T-Matrix Approach to Quarkonia in QGP

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1.) Introduction: **Heavy-Quark Interactions in Medium**

- **strong medium effects, nonpert.:**
  \[ F(r) > 0 \text{ for } T \leq 2T_c \]

- **momentum transfer**
  \[ q^2 = q_0^2 - \bar{q}^2 \approx -\bar{q}^2 \]
  \[ q_0 \sim \bar{q}^2 / 2m_Q << |\bar{q}| \]

- **single heavy quark in QGP**
  \[ p_{th}^2 \sim 2m_Q T >> T^2 \]

- **soft Q-\(\bar{Q}\) and Q-medium interactions static + elastic**

  => common description of quarkonia + heavy-quark transport
  requires bound + scattering states, resummations

  => thermodynamic **T-matrix** approach (e.m. plasmas, nucl. matter)
Outline

1.) **Introduction**

2.) **Thermodynamic T-Matrix**
   - Brueckner Theory of Quark/onia in QGP
   - Heavy-Quark Transport
   - Quarkonia Spectral Functions, Correlators + Widths

3.) **Heavy-Ion Phenomenology**
   - Transport Equation
   - Charmonia at SPS + RHIC
   - Predictions ($v_2$ at RHIC; LHC; $\Upsilon$)

4.) **Conclusions**
2.1 Two-Body Scattering Equation

- Lippmann-Schwinger equation \[ [\text{Mannarelli,Cabrera,Riek+RR '05, '06, '09}] \]

In-Medium

Q-\bar{Q} $T$-Matrix:

\[
T_\alpha(E;q,q') = V_\alpha(q,q') + \int k^2 dk \, V_\alpha(q,k) \, G^0_{Q\bar{Q}}(E,k) \, T_\alpha(E;k,q')
\]

- Q-\bar{Q} propagator:
  \[
  G^0_{Q\bar{Q}}(E,k;T) = T \sum_\nu D_Q(z_\nu,\bar{k})D_{\bar{Q}}(E-z_\nu,-\bar{k})
  \]

  \[
  \text{Im} G^0_{Q\bar{Q}}(E) = -\int \frac{d\omega}{2\pi} \left( \rho_Q(\omega)\rho_{\bar{Q}}(E-\omega)[1 - f^Q(\omega) - f^{\bar{Q}}(E-\omega)]
  + \rho_Q(\omega)\rho_Q(E+\omega)[f^Q(\omega) - f^Q(E+\omega)] \right)
  \]

- Vector channel ($j_0$: density) $\rightarrow$ HQ number susceptibility

  \[
  \chi_c(T) = -\frac{\partial^2 \Omega}{\partial \mu_c^2} = \frac{1}{T} \int \frac{dE}{2\pi} \frac{2}{1-\exp(-E/T)} \rho^{00}_V(E)
  \]

  determined by zero mode!
2.2 Brueckner Theory of Heavy Quarks in QGP

**Input**
- 2-body potential
- Quark selfenergy

**Process**
- Q → Q
  - 0-modes
- Q̅Q
  - T-matrix
- Qq
  - T-matrix

**Output**
- quark-no. susceptibility
  - spectral fcts./ eucl. correlat.
  - Q̅Q evolution (rate equation)
  - Q spectra + v₂ (Langevin)

**Test**
- lattice data
- exp. data
2.3 Field Theoretic Approach to Free Energy in QGP

- effective propagators: Coulomb + string
- fit 4 parameters to lattice-QCD data

\[ D_{00}(k) = \frac{\alpha_s^2}{k^2 + m_D^2} + \frac{m_G^2}{(k^2 + \tilde{m}_D^2)^2} \]

[Megias et al ‘07]

• Corrections to potential
  - Relativistic: magnetic “Breit” correction: [Brown et al ‘52, ‘05]
    \[ V_{Q1Q2}(r) \rightarrow V_{Q1Q2}(r) \left( 1 - v_1 \cdot v_2 \right) \quad \leftrightarrow \text{Poincaré-invariance, pQCD} \]
  - Retardation: - 4-D \rightarrow 3-D reduction of Bethe-Salpeter eq. (off-shell)

[Brown et al ‘52, ‘05]

[Megias et al ‘07]

[Brown et al ‘52, ‘05]

[Riek+RR ‘10]
2.4 Single Charm Quarks in QGP

**Thermal c-\bar{q} T-Matrix**

- meson/diquark resonances for $T < 1.5T_c$
- thermalization 4 (2) times faster using $U$ ($F$) as potential than pert. QCD
- finite-width effects in susceptibility

[Riek+RR ‘10]
2.5 Charmonia in QGP

- **U–potential**, selfconsist. **c**-quark width

- **Spectral Functions**
  - J/ψ melting at \( \sim 1.5T_c \)
  - \( \chi_c \) melting at \( \sim T_c \)
  - \( \Gamma_c \sim 100\text{MeV} \)

- **Correlator Ratios**
  - rough agreement with lattice QCD within uncertainties

[Mocsy+ Petreczky ’05+’08, Wong ’06, Cabrera+RR ’06, Beraudo et al ’06, Satz et al ’08, Lee et al ’09, Riek+RR ’10, …]
2.6 Charmonium Widths in QGP

\[ \Gamma_\psi = \sum_{p=q,\bar{q},g} \int \frac{d^3k}{(2\pi)^3} f^p(\omega_k;T) \sigma_{\psi\psi}^{diss}(s) \]

\[ \rightarrow \text{sensitive to binding energy (i.e., color screening)} \]

- \( E_B \geq T \): gluo-dissociation \hspace{1cm} [Bhanot+Peskin ’79]

- \( E_B < T \): quasi-free dissociation \hspace{1cm} [Grandchamp+RR ’01]

\[ \frac{\Gamma_{\psi}}{T} \]

\[ \alpha_s \sim 0.25 \]

\[ \text{J/\psi lifetime} \sim 1-4 \text{ fm/c} \]
2.6.2 Relation of Quarkonium Widths to EFT

- Singlet-to-octet transition
- Landau damping

![Diagram showing the relation between quarkonium widths and EFT mechanisms, including singlet-to-octet transition and Landau damping.]

- Gluon dissociation
- Quasi-free dissociation
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4.) **Conclusions**
3.) Quarkonium Production in URHICs

- measured in dilepton channel

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- spectra do not reflect in-med. spectral function/thermal dilepton rate!
- relation of yields, $p_t$-spectra + $v_2$ to spectral properties more involved…
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“melting” of $Y'$, $Y''$ !?
3.1 Transport Approach to Quarkonium Evolution

• **Regeneration in QGP + HG:**

  \[
  \text{detailed balance: } \quad J/\psi + g \leftrightarrow c + \bar{c} + X
  \]

  \[
  \frac{dN_{\psi}}{d\tau} = -\Gamma_{\psi} \left( N_{\psi} - N_{\psi}^{eq} \right)
  \]

  - reaction rate (\(\psi\) -width)
  - equilibrium limit \(dN_c / dp_T, m_\psi, m_\bar{c}^*\)

• **Rate Equation:**

• **Input from Thermodynamic T-Matrix (weak/strong binding)**

[References: PBM et al ’01, Gorenstein et al ’02, Thews et al ’01, Grandchamp+RR ’01, Ko et al ’02, Cassing et al ’03, Zhuang et al ’05, …]
3.2 Inputs and Parameters

**Input**
- $J/\psi (\chi_c, \psi')$, $c\bar{c}$ production cross sections, $b$ feeddown [p-p data]
- “Cold Nuclear Matter”: shadowing, nuclear absorption, $p_t$ broadening [p/d-A data, shad. est.]
- Thermal fireball evolution: thermalization time ($\leftrightarrow$ initial $T_0$), expansion rate, lifetime, $T_c$, freezeout … [A-A hadron data, hydrodynamics]

**Parameters**
- strong coupling $\alpha_s$ controls $\Gamma_{\text{diss}}$
- schematic $c$-quark off-equilibrium:
  $N_{\psi}^{\text{eq}}(\tau) \sim N_{\psi}^{\text{therm}}(\tau) \cdot [1 - \exp(-\tau/\tau_c^{\text{eq}})]$
3.3 Inclusive J/ψ in Thermal Media at SPS + RHIC

- thermal rate equation through QGP/Tc/HG for J/ψ, χc, ψ’
- correlation volume
- bottom feeddown
- formation-time effect
- spectral properties $\leftrightarrow \Gamma_\psi, N_\psi^{eq}$

- 2 parameters ($\alpha_s \sim 0.3$, charm relax. $\tau_c^{eq} = 6(3) \text{ fm/c}$)
- different composition in two scenarios

[U-Potential]

[U-Potential]

[F-Potential]
3.4 $J/\psi$ Transverse-Momentum Spectra at RHIC

- Cronin and formation-time effects essential
- slight underestimate at high $p_t$
- similar results for weak-binding scenario (F-potential)

[Zhao+RR ‘08]
3.5 J/$\psi$ Predictions I: Elliptic Flow at RHIC

**Strong Binding**

- Au-Au (200 A GeV)
- $p_T$<0.35
- 20-40%

**Weak Binding**

- Au-Au (200 A GeV)
- $p_T$<0.35
- 20-40%

- Small $v_2$ limits regeneration, but does not exclude it [Zhao+RR '08]
3.6 J/ψ Predictions II: LHC inclusive – ALICE

Mid-Rapidity

Forward Rapidity

- regeneration component increases, still net suppression
- confirmed within main uncertainty of input (shadowing) …
3.7 J/ψ Predictions III: LHC high-p_t – ATLAS+CMS

- underestimate for peripheral (expected from RHIC) (spherical fireball reduces surface effects …)

[Zhao+RR ‘11]
3.8 $\Upsilon$ Predictions at RHIC and LHC

### Weak Binding

- RHIC

### Strong Binding

- LHC

[Grandchamp et al '06, Emerick et al in prep]

- sensitive to screening + early times; regeneration at LHC
4.) Conclusions

• Common framework for quarkonia + heavy-quark transport: thermodynamic T-matrix

• Connect HQ(onium) theory with phenomenology

• Thermal rate-equation approach adequate to describe (+ predict) existing quarkonium data in heavy-ion collisions

• Preference for “strong binding scenario” (also works better for HQ diffusion)

• Open Problems
  - in-medium potential
  - microscopic gain term (realistic HQ spectra)
  - CNM effects at LHC (shadowing, ...), high-$p_T$ dynamics ...

Time for a more systematic comparison of approaches!
4.3 Y(1S) Evolution at RHIC and LHC

- “strong binding”
- “weak binding”

- sensitive to screening + early times; regeneration at LHC

[Grandchamp et al ‘06]
2.3 Single Heavy-Quark Spectral Fct. + Transport

- **Spectral Function (propagator)**
  \[ \rho_Q = -2 \text{Im} D_Q = -2 \text{Im} \frac{1}{\omega - \omega_Q(k) - \Sigma_Q(\omega,k)} \]

- **Scattering Rate (selfenergy)**
  \[ \text{Im} \Sigma_Q(\omega,k) = \int \text{Im} T_{Qq}(\omega + \omega_p) f^q(\omega_p) \]

- **Heavy-Quark Transport**

  - Brownian Motion:
    \[ \frac{\partial f}{\partial t} = \gamma \frac{\partial (pf)}{\partial p} + D \frac{\partial^2 f}{\partial p^2} \]
    thermalization rate
    diffusion coefficient
    \[ T = \frac{D}{\gamma m_Q} \]

  - Fokker Planck Eq.

[Svetitsky ’88, Mustafa et al ’98, Hees+RR ’04, Teaney+Moore ’04, Gubser ‘07, Peshier ’09, Alam et al ’09, …]
2.4 Heavy-Quark Free Energy in Lattice QCD

\[ F_1(r,T) = U_1(r,T) - T S_1(r,T) \]

- Potential?!
  
  (a) Free energy \( F_1 \)
  
  \[ \rightarrow \text{weak} \ \bar{Q}Q \text{ potential, small } Q \text{ “selfenergy” } F_1(r=\infty,T)/2 \]

  (b) Internal Energy \( U_1 \) (\( U = \langle H_{\text{int}} \rangle \))
  
  \[ \rightarrow \text{strong} \ \bar{Q}Q \text{ potential, large } Q \text{ “selfenergy” } U_1(r=\infty,T)/2 \]

  \[ \rightarrow \text{compensation in eucl. correlators} \]

- \( F, U, S \) thermodynamic quantities
  
  (0-point functions), potential 4-point function

[Kaczmarek+Zantow ’05]
4.4 Heavy-Quark Transport + e± Spectra at RHIC

\( e^\pm \) Decays from c/b Langevin in Hydro

- hadronic resonances at \( \sim T_c \) ↔ quark coalescence!
- connects 2 “pillars” of RHIC: strong coupling + coalescence

\[ \text{T-matrix interaction} \]

[He et al. in prep.]
1.) Introduction I: A “Calibrated” QCD Force

- $\sigma \sim 1\text{GeV/fm}$ nonperturbative (gluonic condensate)
- $V(r_0)=0 \Rightarrow r_0 \sim 0.8\text{GeV}^{-1}$

- Charm- + Bottomonium spectroscopy well described (effective potential theory, $1/m_Q$ expansion)

- Confining term crucial: $E_B^{\text{Coul}}(J/\psi) \sim 0.05\text{ GeV} \text{ vs. } 0.6\text{ GeV expt.}$

- Medium modifications $\leftrightarrow$ QCD phase structure (de-/confinement)

$V_{Q\bar{Q}}(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r$

[Matsui+Satz ‘86]
4.3 $J/\psi$ at Forward Rapidity at RHIC

[Zhao+ RR ’10]
2.4.1 Constraints I: Vacuum Spectroscopy

Quarkonia

- no hyperfine splitting
- (bare) masses adjusted to ground state
- \( \pm 50 \text{ MeV} \) accuracy

D-Mesons

- no hyperfine splitting
- (bare) masses adjusted to ground state
- \( \pm 50 \text{ MeV} \) accuracy

<table>
<thead>
<tr>
<th>Potential</th>
<th>BbS-scheme</th>
<th>Th-scheme</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.355 GeV</td>
<td>1.264 GeV</td>
</tr>
<tr>
<td>2</td>
<td>1.402 GeV</td>
<td>1.293 GeV</td>
</tr>
</tbody>
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\( m_q = 0.4 \text{ GeV}, m_s = 0.55 \text{ GeV} \)
2.4.2 Constraints II: High-Energy Scattering

Born Approximation compared to Perturbative QCD

- Breit correction essential