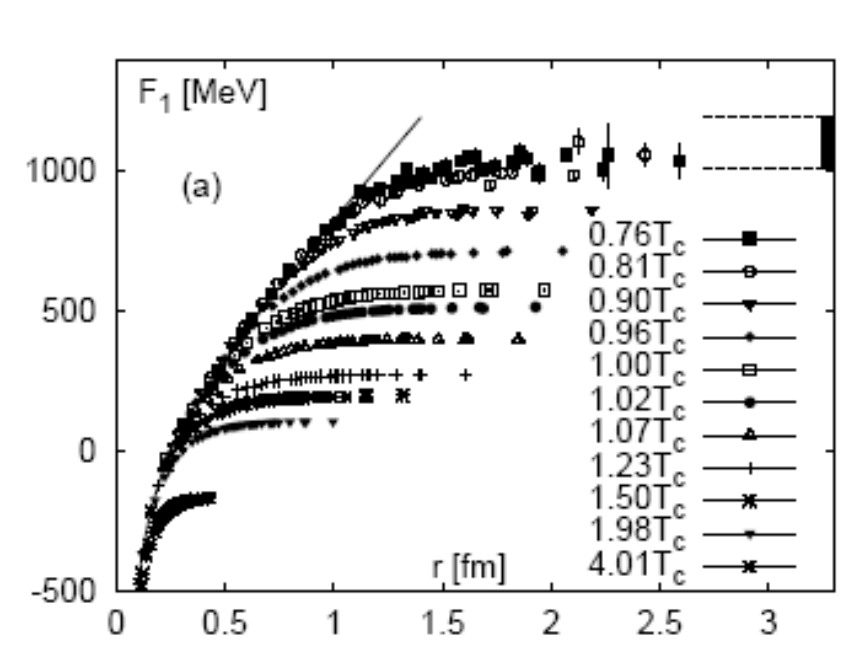
T-Matrix Approach to Quarkonia in QGP

Ralf Rapp
Dept. of Physics & Astronomy
Texas A&M University
College Station, TX, USA

EMMI Workshop on "Quarkonia in Deconfined Matter"
Acitrezza (Sicily, Italy), 28.-30.09.11

1.) Introduction: Heavy-Quark Interactions in Medium



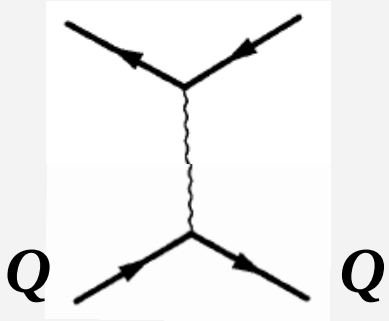
- strong medium effects, nonpert.:

  $F(r) > 0$ for $T \leq 2T_c$

- momentum transfer

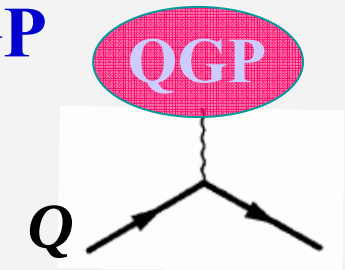
  $q^2 = q_0^2 - \vec{q}^2 \approx -\vec{q}^2$

  $q_0 \sim \vec{q}^2 / 2m_Q << |\vec{q}|$



- single heavy quark in QGP

  $p_{th}^2 \sim 2m_Q T >> T^2$



- soft Q-$\bar{Q}$ and Q-medium interactions static + elastic

=> common description of quarkonia + heavy-quark transport

requires bound + scattering states, resummations

=> thermodynamic T-matrix approach (e.m. plasmas, nucl. matter)

Outline

1.) **Introduction**

2.) **Thermodynamic T-Matrix**

- Brueckner Theory of Quark/onia in QGP
- Heavy-Quark Transport
- Quarkonia Spectral Functions, Correlators + Widths

3.) **Heavy-Ion Phenomenology**

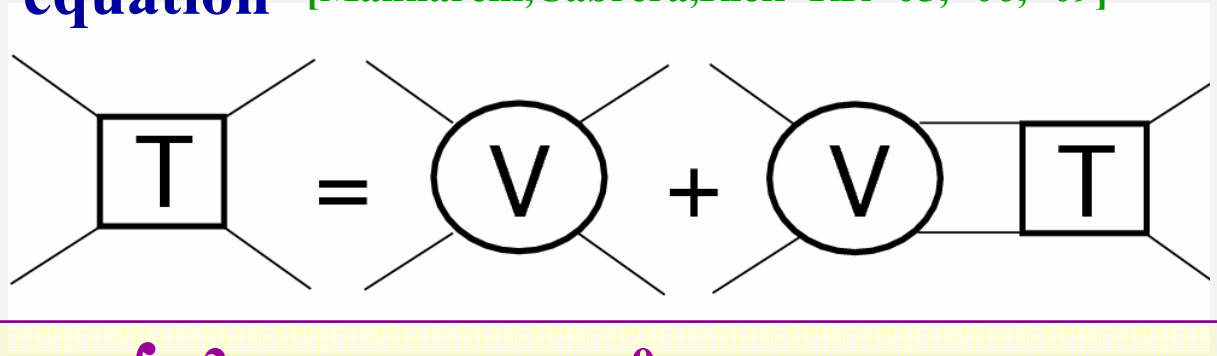
- Transport Equation
- Charmonia at SPS + RHIC
- Predictions (v_2 at RHIC; LHC; Υ)

4.) **Conclusions**

2.1 Two-Body Scattering Equation

- **Lippmann-Schwinger equation** [Mannarelli,Cabrera,Riek+RR '05, '06, '09]

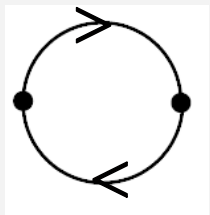
In-Medium Q-Q̄ *T*-Matrix:



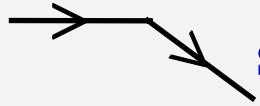
$$T_\alpha(E;q,q') = V_\alpha(q,q') + \int k^2 dk\, V_\alpha(q,k)\, G^0_{Q\bar{Q}}(E,k)\, T_\alpha(E;k,q')$$

- **Q-Q̄ propagator:** $G^0_{Q\bar{Q}}(E,k;T) = T\sum_\nu D_Q(z_\nu,\vec{k}) D_{\bar{Q}}(E-z_\nu,-\vec{k})$

$$\mathrm{Im}G^0_{Q\bar{Q}}(E) = -\int \frac{d\omega}{2\pi}\Big(\rho_Q(\omega)\rho_{\bar{Q}}(E-\omega)[1-f^Q(\omega)-f^{\bar{Q}}(E-\omega)] + \rho_Q(\omega)\rho_Q(E+\omega)[f^Q(\omega)-f^Q(E+\omega)]\Big)$$



Q-Q̄ propagation



Q → Q scattering

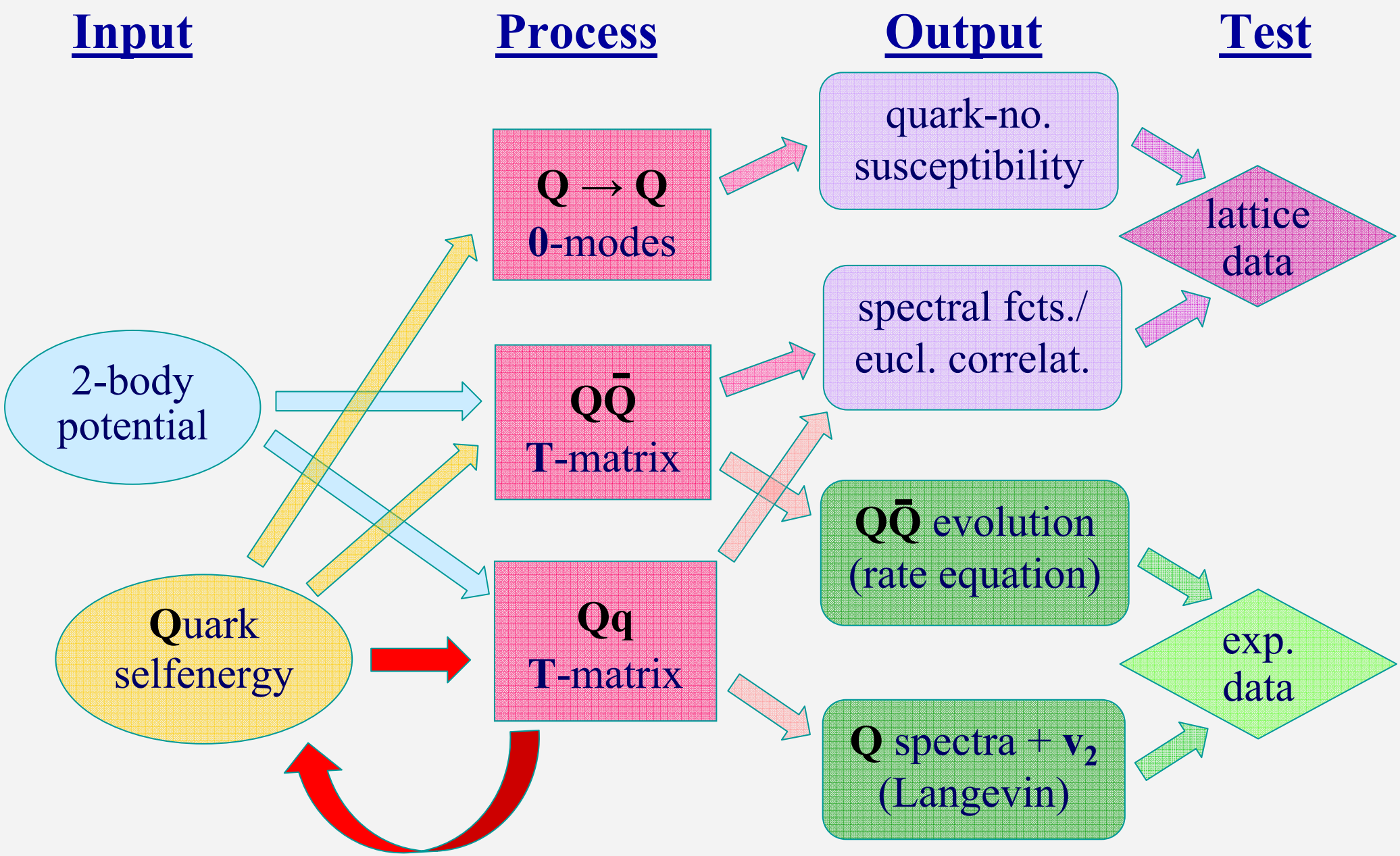
zero mode!

- Vector channel (j_0: density) → **HQ number susceptibility**

$$\chi_c(T) = -\frac{\partial^2\Omega}{\partial\mu_c^2} = \frac{1}{T}\int \frac{dE}{2\pi}\frac{2}{1-\exp(-E/T)}\rho_V^{00}(E)$$

determined by zero mode!

2.2 Brueckner Theory of Heavy Quarks in QGP

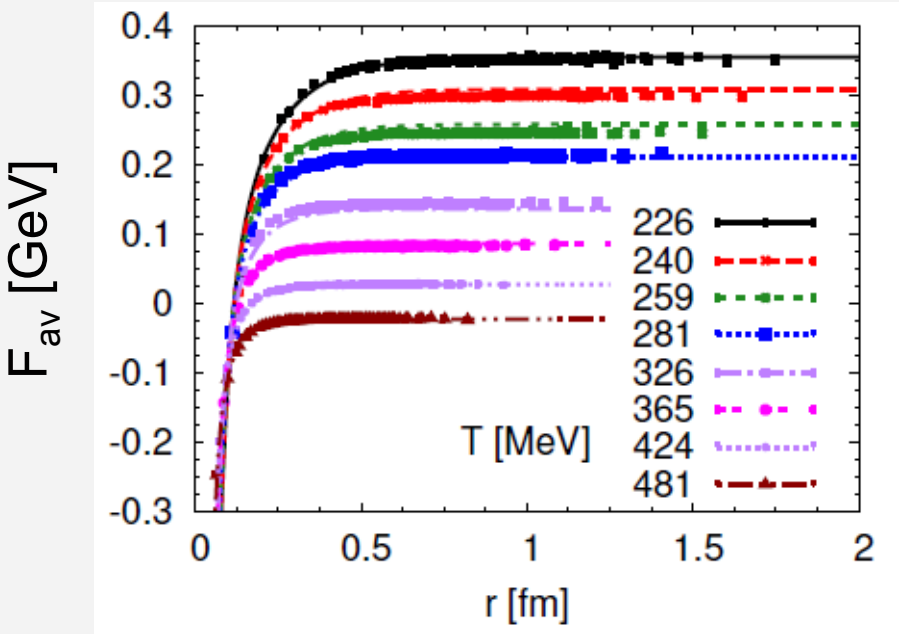


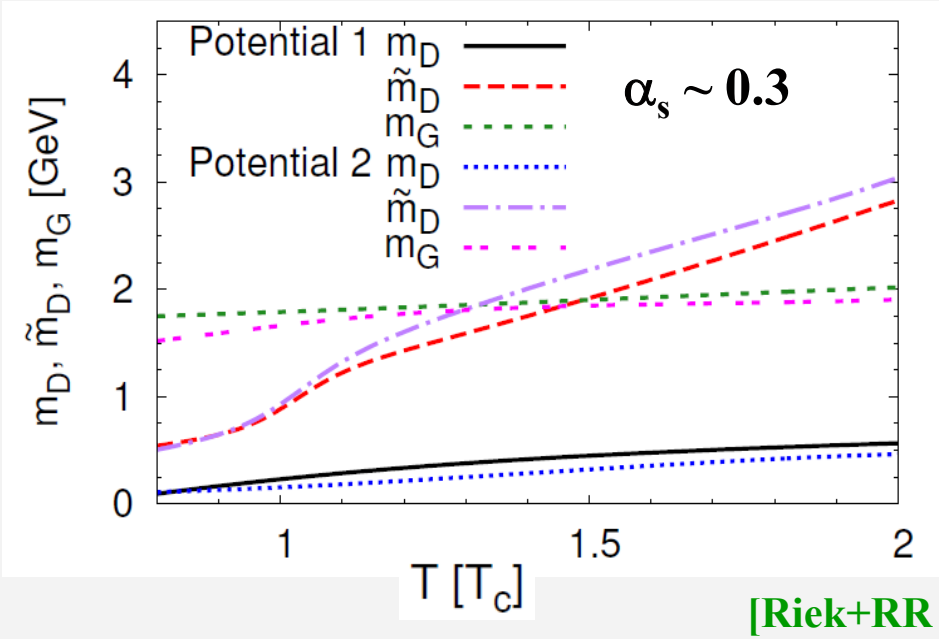
2.3 Field Theoretic Approach to Free Energy in QGP

- effective propagators: Coulomb + string
- fit 4 parameters to lattice-QCD data

$$D_{00}(k) = \frac{\alpha_s^2}{k^2 + m_D^2} + \frac{m_G^2}{(k^2 + \tilde{m}_D^2)^2}$$

[Megias et al '07]





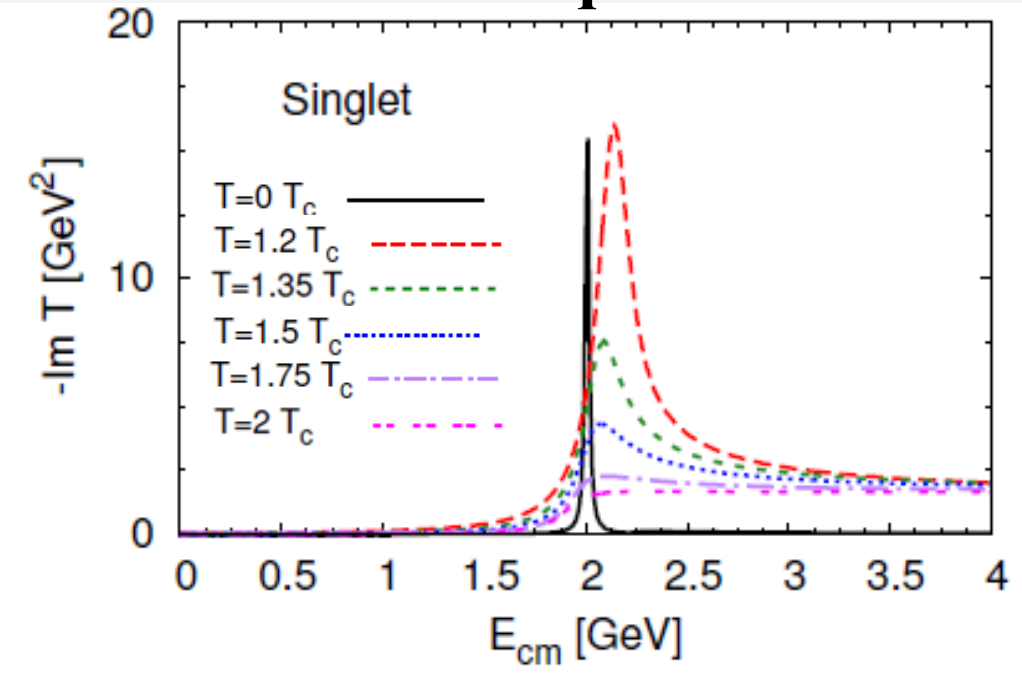
[Riek+RR '10]

- **Corrections to potential**
  - **Relativistic:** magnetic "Breit" correction: [Brown et al '52, '05]

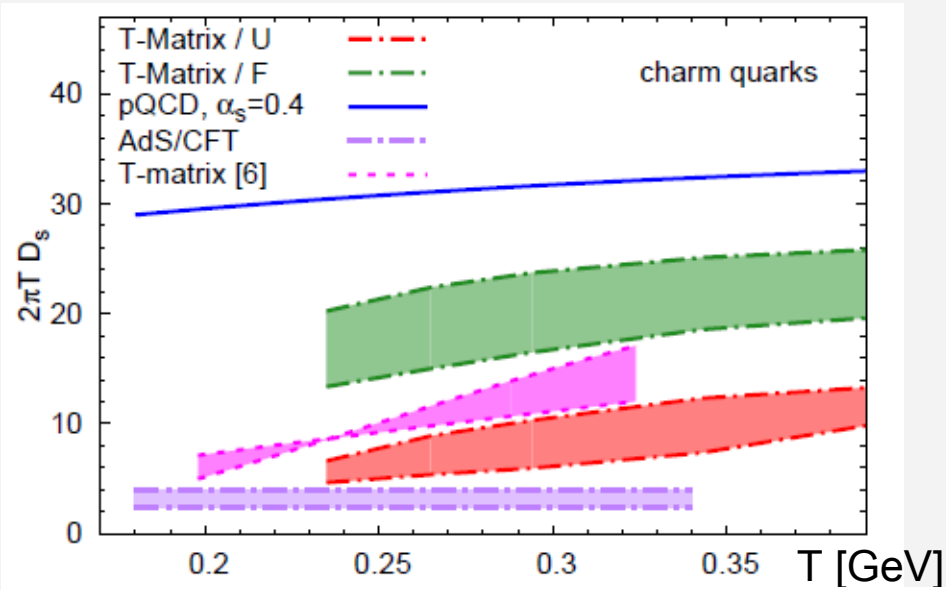
    $V_{Q1Q2}(r) \rightarrow V_{Q1Q2}(r)\ (1 - v_1 \cdot v_2)$ (↔ Poincaré-invariance, pQCD)
  - **Retardation:** - **4-D → 3-D** reduction of Bethe-Salpeter eq. (off-shell)

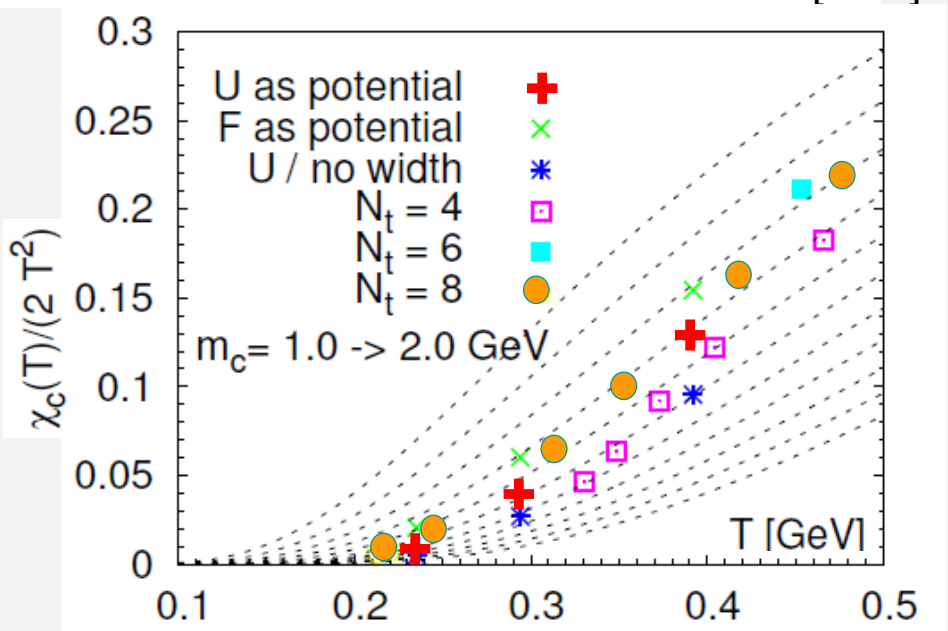
2.4 Single Charm Quarks in QGP

Thermal c-q̄ T-Matrix



Diffusion Coefficient





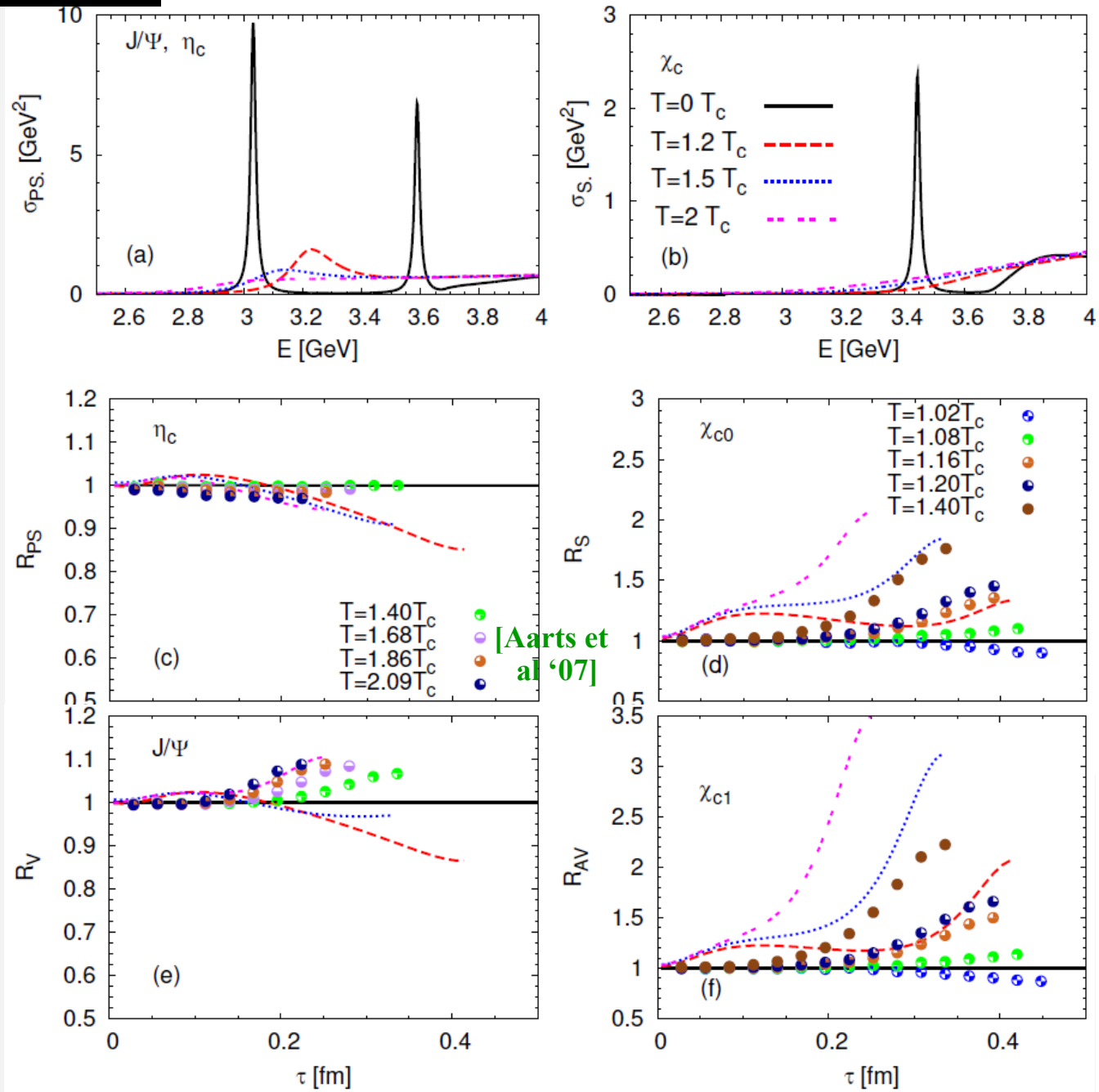
- meson/diquark resonances for **T < 1.5 T_c**
- thermalization **4** (**2**) times faster using **U** (**F**) as potential than pert. QCD
- finite-width effects in susceptibility

[Riek+RR '10]

2.5 Charmonia in QGP

- U–potential, selfconsist. **c**-quark width
- **Spectral Functions**
  - **J/ψ** melting at **~1.5T$_c$**
  - **χ$_c$** melting at **~T$_c$**
  - **Γ$_c$ ~ 100MeV**
- **Correlator Ratios**
  - rough agreement with lattice QCD within uncertainties

[Mocsy+ Petreczky '05+'08, Wong '06, Cabrera+RR '06, Beraudo et al '06, Satz et al '08, Lee et al '09, Riek+RR '10, …]



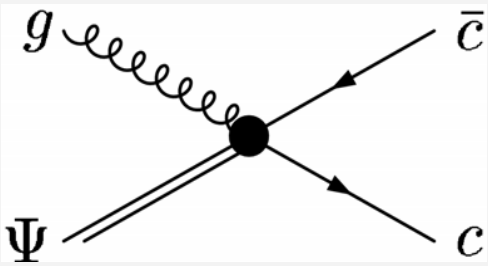
[Aarts et al '07]

2.6 Charmonium Widths in QGP

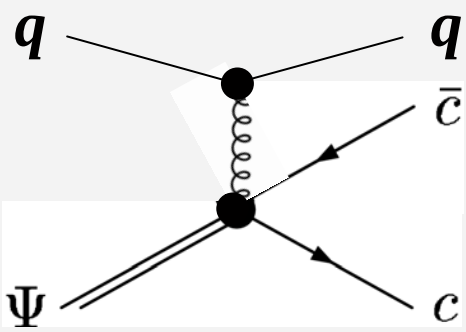
$$\Gamma_\psi = \sum_{p=q,\bar{q},g} \int \frac{d^3k}{(2\pi)^3} f^p(\omega_k;T)\, \sigma^{diss}_{p\psi}(s)$$

→ sensitive to binding energy (i.e., **color screening**)

- $E_B \geq T$: **gluo-dissociation** [Bhanot+Peskin '79]

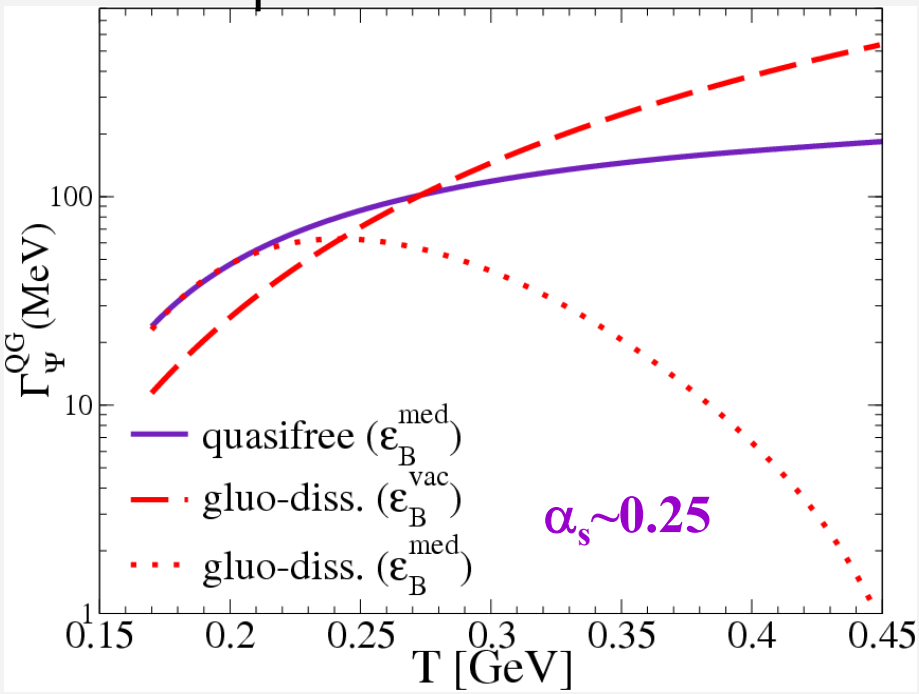


- $E_B < T$: **quasi-free dissociation**



[Grandchamp+RR '01]

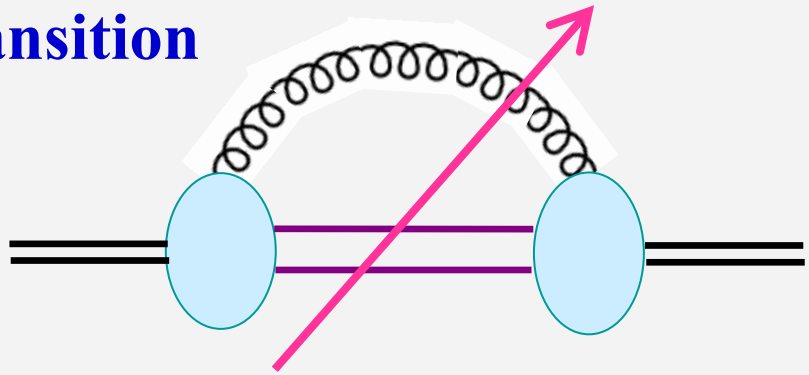
J/ψ Dissociation Rates

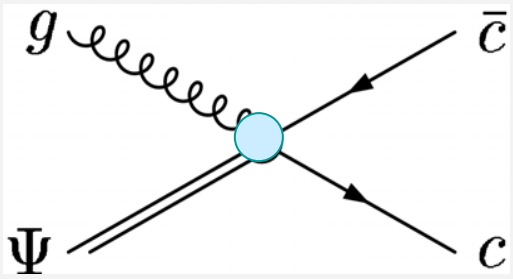


- **J/ψ lifetime ~ 1-4 fm/c**

2.6.2 Relation of Quarkonium Widths to EFT

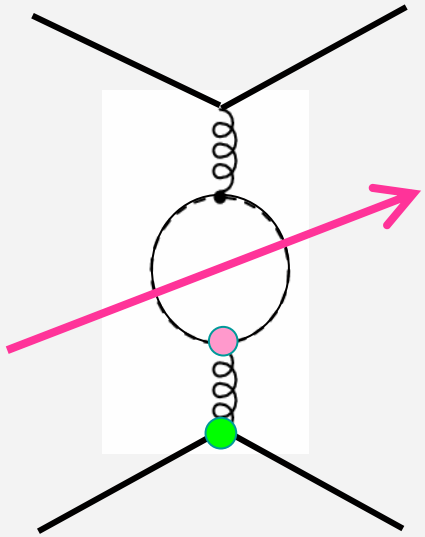
- **Singlet-to-octet transition**

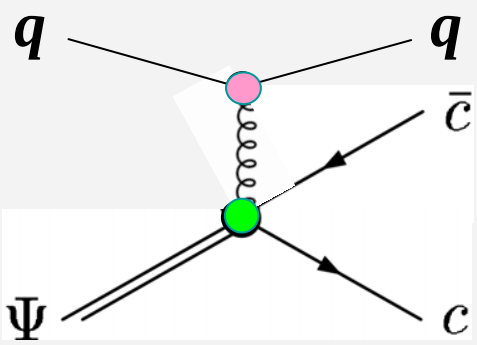




gluo-dissociation

- **Landau damping**





quasi-free dissociation

Outline

1.) **Introduction**

2.) **Thermodynamic T-Matrix**

- Brueckner Theory of Quark/onia in QGP
- Heavy-Quark Transport
- Quarkonia Spectral Functions, Correlators + Widths

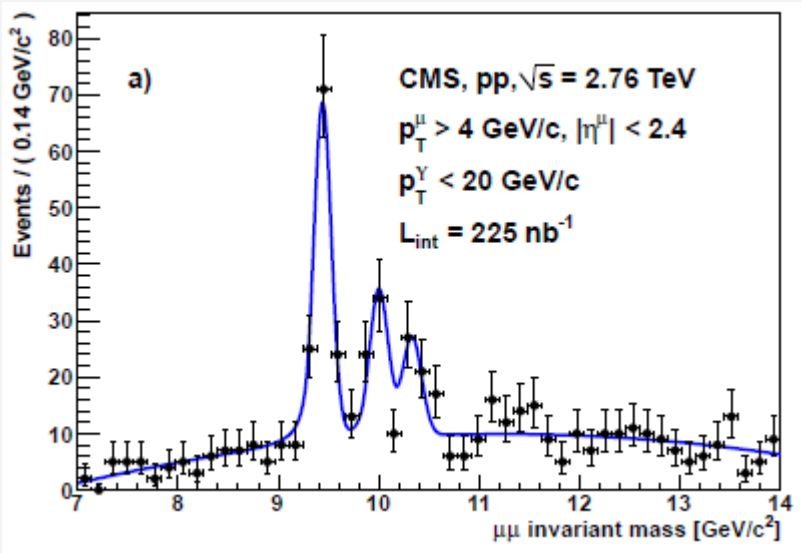
3.) **Heavy-Ion Phenomenology**

- Transport Equation
- Charmonia at SPS + RHIC
- Predictions (v_2 at RHIC; LHC; Υ)

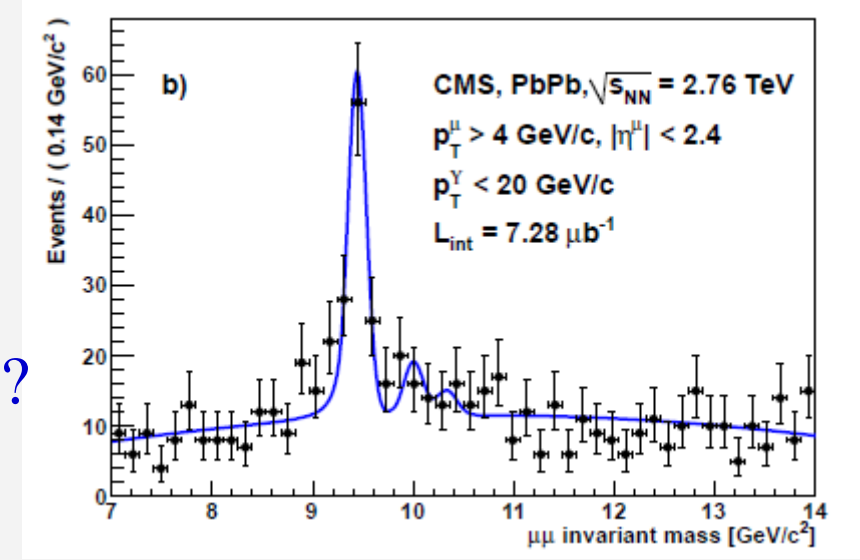
4.) **Conclusions**

3.) Quarkonium Production in URHICs

- measured in dilepton channel



"melting"
of **Υ' , Υ"** !?



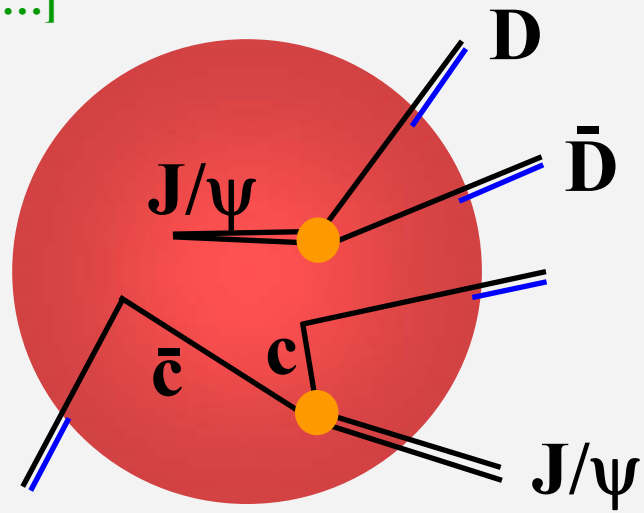
- spectra do **not** reflect in-med. spectral function/thermal dilepton rate!
- relation of **yields**, **p_t-spectra** + **v_2** to spectral properties more involved…

3.1 Transport Approach to Quarkonium Evolution

- **Regeneration in QGP + HG:**

[PBM et al '01, Gorenstein et al '02,Thews et al '01, Grandchamp+RR '01, Ko et al '02, Cassing et al '03, Zhuang et al '05, …]

detailed balance: $J/\psi + g \rightleftarrows c + \bar{c} + X$



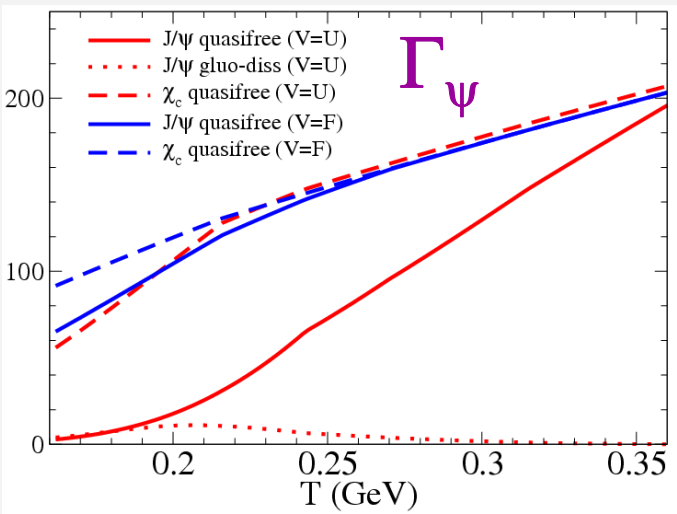
- **Rate Equation:**

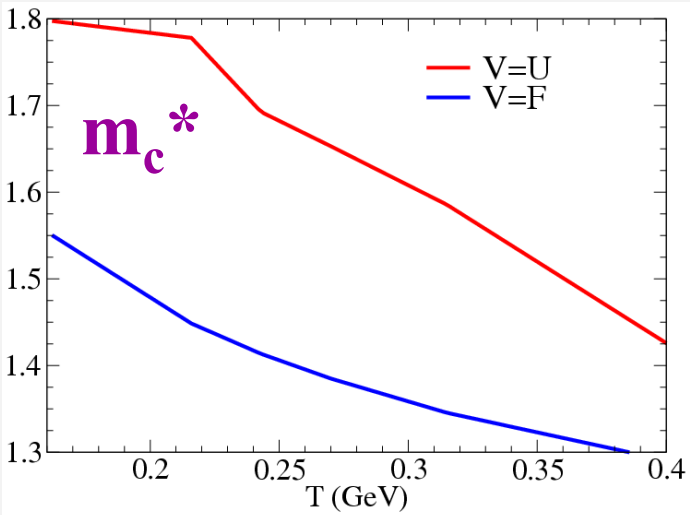
$$\frac{dN_\psi}{d\tau} = -\Gamma_\psi \left(N_\psi - N_\psi^{eq} \right)$$

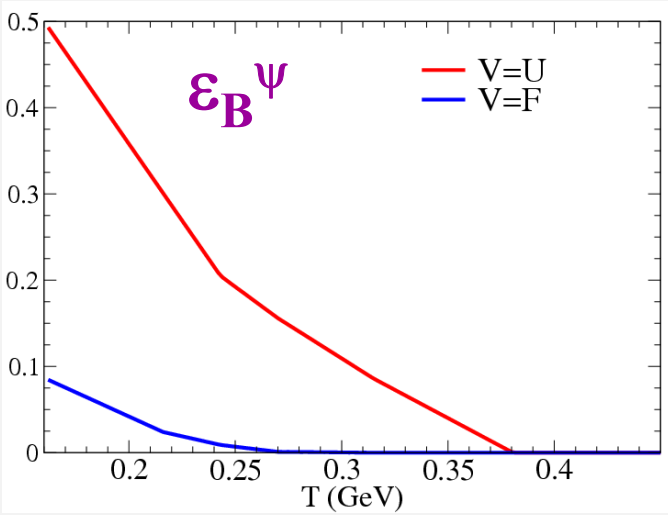
reaction rate (ψ -width)

equilibrium limit $(dN_c / dp_T, m_\psi, m_c^*)$

- **Input from Thermodynamic T-Matrix (weak/strong binding)**







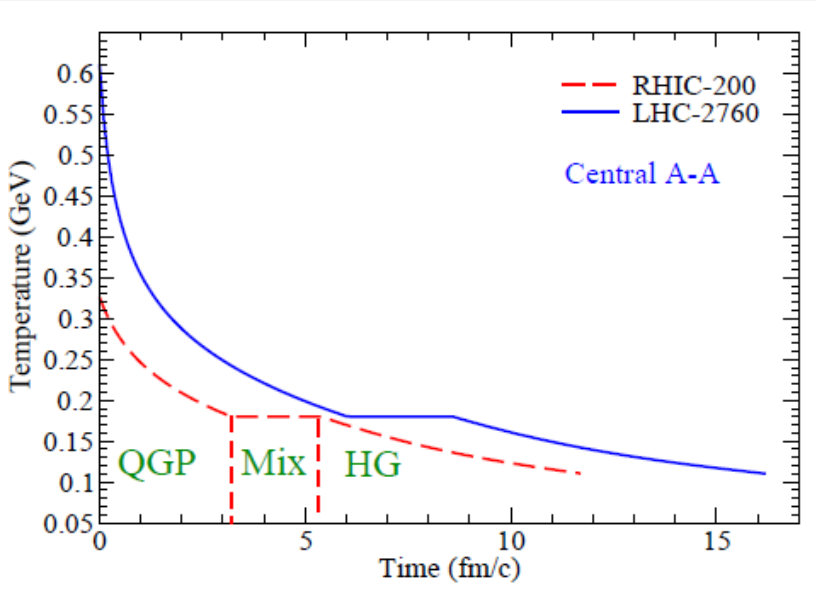
3.2 Inputs and Parameters

- **Input**

  - **J/ψ** (χ_c, ψ'), **c-$\bar{c}$** production cross sections, **b** feeddown [**p-p** data]

  - "Cold Nuclear Matter": shadowing, nuclear absorption, p_t broadening [**p/d-A** data, shad. est.]

  - Thermal fireball evolution: thermalization time (↔ initial T_0), expansion rate, lifetime, T_c, freezeout … [**A-A** hadron data, hydrodynamics]

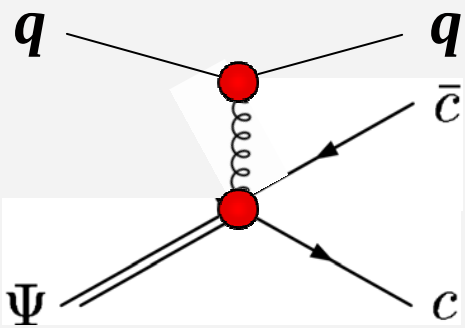


- **Parameters**

  - strong coupling α_s controls Γ_{diss}

  - schematic **c**-quark off-equilibrium:

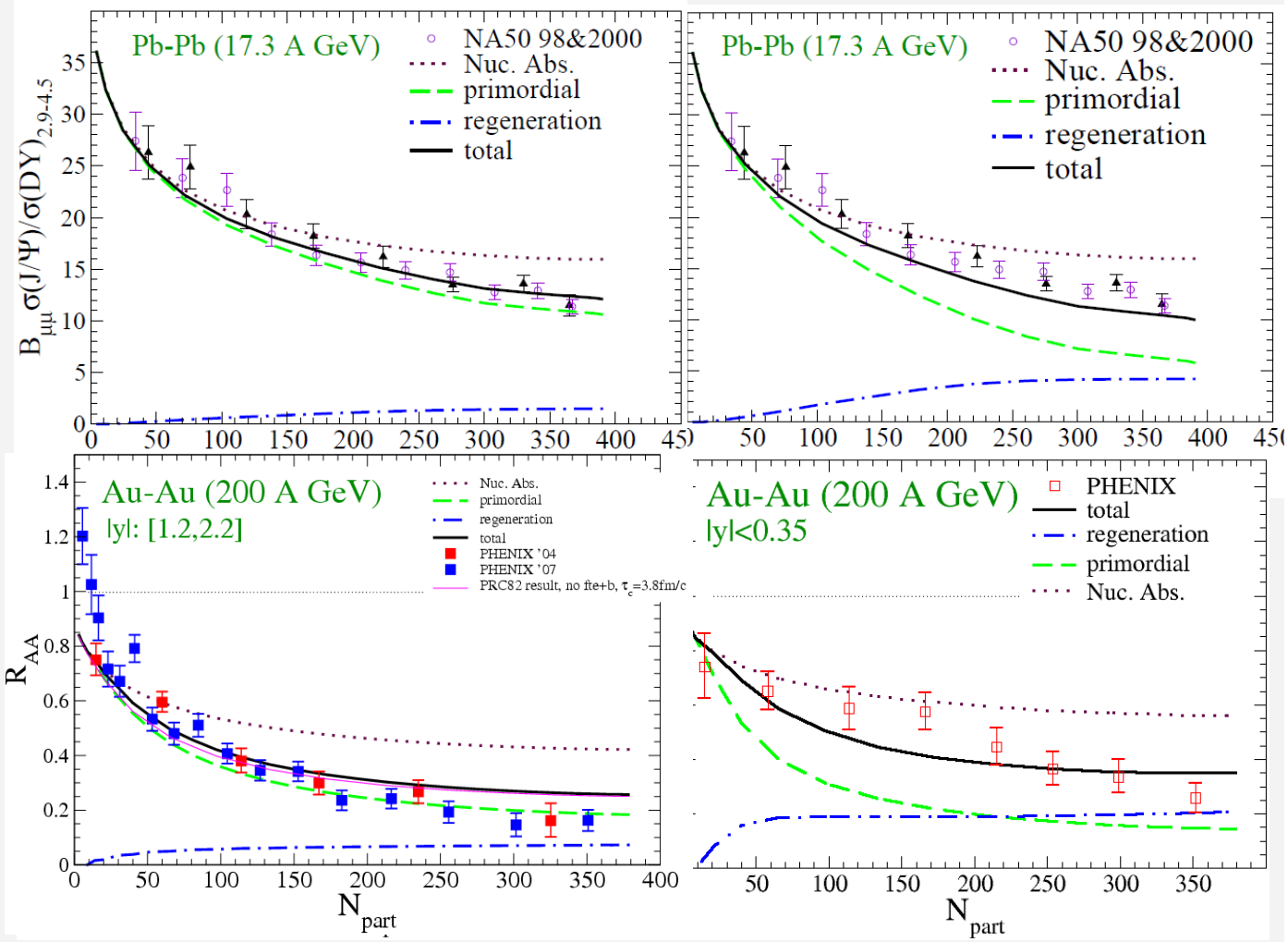
    $N_\psi^{eq}(\tau) \sim N_\psi^{therm}(\tau) \cdot [1-\exp(-\tau/\tau_c^{eq})]$



3.3 Inclusive J/ψ in Thermal Media at SPS + RHIC

- thermal rate equation through **QGP/T_c/HG** for **J/ψ , χ_c , ψ'**
- correlation volume
- bottom feeddown
- formation-time effect
- spectral properties ↔ Γ_ψ , N_ψ^{eq}

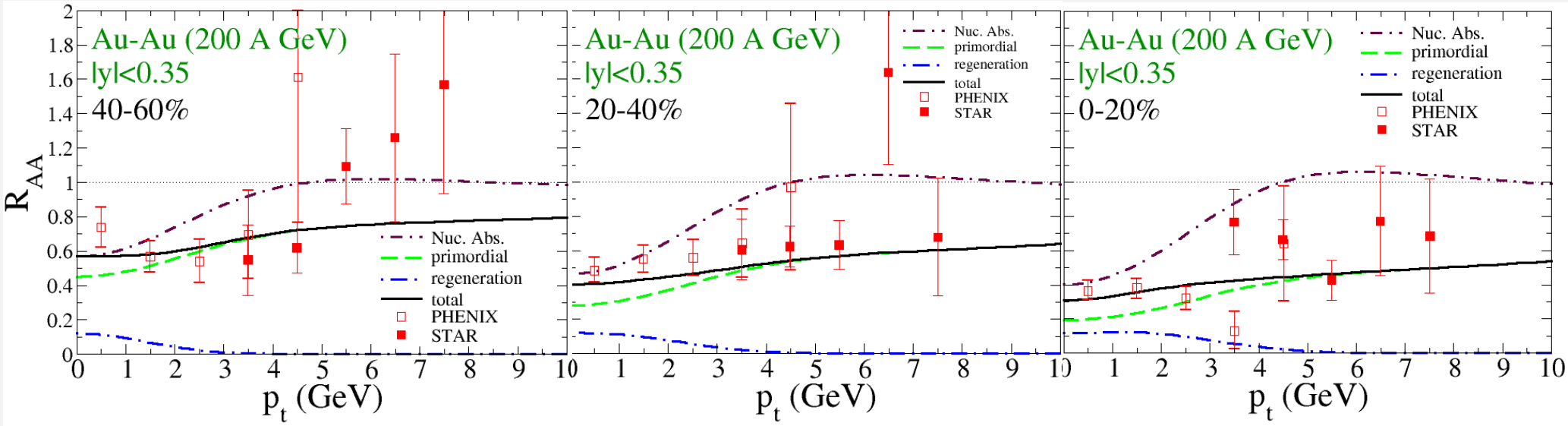
U-Potential **F-Potential**



- 2 parameters (α_s~**0.3**, charm relax. τ_c^{eq}**=6(3) fm/c**)
- different composition in two scenarios

[Zhao+ RR '10]

3.4 J/ψ Transverse-Momentum Spectra at RHIC

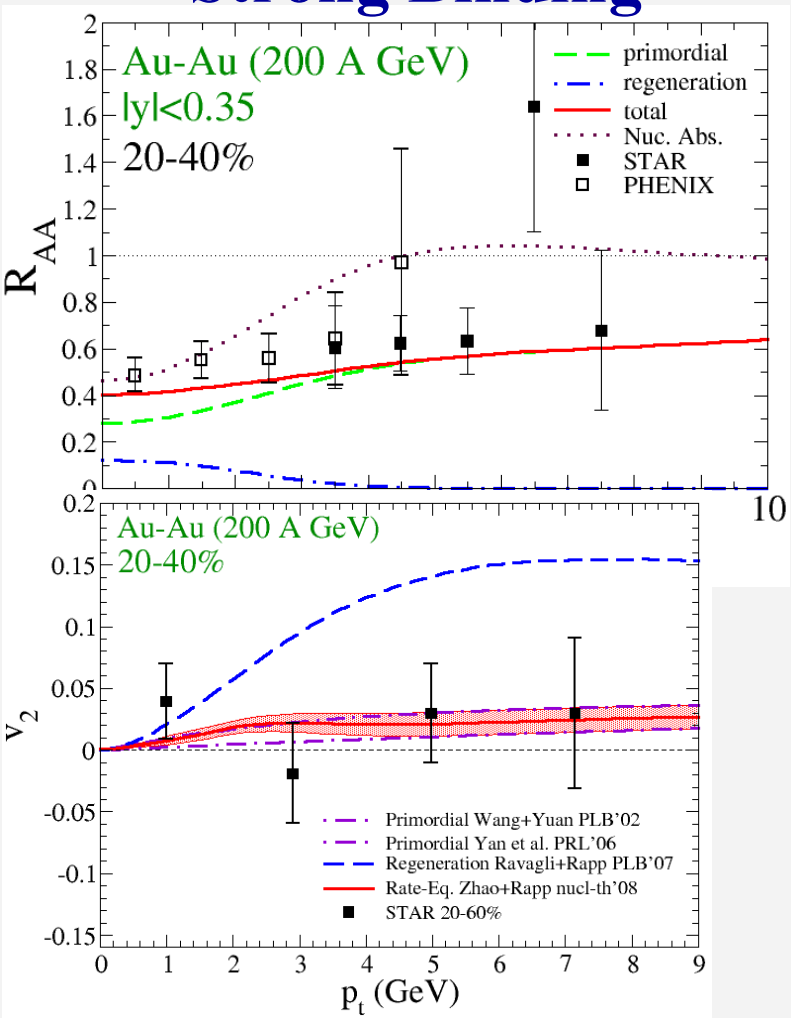


[Zhao+RR '08]

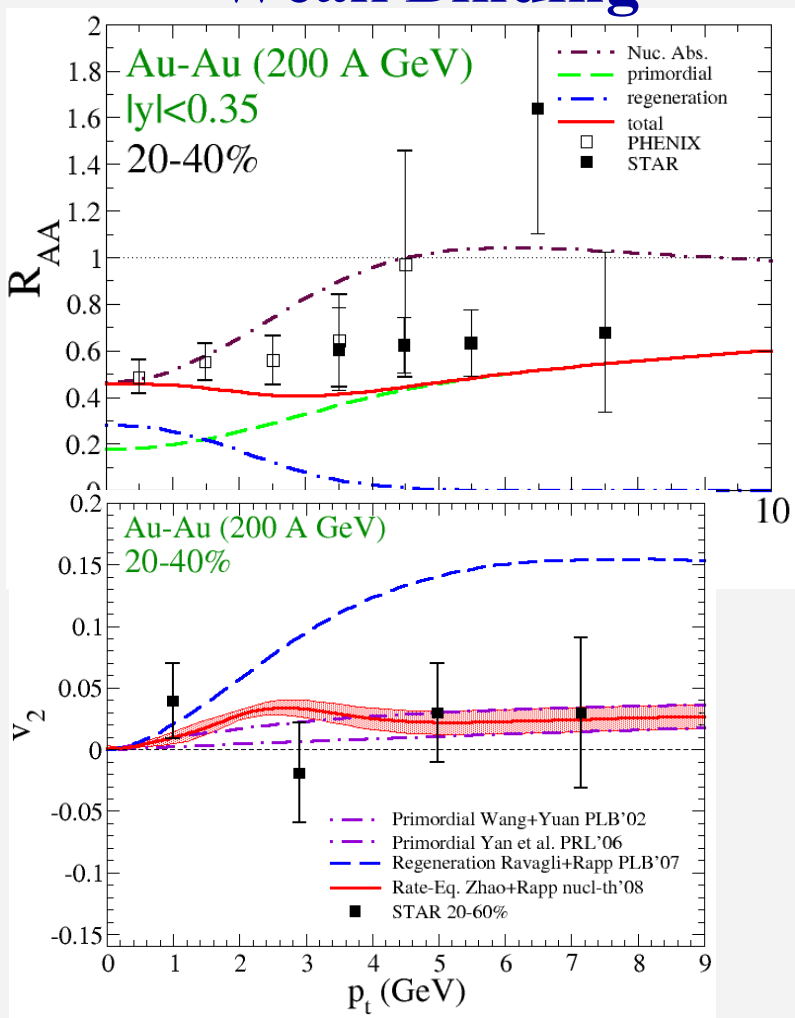
- Cronin and formation-time effects essential
- slight underestimate at high p_t
- similar results for weak-binding scenario (**F**-potential)

3.5 J/ψ Predictions I: Elliptic Flow at RHIC

Strong Binding



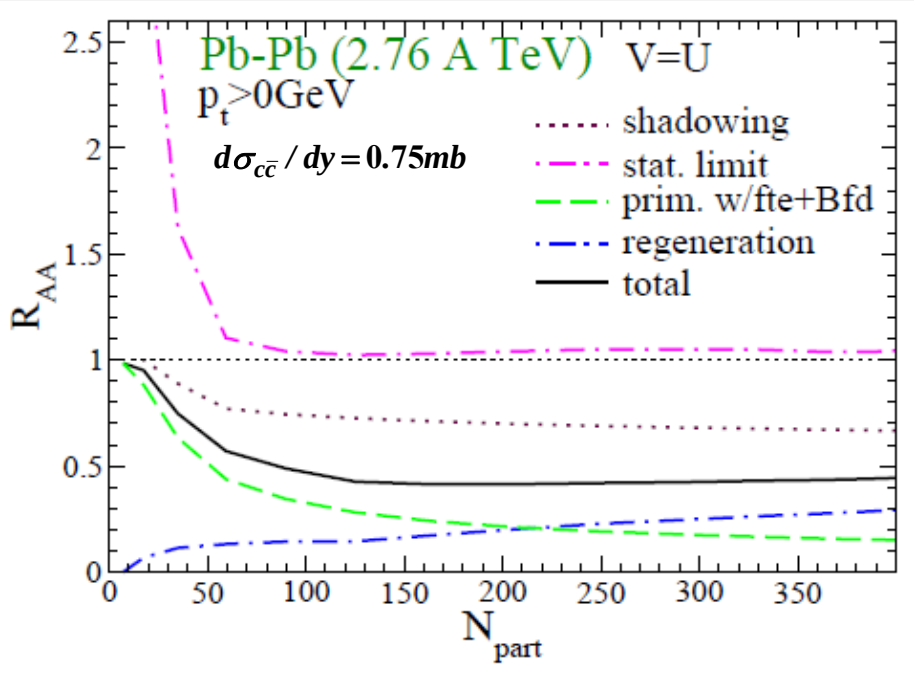
Weak Binding



- small v_2 **limits** regeneration, but does **not exclude** it **[Zhao+RR '08]**

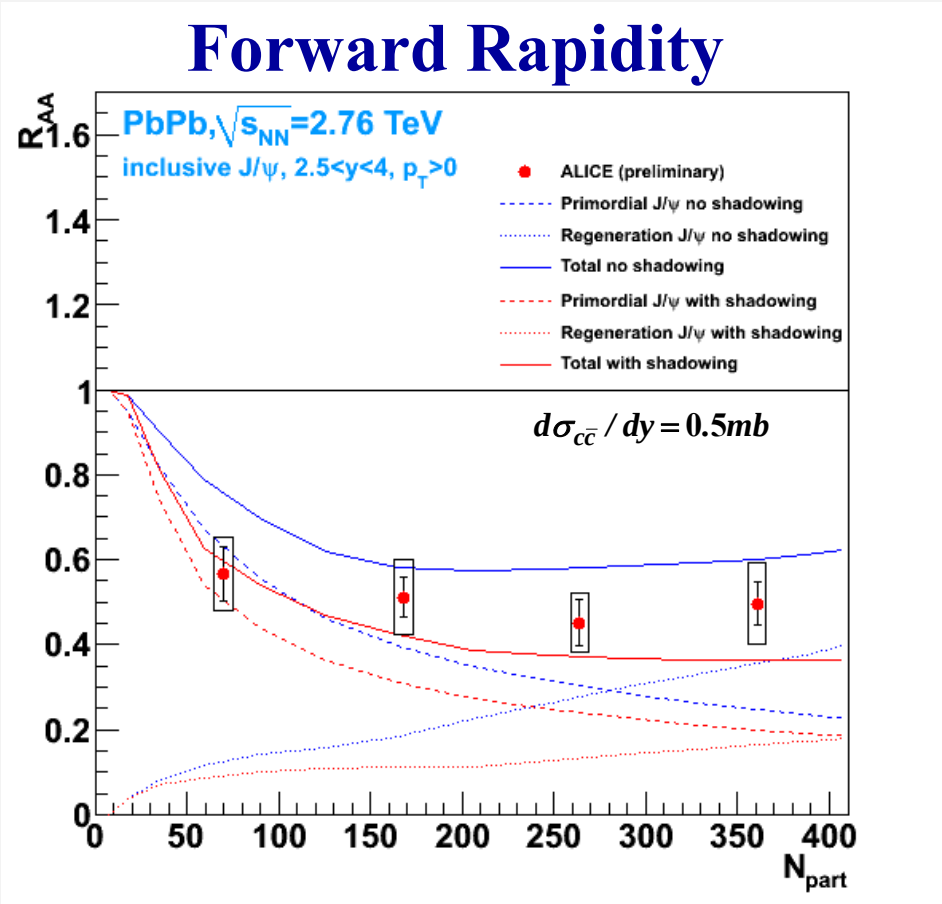
3.6 J/ψ Predictions II: LHC inclusive – ALICE

Mid-Rapidity



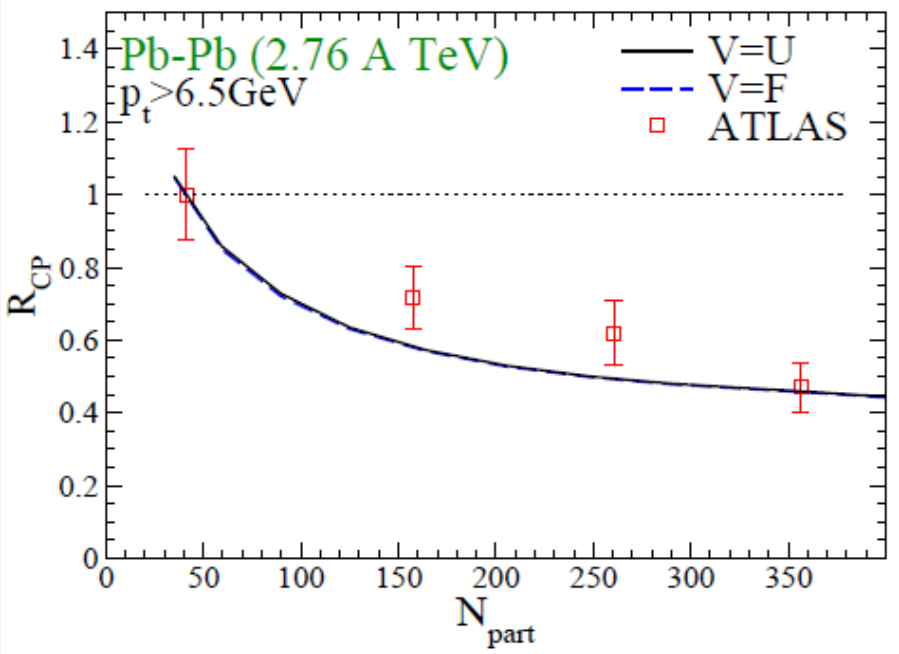
[Zhao+RR '11]

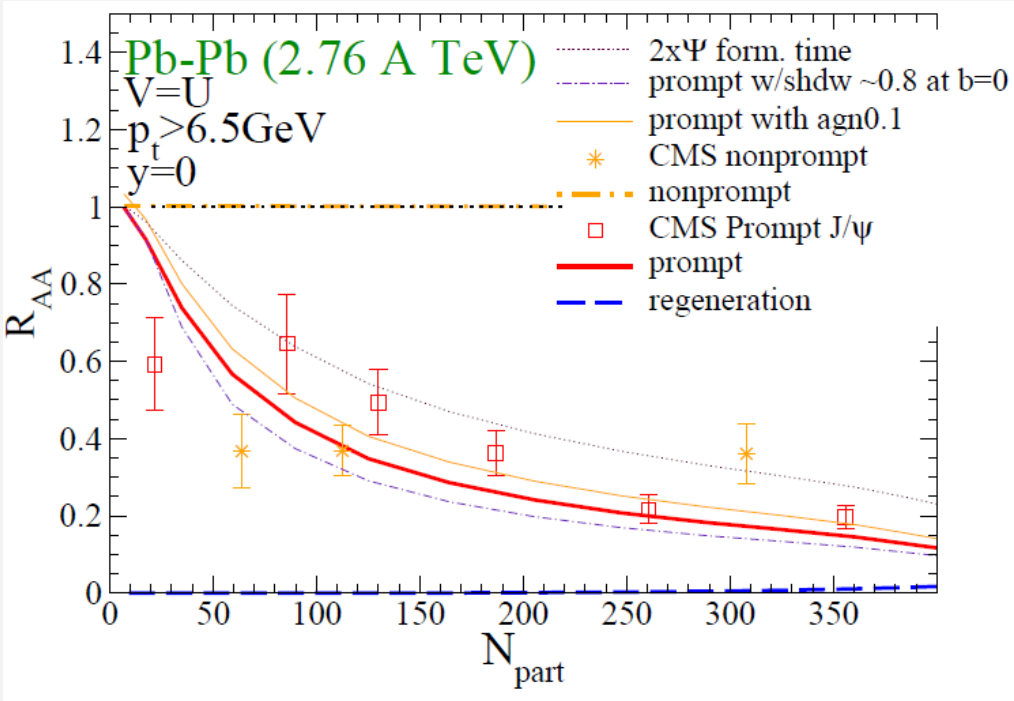
Forward Rapidity



- regeneration component increases, still **net suppression**
- confirmed within main uncertainty of input (shadowing) …

3.7 J/ψ Predictions III: LHC high-p_t – ATLAS+CMS





[Zhao+RR '11]

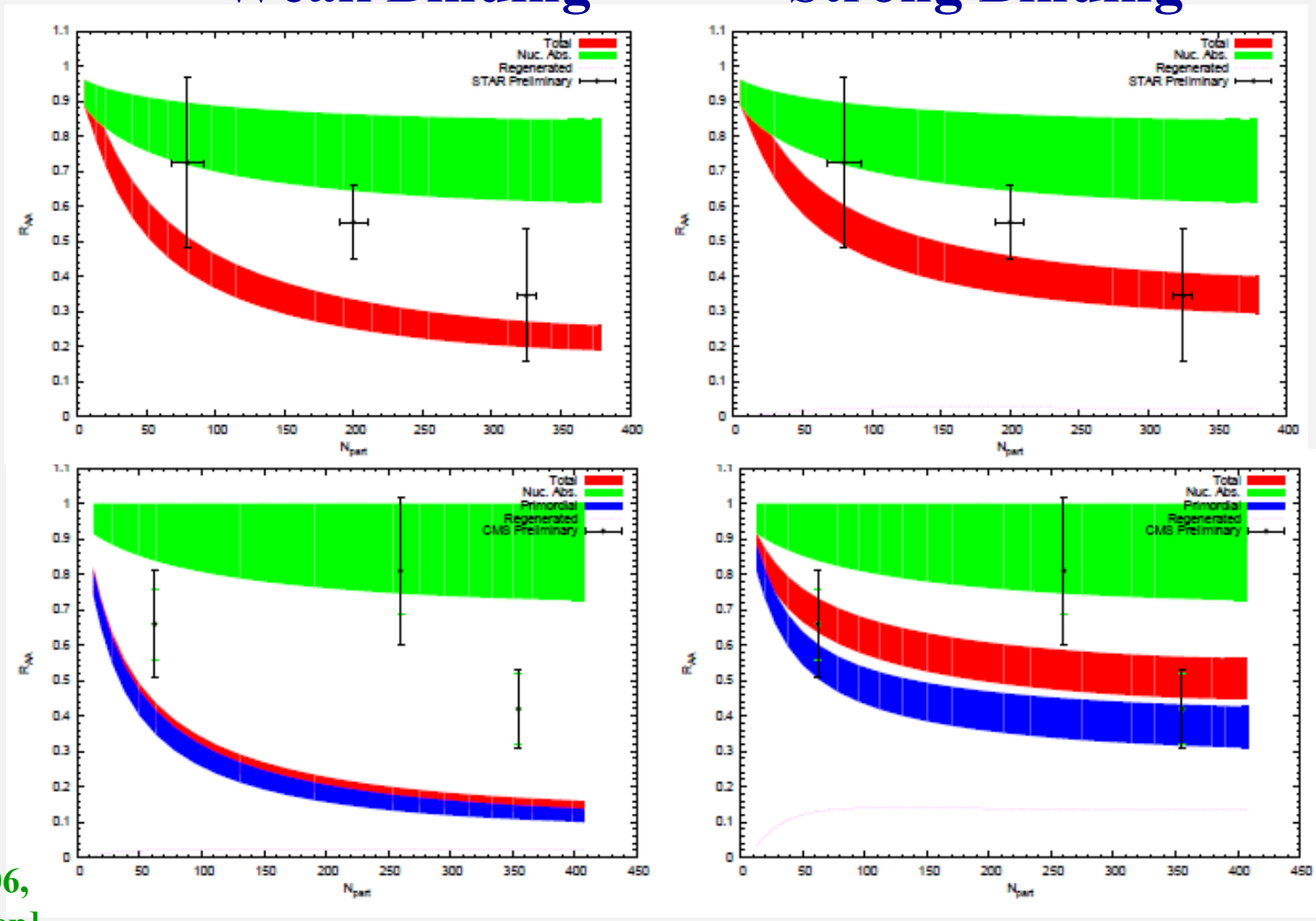
- underestimate for peripheral (expected from **RHIC**)
  (spherical fireball reduces surface effects …)

3.8 ϒ Predictions at RHIC and LHC

Weak Binding **Strong Binding**

RHIC →

LHC →



[Grandchamp et al '06, Emerick et al in prep]

- sensitive to screening + early times; regeneration at LHC

4.) Conclusions

- Common framework for quarkonia + heavy-quark transport: thermodynamic T-matrix
- Connect HQ(onium) theory with phenomenology
- Thermal rate-equation approach adequate to describe (+ predict) existing quarkonium data in heavy-ion collisions
- Preference for "strong binding scenario" (also works better for HQ diffusion)
- Open Problems
  - in-medium potential
  - microscopic gain term (realistic HQ spectra)
  - CNM effects at LHC (shadowing, …), high-p_T dynamics …

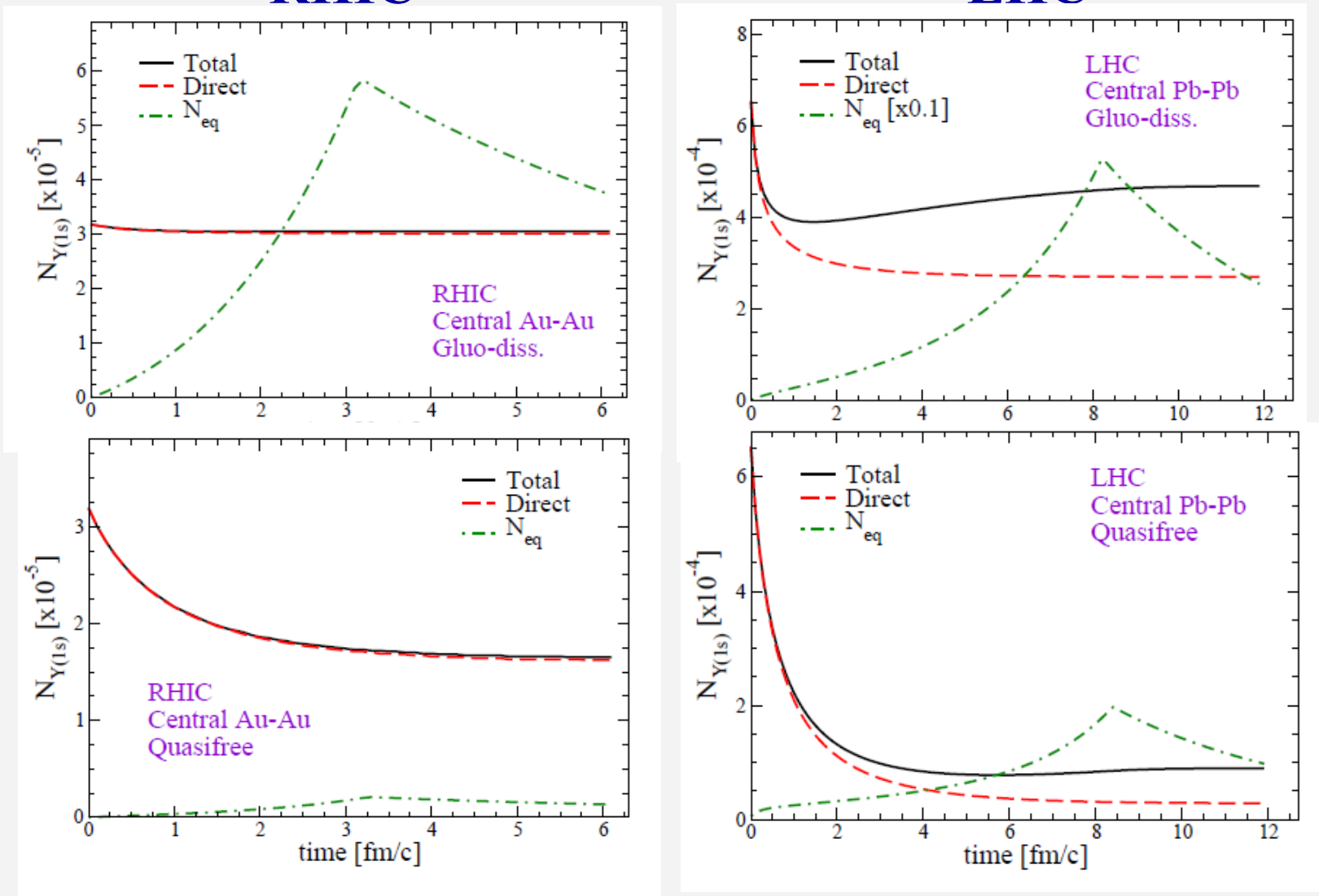
Time for a more systematic comparison of approaches!

4.3 Y(1S) Evolution at RHIC and LHC

RHIC **LHC**

"strong binding" →

"weak binding" →



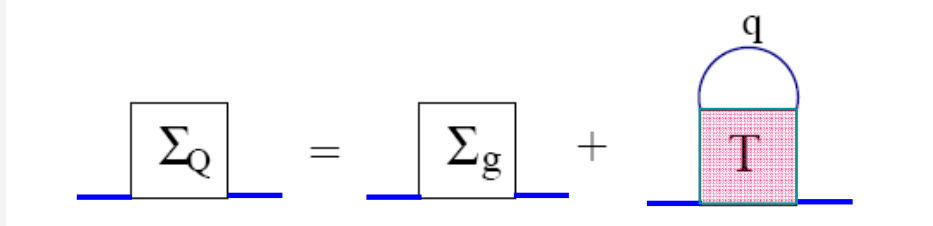
[Grandchamp et al '06]

- sensitive to screening + early times; regeneration at LHC

2.3 Single Heavy-Quark Spectral Fct. + Transport

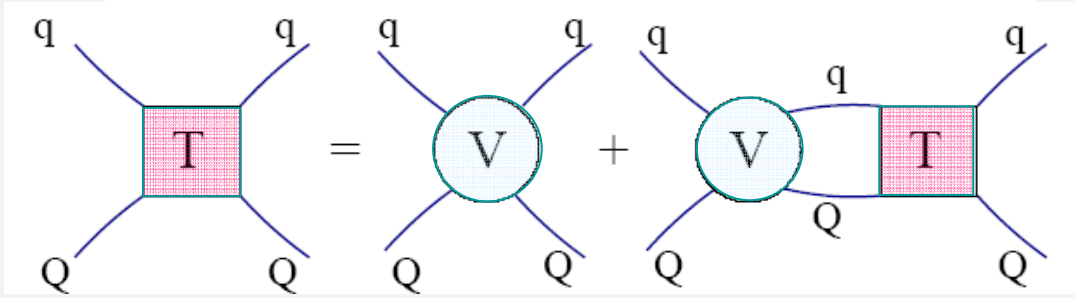
- **Spectral Function (propagator)**

$$\rho_Q = -2\,\mathrm{Im}D_Q = -2\,\mathrm{Im}\frac{1}{\omega-\omega_Q(k)-\Sigma_Q(\omega,k)}$$

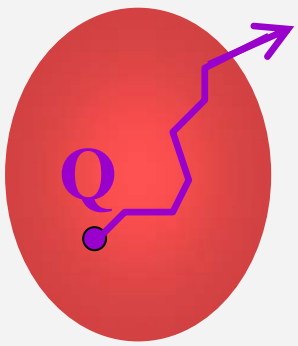


- **Scattering Rate (selfenergy)**

$$\mathrm{Im}\Sigma_Q(\omega,k) = \int \mathrm{Im}T_{Qq}(\omega+\omega_p)\, f^q(\omega_p)$$



- **Heavy-Quark Transport**



- Brownian Motion:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial(pf)}{\partial p} + D\frac{\partial^2 f}{\partial p^2}$$

Fokker Planck Eq.

thermalization rate

$$\gamma p = \int d^3k\, w_Q(k,p)\, k$$

$$\sim \int |T_{Qq}|^2 (1-\cos\theta)\, f^q$$

diffusion coefficient

$$D = \frac{1}{2}\int d^3k\, w_Q(k,p)\, k^2$$

$$T = \frac{D}{\gamma\, m_Q}$$

[Svetitsky '88, Mustafa et al '98, Hees+RR '04, Teaney+Moore '04, Gubser '07, Peshier '09, Alam et al '09, …]

2.4 Heavy-Quark Free Energy in Lattice QCD

$$F_1(r,T) = U_1(r,T) - T\,S_1(r,T)$$

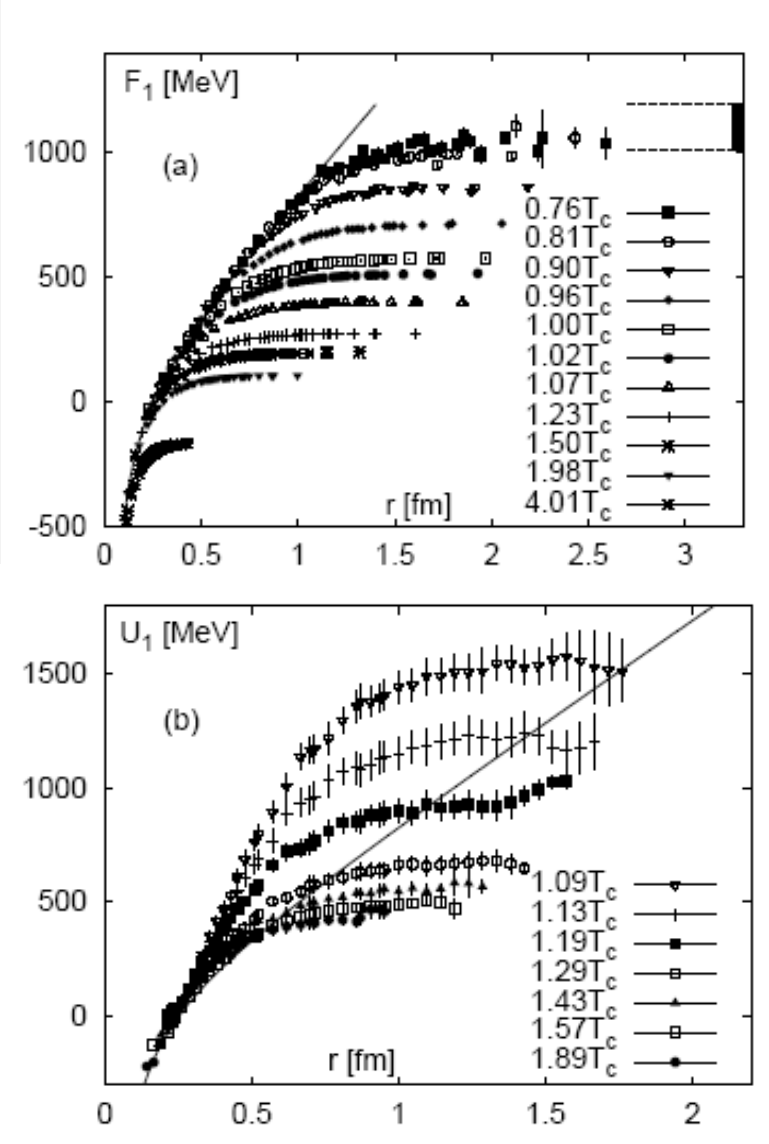
- **Potential?!**

  (a) Free energy F_1
  => weak $\bar{Q}Q$ potential,
  small Q "selfenergy" $F_1(r=\infty,T)/2$

  (b) Internal Energy U_1 $(U = \langle H_{int} \rangle)$
  => strong $\bar{Q}Q$ potential,
  large Q "selfenergy" $U_1(r=\infty,T)/2$

  → compensation in eucl. correlators

- **F, U, S thermodynamic quantities**
  (0-point functions),
  potential 4-point function

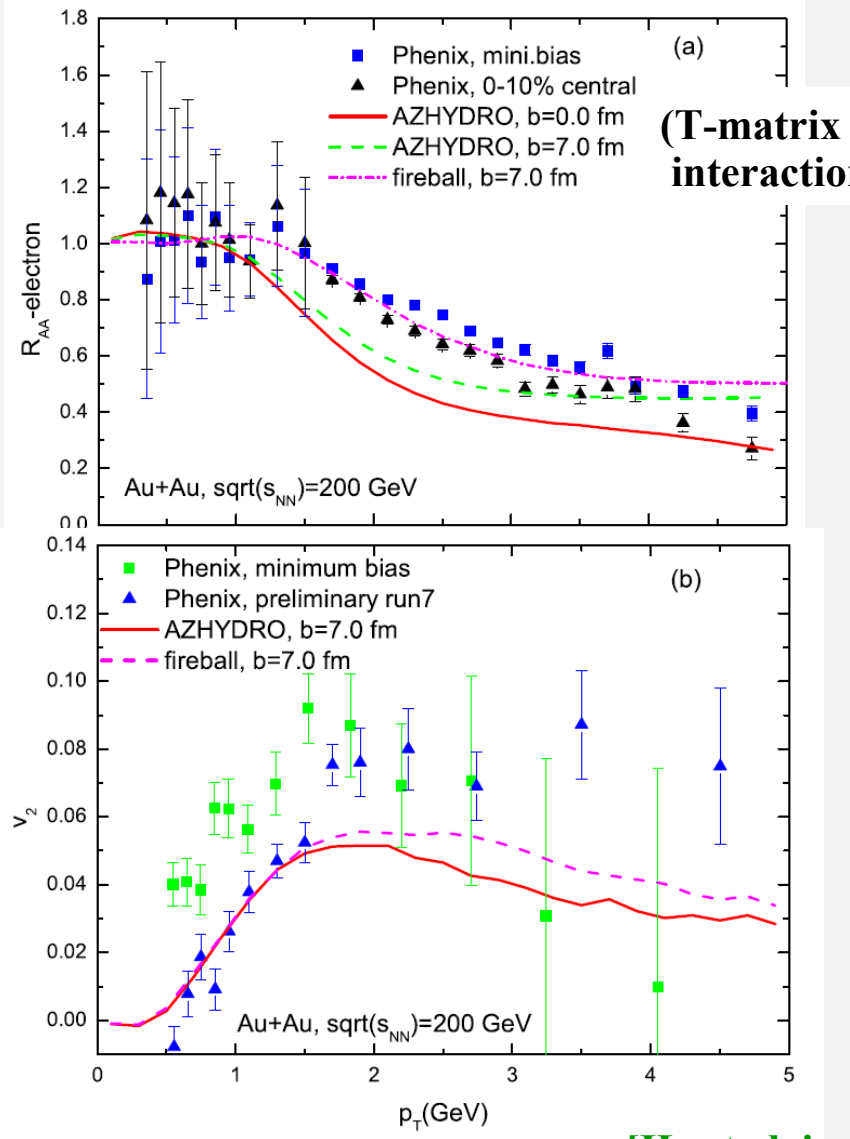


[Kaczmarek+Zantow '05]

4.4 Heavy-Quark Transport + $e^{\pm}$ Spectra at RHIC

$e^{\pm}$ Decays from c/b Langevin in Hydro

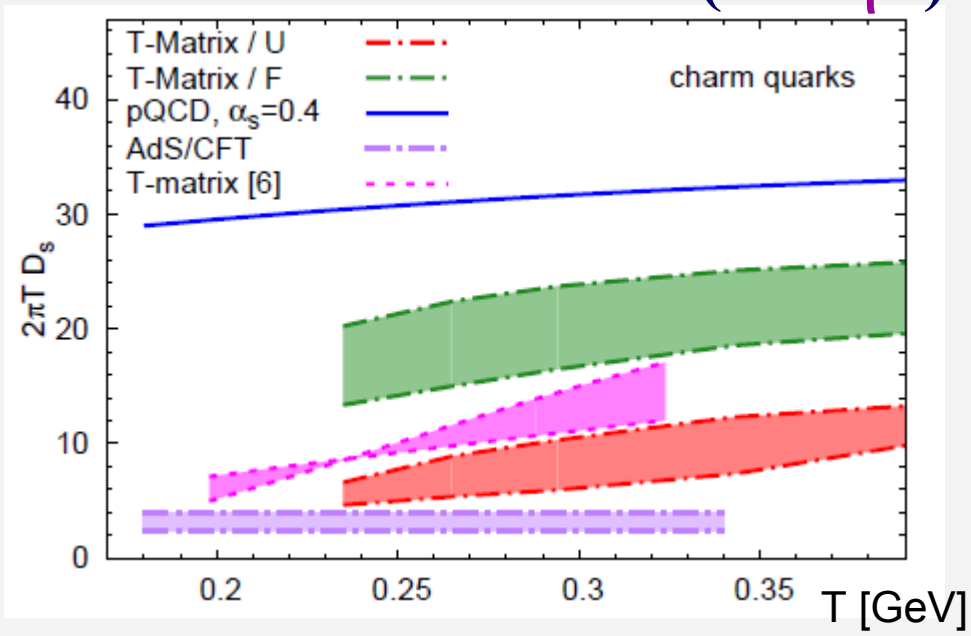
[Teaney+Moore '04, Mustafa '05, Hees et al '05, Gossiaux et al '09, Akamatsu et al '09, Alam et al '10, Beraudo et al '10, …]



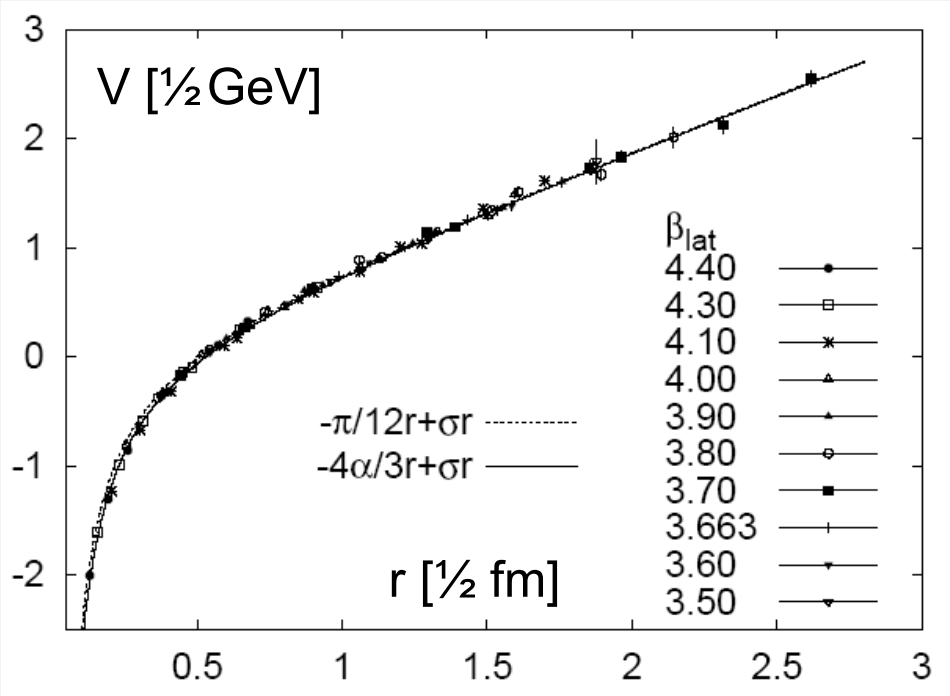
[He et al in prep.]

- hadronic resonances at ~T_c ↔ quark coalescence!
- **connects 2 "pillars" of RHIC: strong coupling + coalescence**

Diffusion Coefficient (~30 η/s)



1.) Introduction I: A "Calibrated" QCD Force



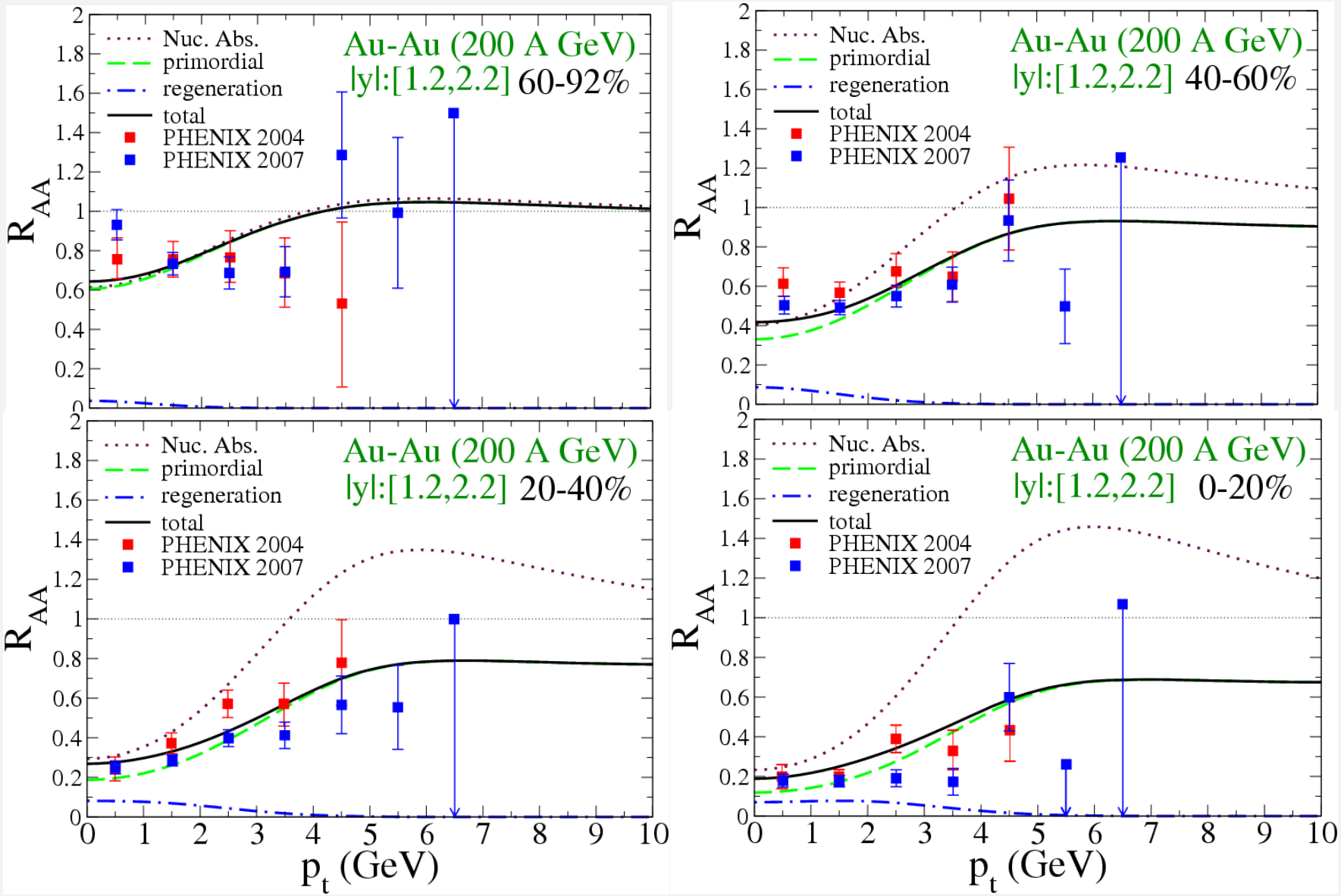
$$V_{Q\bar{Q}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$$

- **$\sigma \sim 1$GeV/fm** nonperturbative (gluonic condensate)
- **$V(r_0)=0 \Rightarrow r_0 \sim ¼$ fm $\sim (0.8\text{GeV})^{-1}$**

- Charm- + Bottomonium spectroscopy well described (effective potential theory, **$1/m_Q$** expansion)
- confining term crucial: **$E_B^{Coul}(J/\psi) \sim 0.05$ GeV** vs. **0.6 GeV expt.**
- **Medium modifications ↔ QCD phase structure (de-/confinement)**

[Matsui+Satz '86]

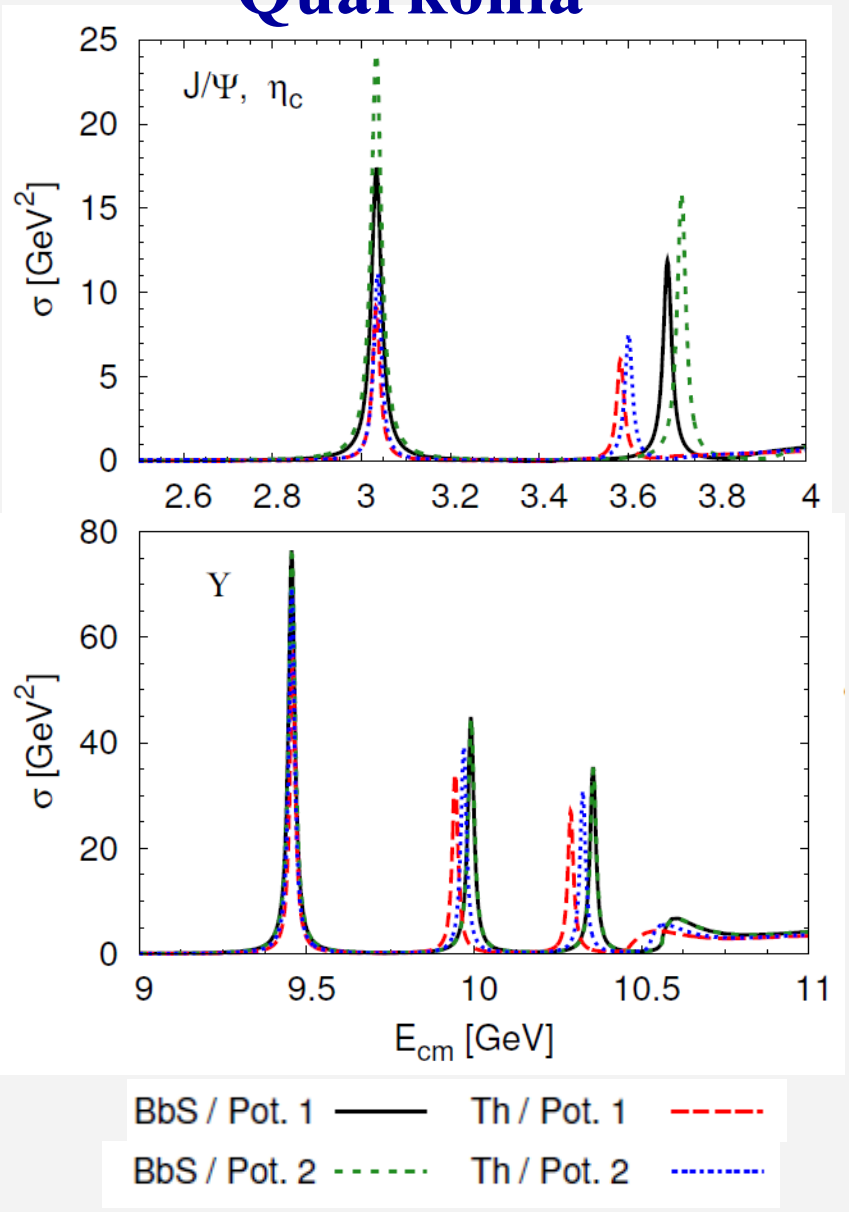
4.3 J/ψ at Forward Rapidity at RHIC



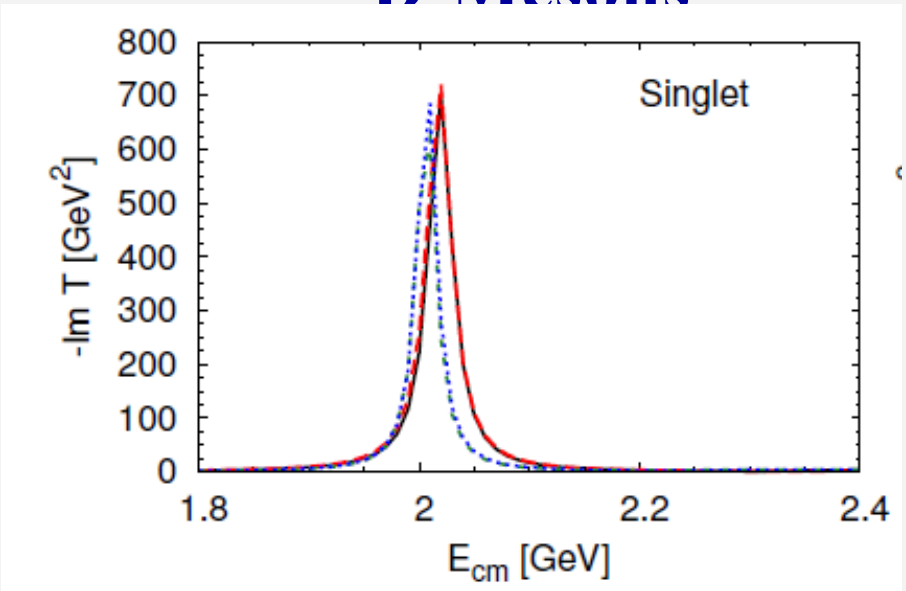
[Zhao+ RR '10]

2.4.1 Constraints I: Vacuum Spectroscopy

Quarkonia



D-Mesons



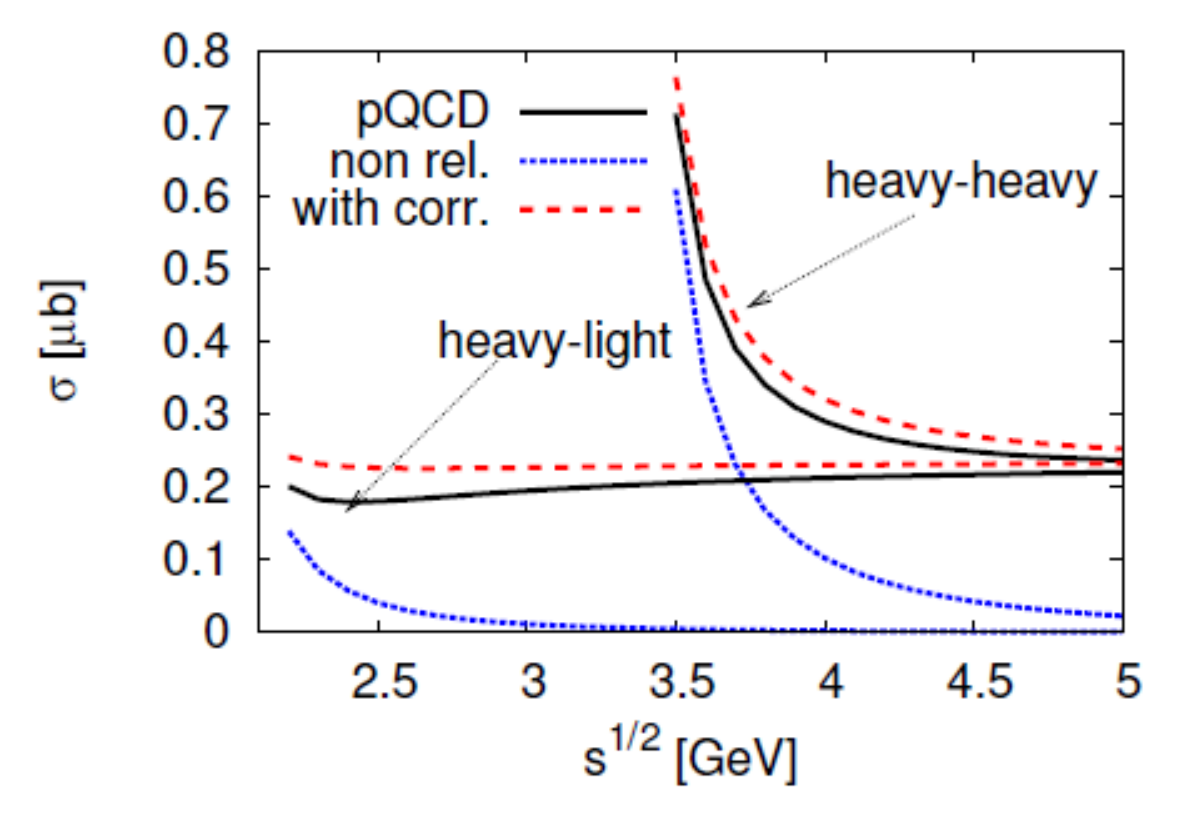
- no hyperfine splitting
- (bare) masses adjusted to ground state
- ~ **±50 MeV** accuracy

		BbS-scheme	Th-scheme
Potential 1	m_c^0	1.355 GeV	1.264 GeV
	m_b^0	4.712 GeV	4.662 GeV
Potential 2	m_c^0	1.402 GeV	1.293 GeV
	m_b^0	4.718 GeV	4.668 GeV

$m_q = 0.4$ GeV, $m_s = 0.55$ GeV

2.4.2 Constraints II: High-Energy Scattering

Born Approximation compared to Perturbative QCD



- Breit correction essential