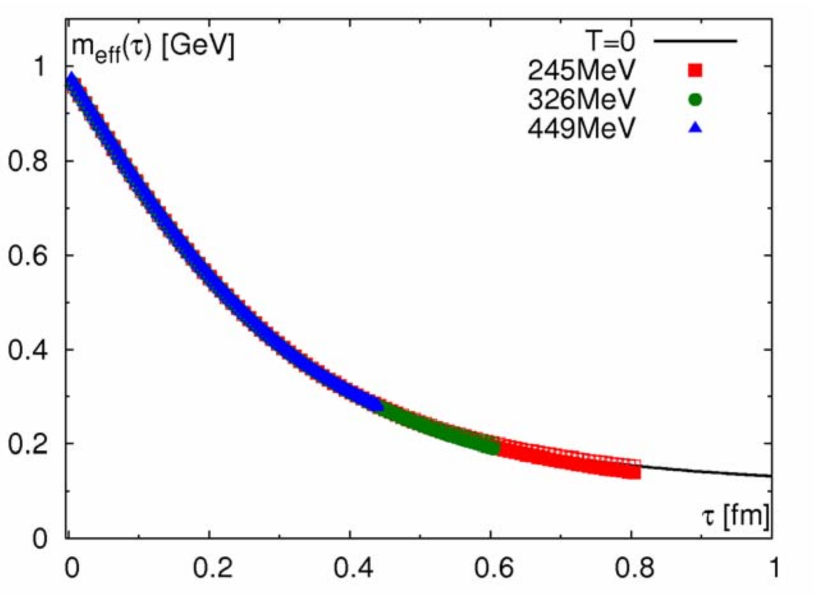


small temperature dependence of the Euclidean correlators, inclusion of the imaginary part and the consequent dissolutions of quarkonium states only lead to (1-4)% reduction of the correlators

Non-relativistic bottomonium correlators

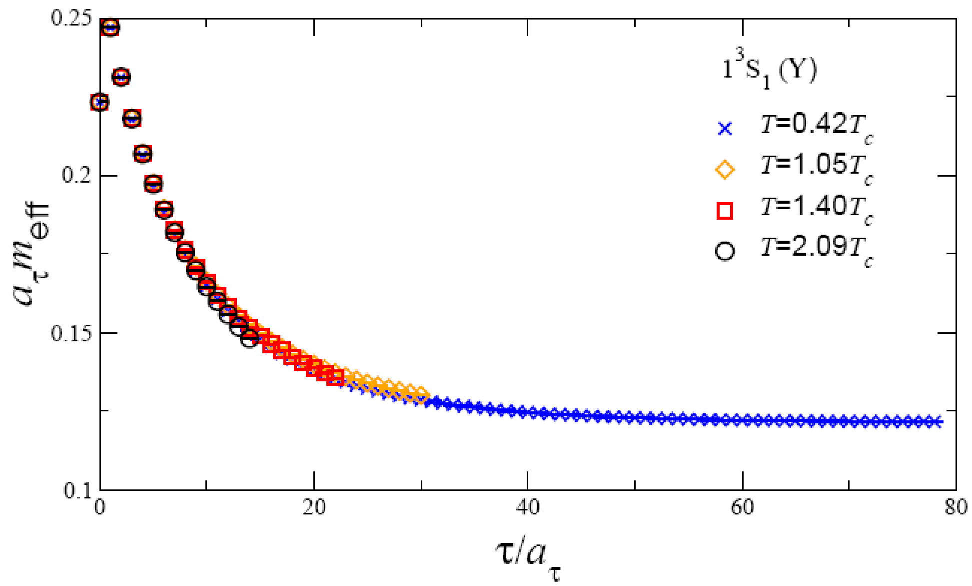
In NRQCD calculations p.b.c are not implemented and and correlators can be studied to twice larger separations $\tau_{max}=1/(2\,T) \rightarrow \tau_{max}=1/T$

$$m_{eff}(\tau) = -\ln(G(\tau+d\tau)/G(\tau))/d\tau - 2m_b$$



lattice NRQCD calculations :

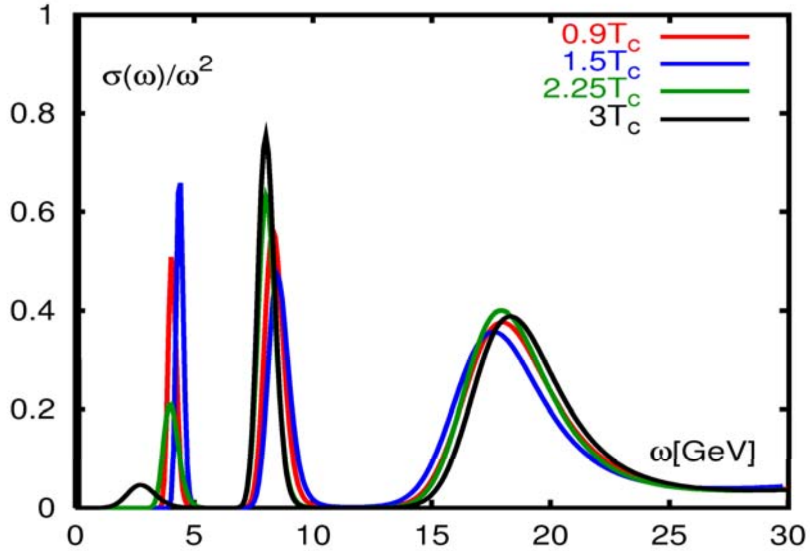
Aarts et al, PRL106 (2011) 061602

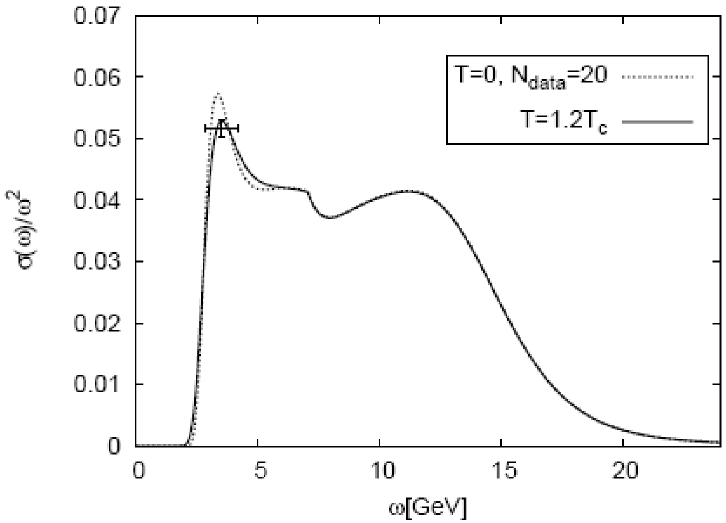


The potential model calculations can reproduce quite well the very small T-dependence of the S-wave correlation function calculated in NRQCD despite of dissolution of *2S* and *3S* state and large broadening of *1S* bottomonium states

Charmoniu spectral functions from MEM

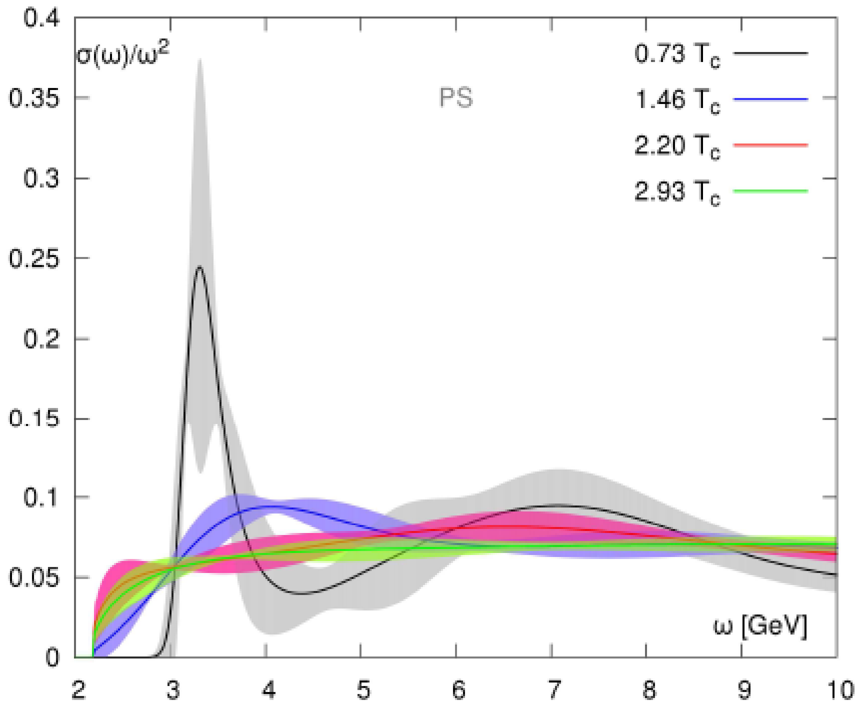
Old isotropic lattice : Datta, Karsch, P.P, Wetzorke, PRD 69 (2004) 094507, N_τ=12-40, a^{-1}=9.72GeV





asitropic lattice : Jakovác, P.P., Petrov, Velytsky, PRD

state of the art isotropic lattice :
H.-T. Ding, A. Francis, O. Kaczmarek, F. Karsch, H. Satz, W. Soeldner, PoS Lattice2010 180
N_τ=24-96, a^{-1}=18.97GeV



No clear evidence for charmonium bound state peaks from MEM spectral functions !

Summary

• Temporal meson correlation function for charmonium are not sensitive to the medium modification of the quarkoniuum spectral functions and thus show almost no change across the deconfinement transition

• The study of the spatial meson correlation functions provides the 1st direct lattice QCD evidence for melting of the *1S* charmonium for $T>300$ MeV :

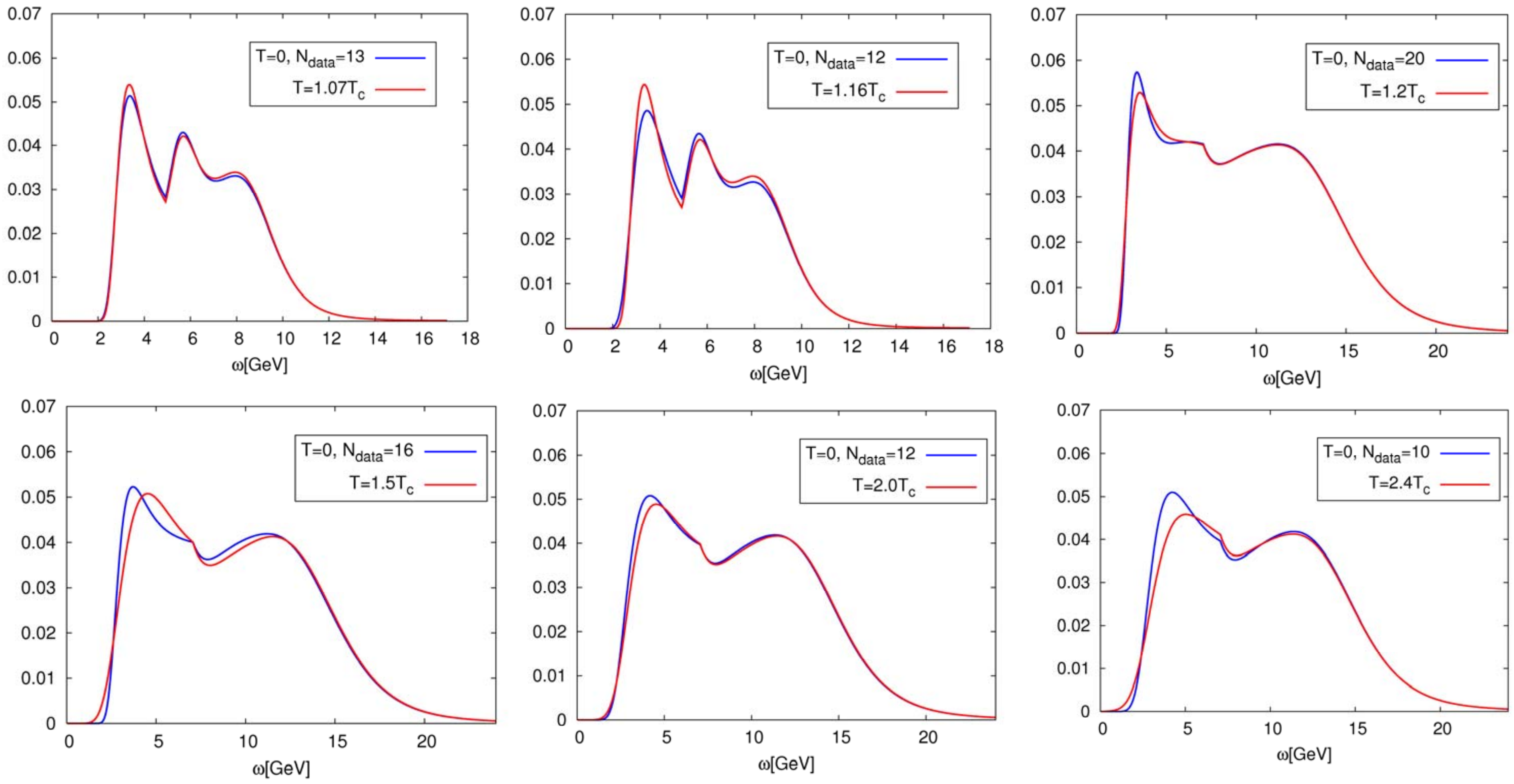
1) screening masses are compatible with the free/unbound value
2) strong dependence on the boundary conditions

• The imaginary part of the potential plays a prominent role as a quarkonium dissolution mechanism. Even for the most binding potential allowed by lattice QCD it leads to the dissolution of the 1S charmonium and excited bottomonium states for $T \approx 250$ MeV and dissolution of the 1S bottomonium states for $T \approx 450$ MeV consistent with previous findings

• Improved MEM determination of the charmonium spectral functions show no evidence for bound state peaks in the deconfined phase and thus are not consistent with potential model calculations

Back-up: Charmonium spectral functions at finite temperature

Jakovác, P.P., Petrov, Velytsky, PRD 75 (07) 014506



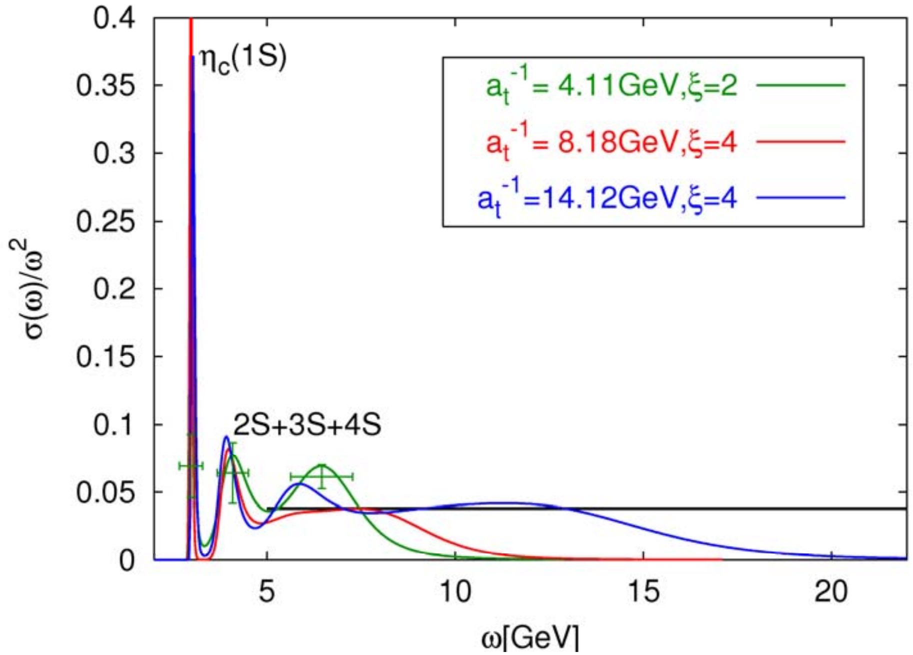
no large T-dependence but details are not resolved

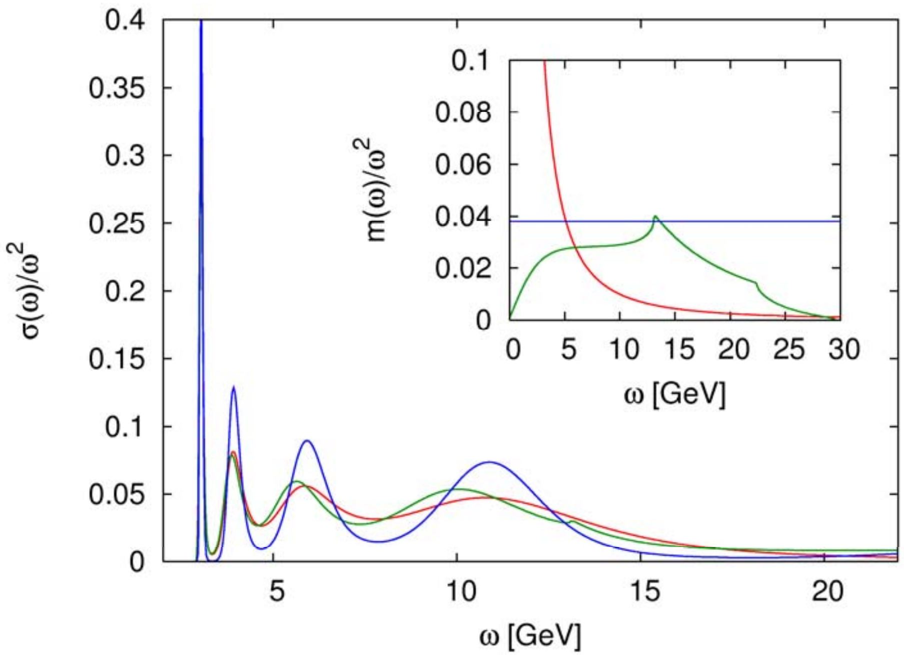
Back-up: Charmonia spectral functions at T=0

Anisotropic lattices: $16^3 \times 64, \xi = 2$ $16^3 \times 96, \xi = 4$, $24^3 \times 160, \xi = 4$
$L_s = 1.35 - 1.54$fm, #configs=500-930;
Wilson gauge action and Fermilab heavy quark action

Jakovác, P.P., Petrov, Velytsky, PRD 75 (07) 014506

Pseudo-scalar (PS) $\rightarrow$ S-states





For $\omega > 5$ GeV the spectral function is sensitive to lattice cut-off ;
Strong default model dependence in the continuum region