QUARKONIA IN DECONFINED MATTER

September 28 - 30, 2011 Acitrezza (Italy)

Progress in lattice QCD relevant for charmonium physics

- From charmonium-hadron interaction to structure of charmonia -

Shoichi Sasaki (Univ. of Tokyo)

SCHOOL OF SCIENCE
THE UNIVERSITY OF TOKYO

Recent studies

✳ Charmonium-hadron interaction

- J/ψ-hadron elastic scattering at low energy
- Charmonium-nucleon potential

✳ Structure of charmonia

- Interquark potential at finite quark mass
- Charmonium potential from full QCD

✳ In-medium heavy quark potential

- Complex potential at finite temperature

➡ A. Rothkopf's talk (1st day)

Contents of this talk

✳ Charmonium-hadron interaction

- J/ψ-hadron elastic scattering at low energy
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✳ Structure of charmonia

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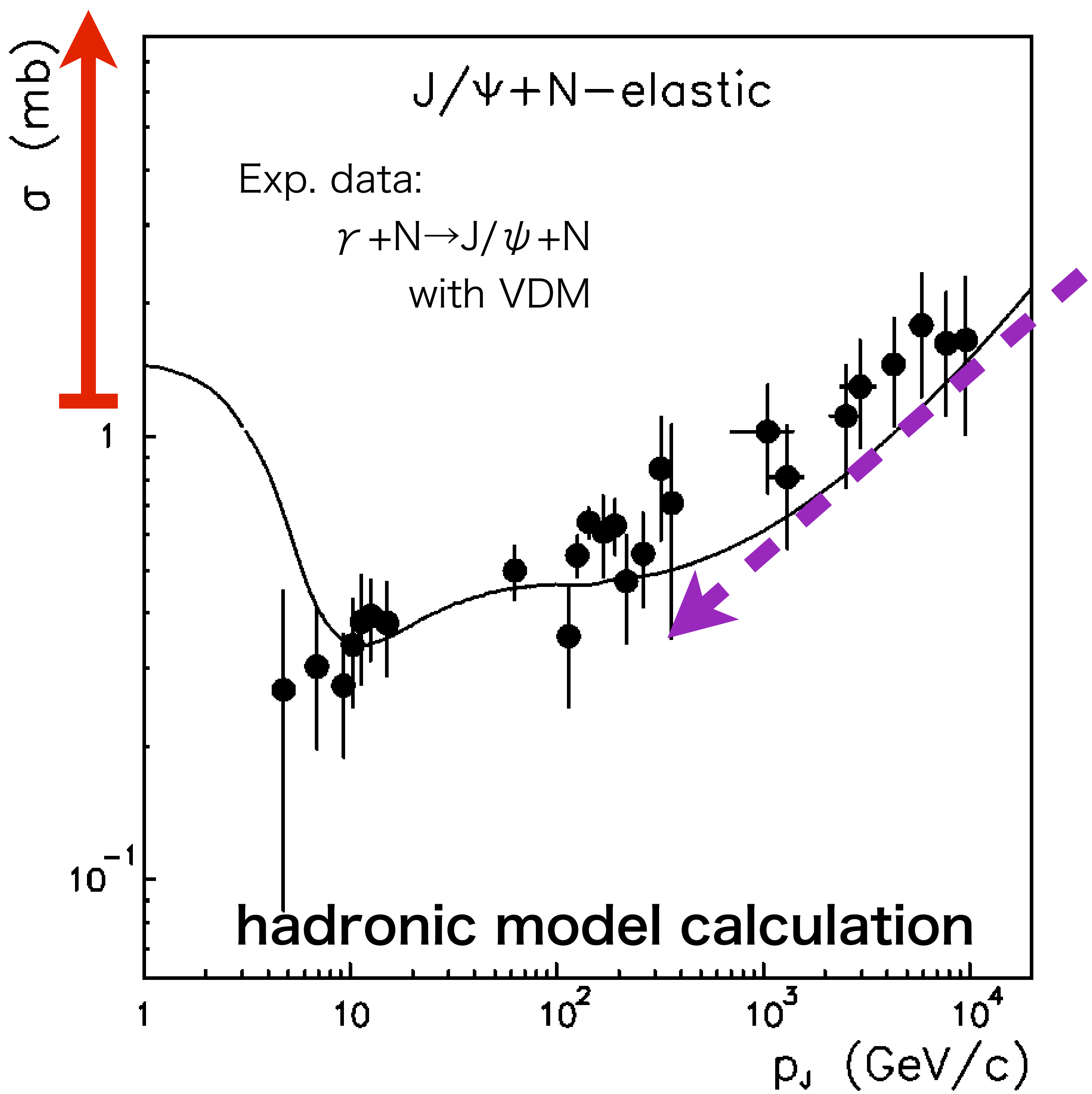
Charmonium-hadron interaction

Total elastic J/ψ-N cross section



Sibirtsev et al. PRC 63 (2001) 044906

at high energies

Regge theory

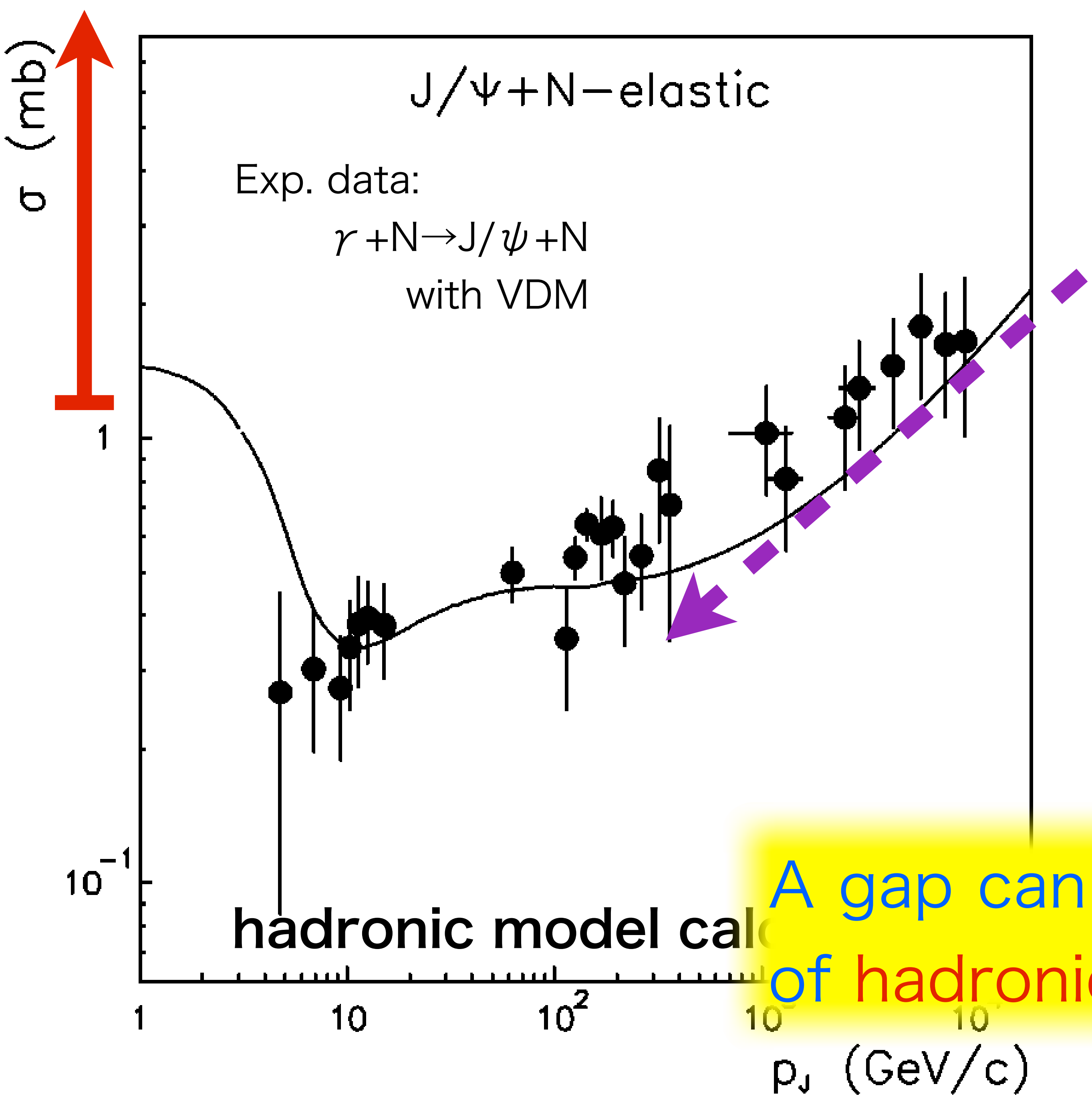
Donnachie-Landshoff, PLB470 (1999) 243

at low energy

$$\lim_{p_J \to 0} \sigma_{\rm el}(p_J) = 4\pi a^2_{J/\psi N} \sim 1 \text{ mb}$$

scattering length

Total elastic J/ψ-N cross section



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Regge theory

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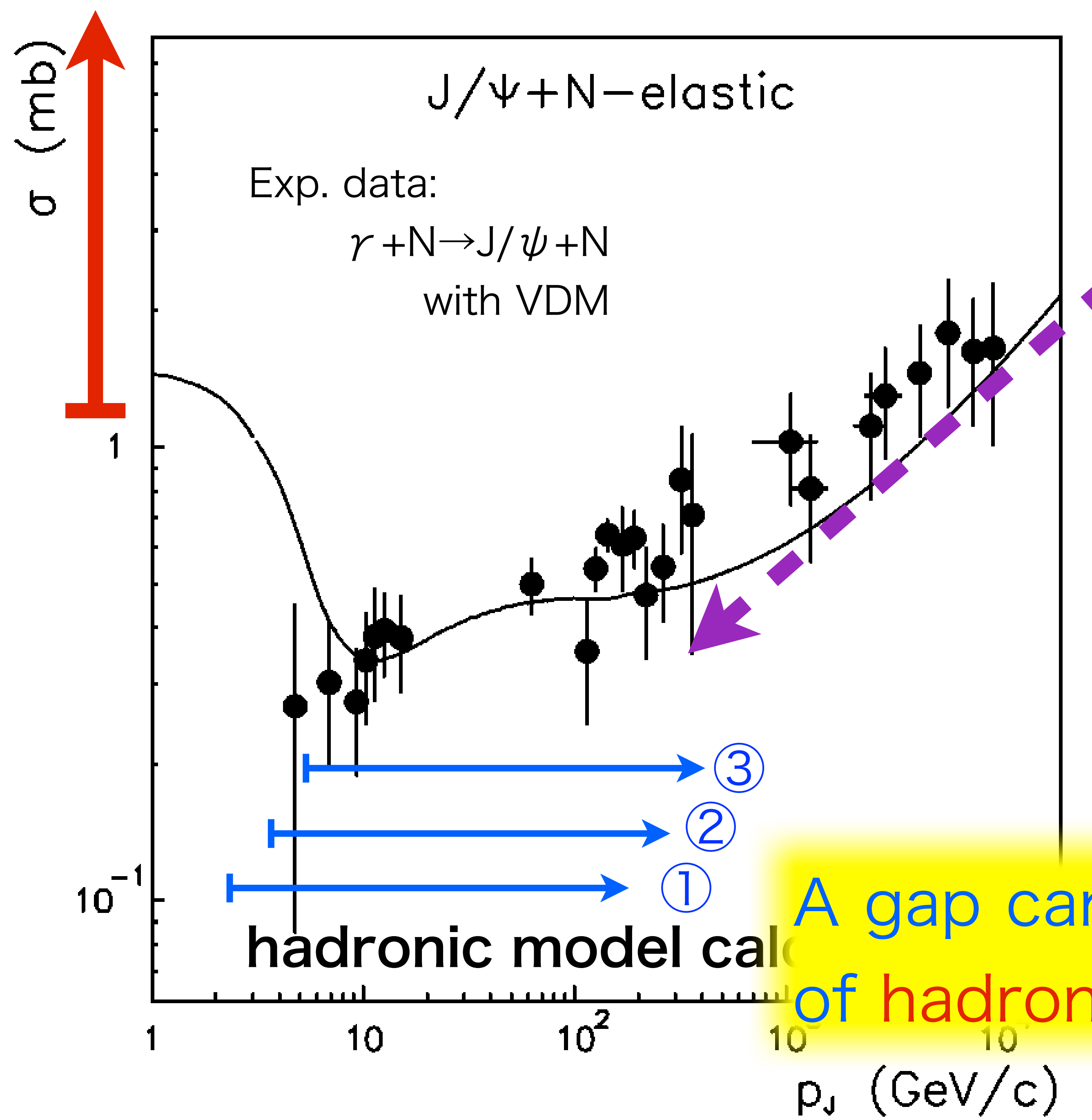
$$\lim_{p_J \to 0} \sigma_{\mathrm{el}}(p_J) = 4\pi a^2_{J/\psi N} \sim 1\ \mathrm{mb}$$

scattering length

A gap can be modulated by the strength of hadronic J/ψ dissociation

Total elastic J/ψ-N cross section



① $p_J > 1.88$ GeV/c

$$J/\psi + N \to \Lambda_c + \bar{D}$$

② $p_J > 2.92$ GeV/c

$$J/\psi + N \to \Lambda_c + \bar{D}^*$$

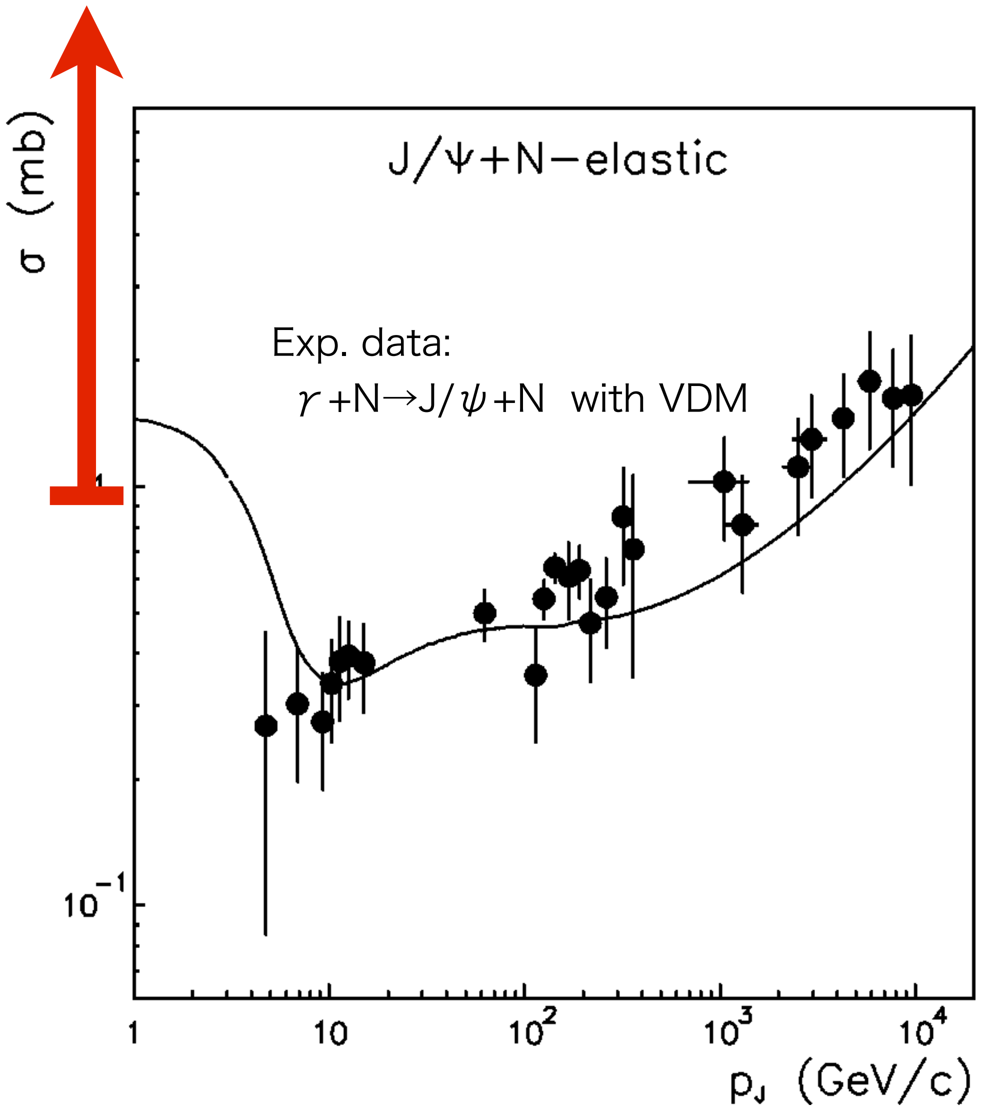
③ $p_J > 5.24$ GeV/c

$$J/\psi + N \to N + D + \bar{D}$$

A gap can be modulated by the strength of hadronic J/ψ dissociation

Sibirtsev et al. PRC 63 (2001) 044906

Total elastic J/ψ-N cross section

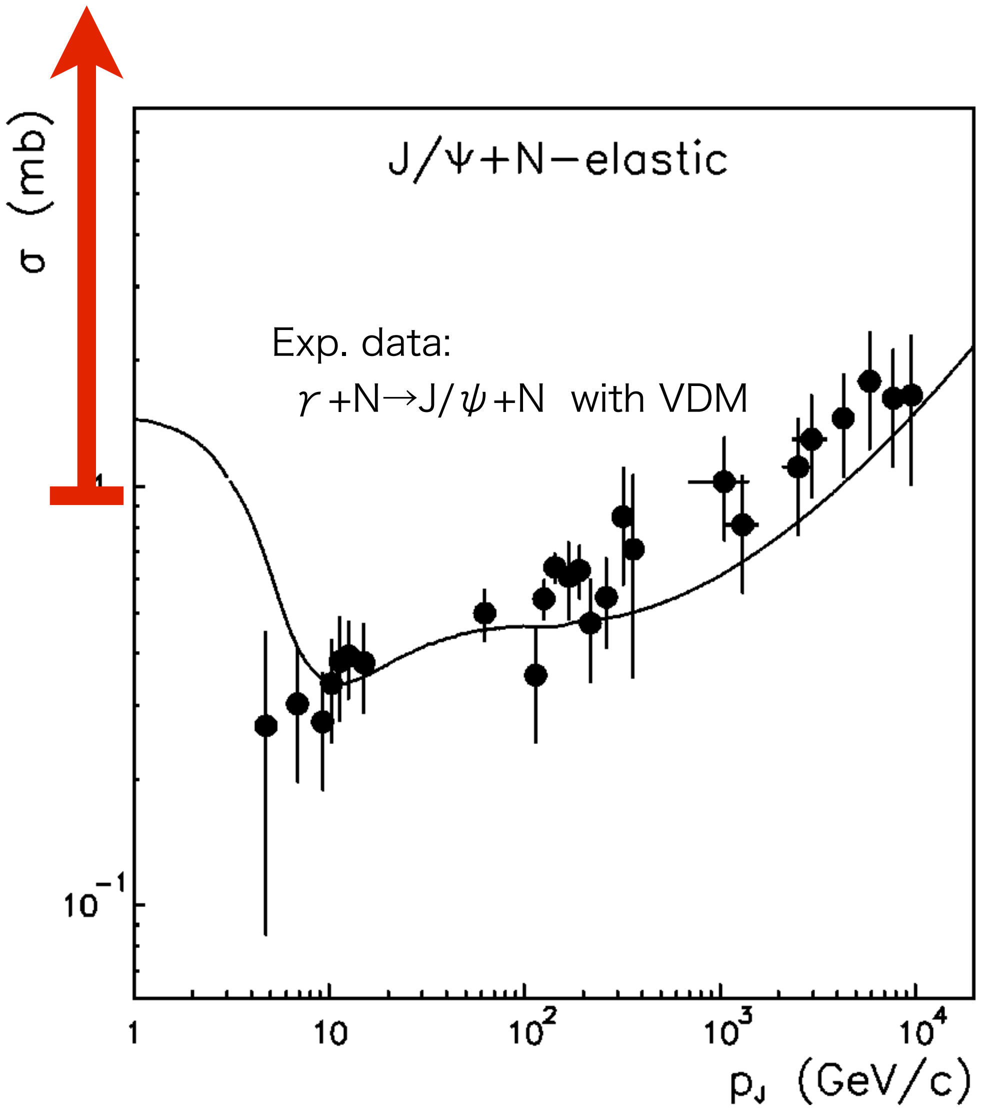


$$\lim_{p_J \to 0} \sigma_{\rm el}(p_J) = 4\pi a^2_{J/\psi N}$$

	$a_{J/\psi N}$	$\sigma_{\rm el}$
QCDSR	0.1 fm	1.3 mb
van der Waals interaction	0.25 fm	7.9 mb
QCD multipole expansion	> 0.37 fm	17 mb

- QCDSR: A. Hayashigaki, Prog. Theor. Phys. 101 (1999) 923.
- gluonic van der Waals interaction: S. Brodsky and G.A. Miller, Phys. Lett. B412 (1997) 125.
- QCD mulutipole expansion: A. Sibirtsev and M.B. Voloshin, Phys. Rev. D71 (2005) 076005.

Total elastic J/ψ-N cross section



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Quench LQCD	0.71(48) fm	64 mb

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- Quench Lattice QCD: K. Yokokawa, S.S., T. Hatsuda, A. Hayashigaki, Phys. Rev. D74 (2006) 034504.

Special features of cc^{bar}-hadron interaction

- No quark exchange
  - The cc^{bar} states do not share the same quark flavor with the light hadrons

- Mainly multiple-gluon exchange
  - Interaction is induced by the genuine QCD effect
  - color van der Waals type force, which should be weakly attractive.

Finite size effect on two-particle total energy

$$E_{2-\mathrm{body}} = M_1 + M_2 + \Delta E \neq M_1 + M_2$$

Scattering length

$$M_{\mathrm{red}} = M_1 M_2 / (M_1 + M_2)$$

$$\Delta E = \boxed{-\frac{2\pi a_0}{M_{\mathrm{red}} L^3}} \left[1 + c_1 \frac{a_0}{L} + c_2 \left(\frac{a_0}{L} \right)^2 \right] + \mathcal{O}(L^{-6})$$

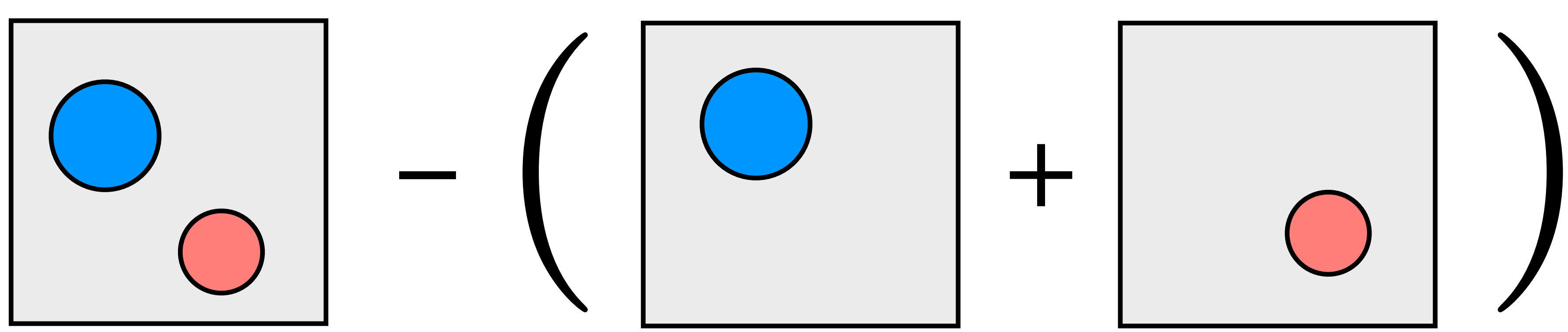
Universal

Volume

depends on volume-shape
and boundary condition

L^3 box+Periodic BC :
Lüscher (86)

ΔE =



Lüscher's finite size method, Commun. Math. Phys. 104, 177 (1986)

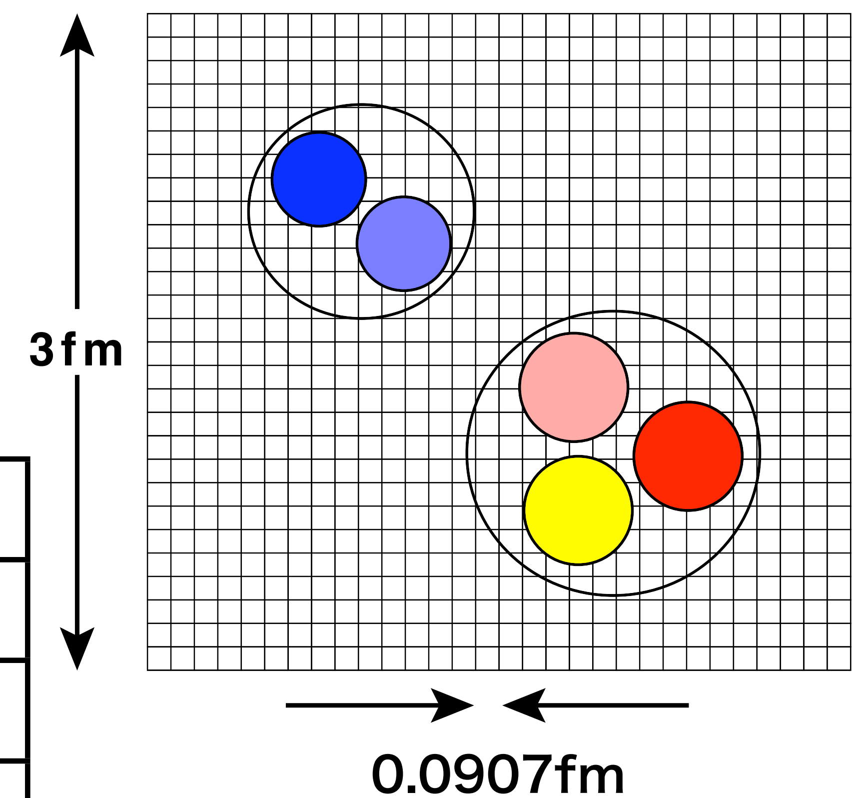
Simulation setup

- 2+1 flavor PACS-CS gauge configurations
- Lattice size : $L^3 x T = 32^3 x 64$ at $1/a = 2.17$ GeV
- Iwasaki gauge action

  +NP Clover fermions (u,d quarks)

  +RHQ action (charm quark)
- charm: $\kappa_{charm} = 0.106787$, ($m_{J/\psi} \sim 3.0$ GeV)
- light:

κ	0.13770	0.13754	0.13727	0.13700
m_π [GeV]	0.29	0.41	0.57	0.70
m_N [GeV]	1.08	1.19	1.40	1.57
Statistics	799	450	400	399



4-point correlator of two hadron states

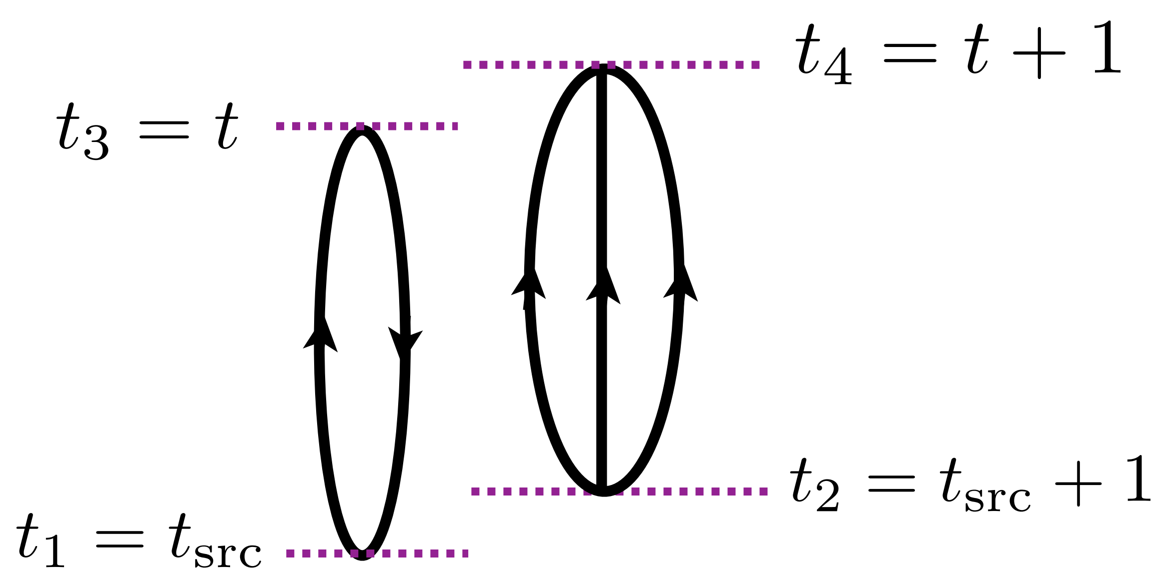
4-point correlator

$$G^{h_1\text{-}h_2}(t_4, t_3; t_2, t_1) = \langle \mathcal{O}^{h_1}(t_4)\mathcal{O}^{h_2}(t_3) \left(\mathcal{O}^{h_1}(t_2)\mathcal{O}^{h_2}(t_1)\right)^\dagger \rangle$$

$$\mathcal{O}^h(t) = \sum_{\mathbf{x}} \mathcal{O}^h(\mathbf{x}, t)$$ projected on the lowest mode

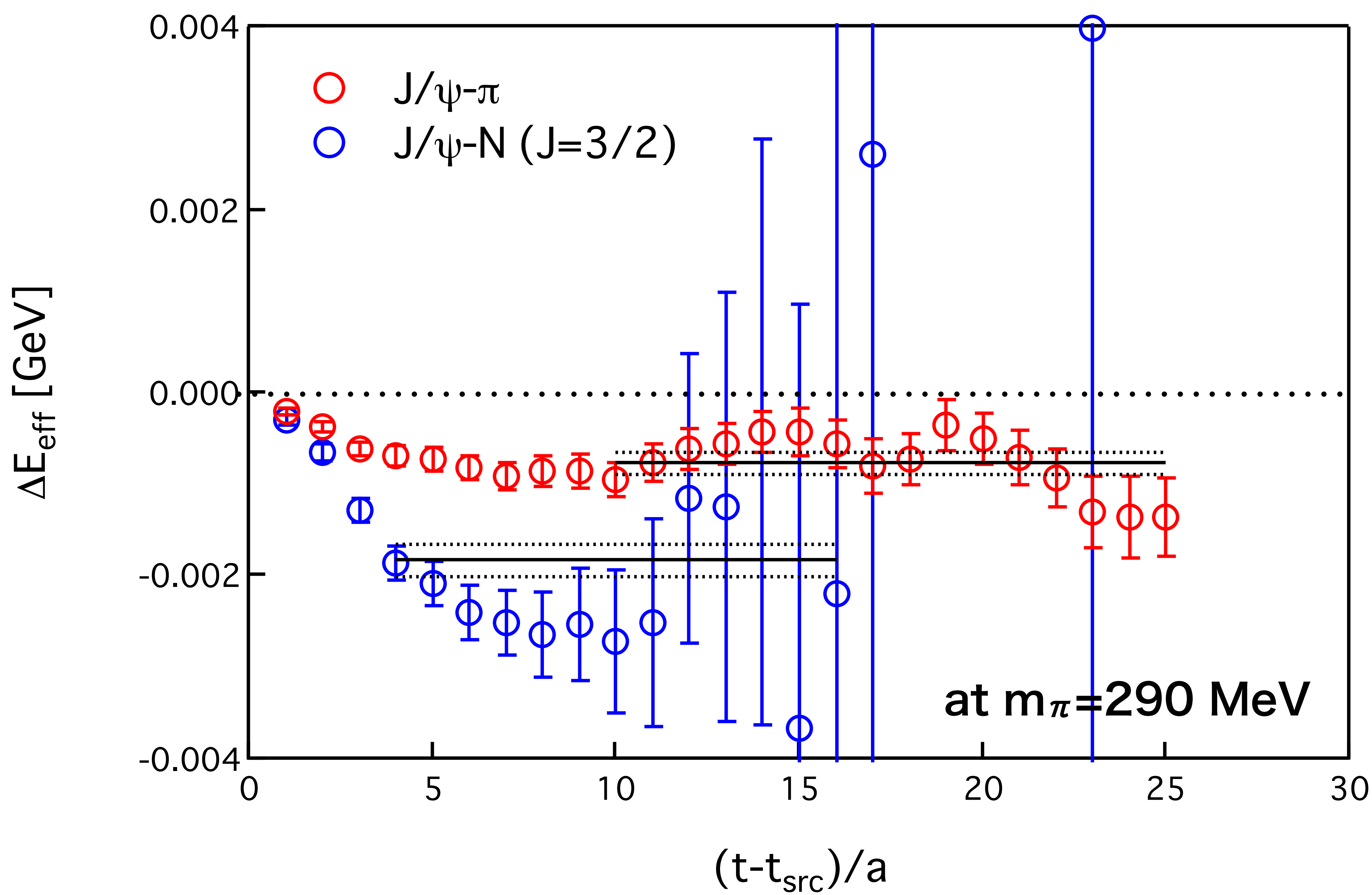
2-point correlator

$$G_h(t, t_{\rm src}) = \langle \mathcal{O}^h(t)\mathcal{O}^h(t_{\rm src})\rangle$$



$$R_{h_1\text{-}h_2}(t) = \frac{G_{h_1\text{-}h_2}(t, t_{\rm src})}{G_{h_1}(t, t_{\rm src})G_{h_2}(t, t_{\rm src}+1)} \to \exp(-\Delta E \cdot t)$$

Measurement of Energy Shift ΔE



J/ψ-π and J/ψ-N channels clearly exhibit
attractive interaction

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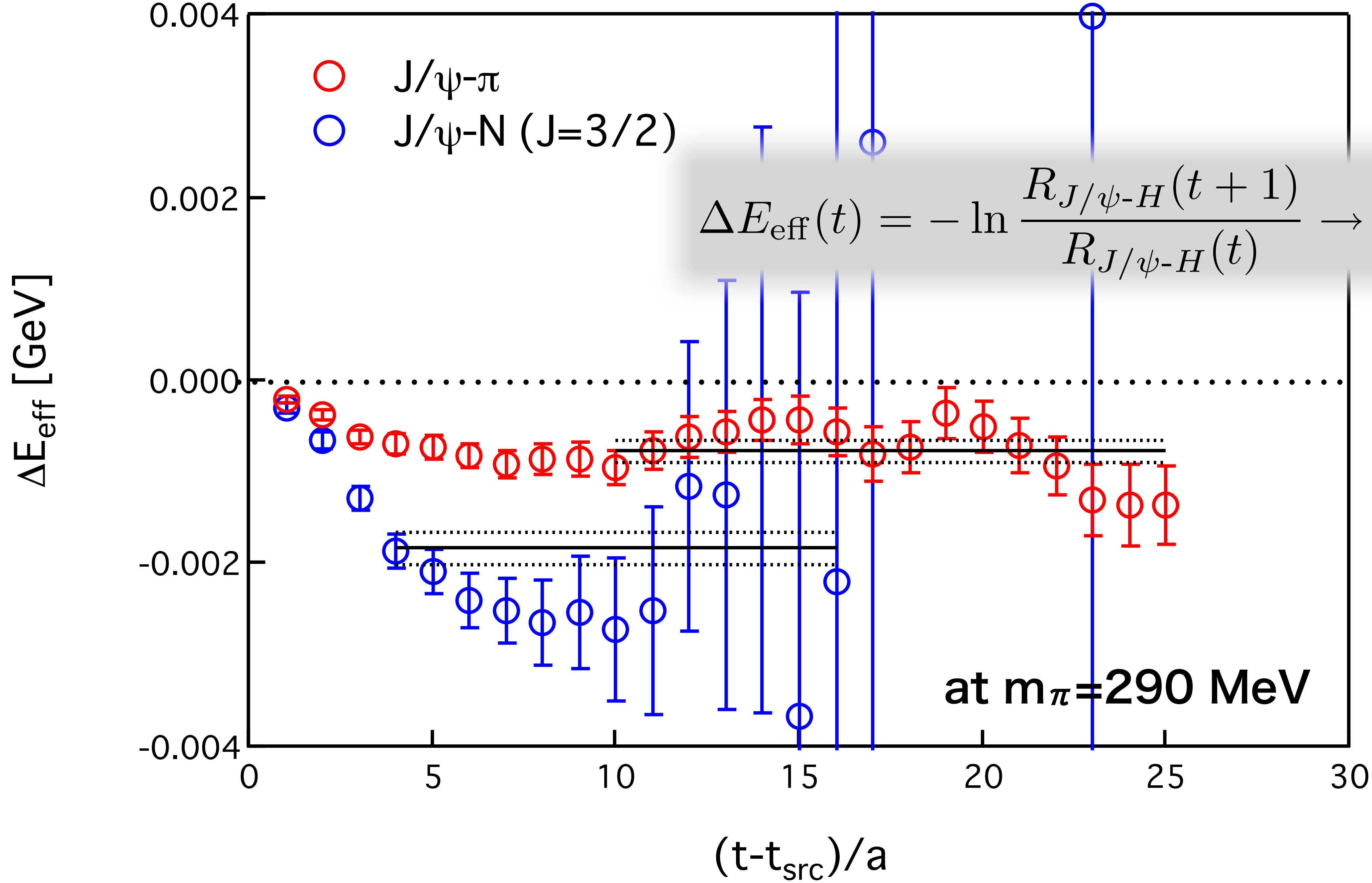

$$\Delta E_{\rm eff}(t) = -\ln \frac{R_{J/\psi\text{-}H}(t+1)}{R_{J/\psi\text{-}H}(t)} \rightarrow \Delta E \quad \text{for } t \gg t_{\rm src}$$

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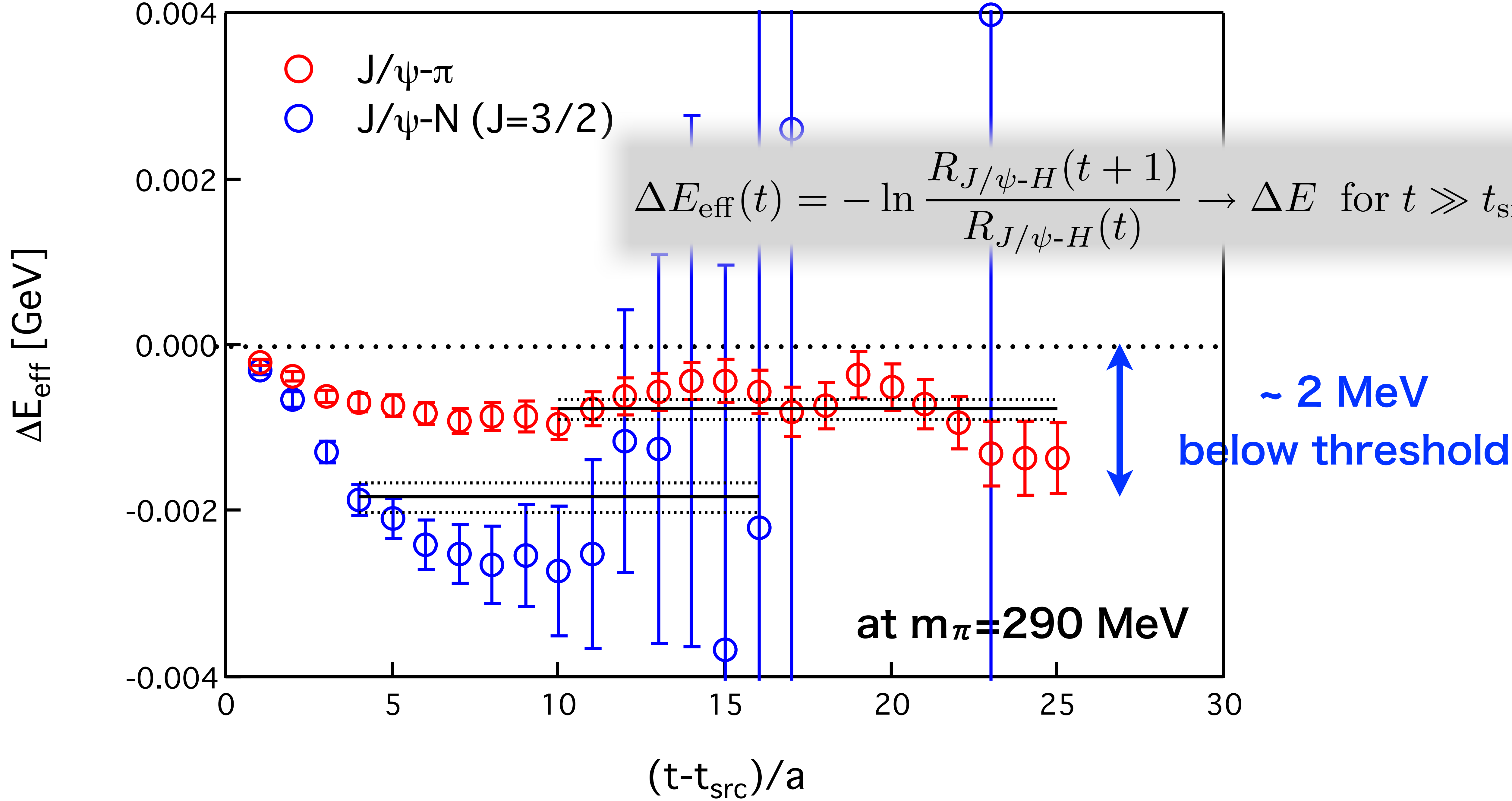

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Does J/ψ-N(π) channel form a shallow bound state ?

What is a bound state?

S.S and T. Yamazaki, Phys. Rev. D74 (2006) 114507

In quantum mechanics,

- The energy spectrum of a bound state is discrete.
- Their energy is negative (below threshold).

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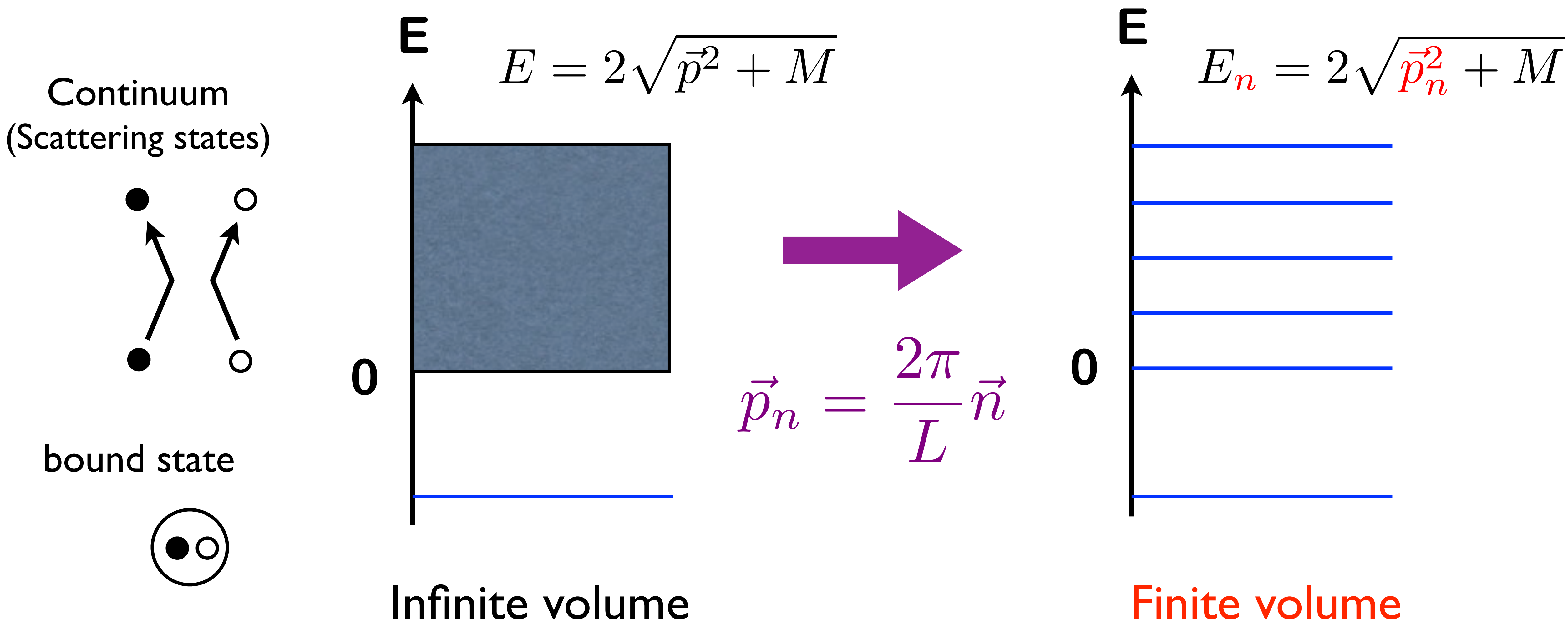

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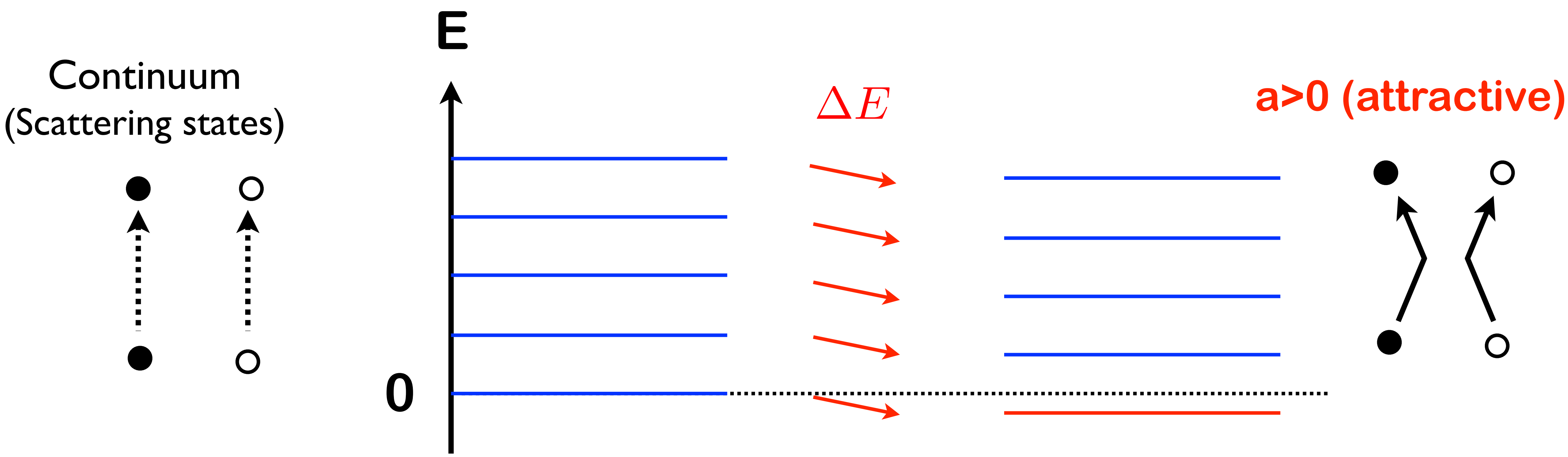

Lüscher Formula for two-body system:

$$\Delta E = -\frac{2\pi a}{\mu L^3}\left\{1 + c_1 \frac{a}{L} + c_2 \frac{a^2}{L^2}\right\}$$

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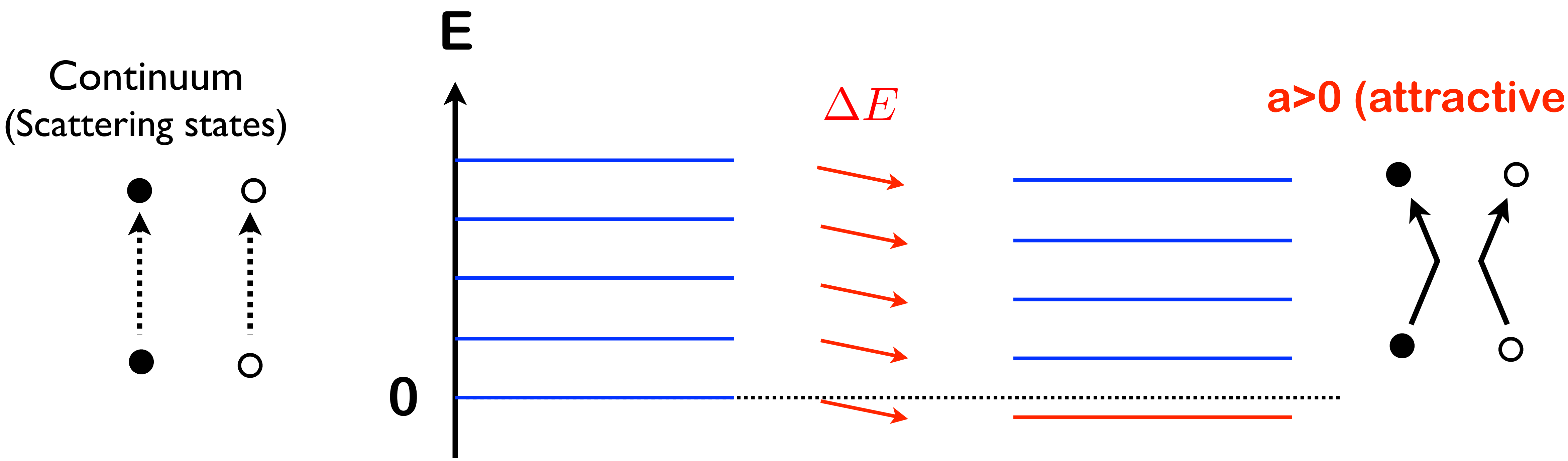


How can we distinguish the shallow bound state from the lowest energy level of the scattering state in the finite volume simulation?

Bethe-Salpeter (BS) wave function

- One-particle correlation (2pt-function)

$$\langle 0|\phi(\vec{x},t)\phi(\vec{0},0)|0\rangle = \sum_n \langle 0|\phi(\vec{x})|E_n\rangle e^{-E_n \cdot t} \underbrace{\langle E_n|\phi(\vec{0})|0\rangle}_{\text{constant}}$$

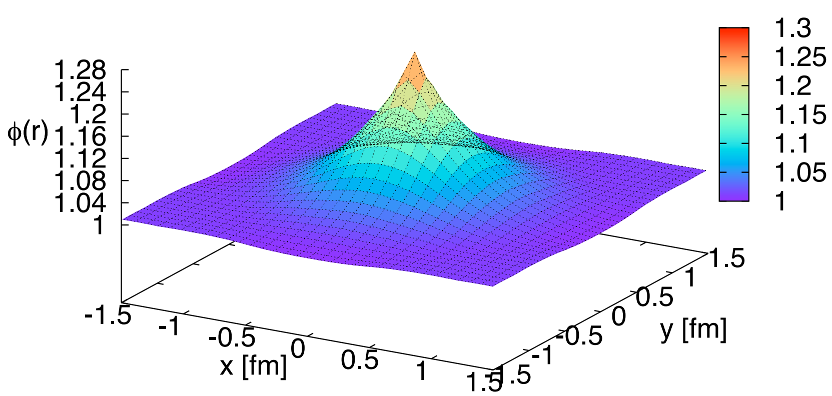
- Two-particles correlation (4pt-function)

$$\psi(\vec{x},\vec{y};t) = \phi_1(\vec{x},t)\phi_2(\vec{y},t)$$

$$\langle 0|\psi(\vec{x},\vec{y};t)\psi(\vec{0},\vec{0};0)|0\rangle = \sum_n \boxed{\langle 0|\psi(\vec{x},\vec{y})|E_n\rangle} e^{-E_n \cdot t} \underbrace{\langle E_n|\psi(\vec{0},\vec{0})|0\rangle}_{\text{constant}}$$

$$= \varphi_{E_n}(\vec{x},\vec{y})$$

equal-time BS amplitude



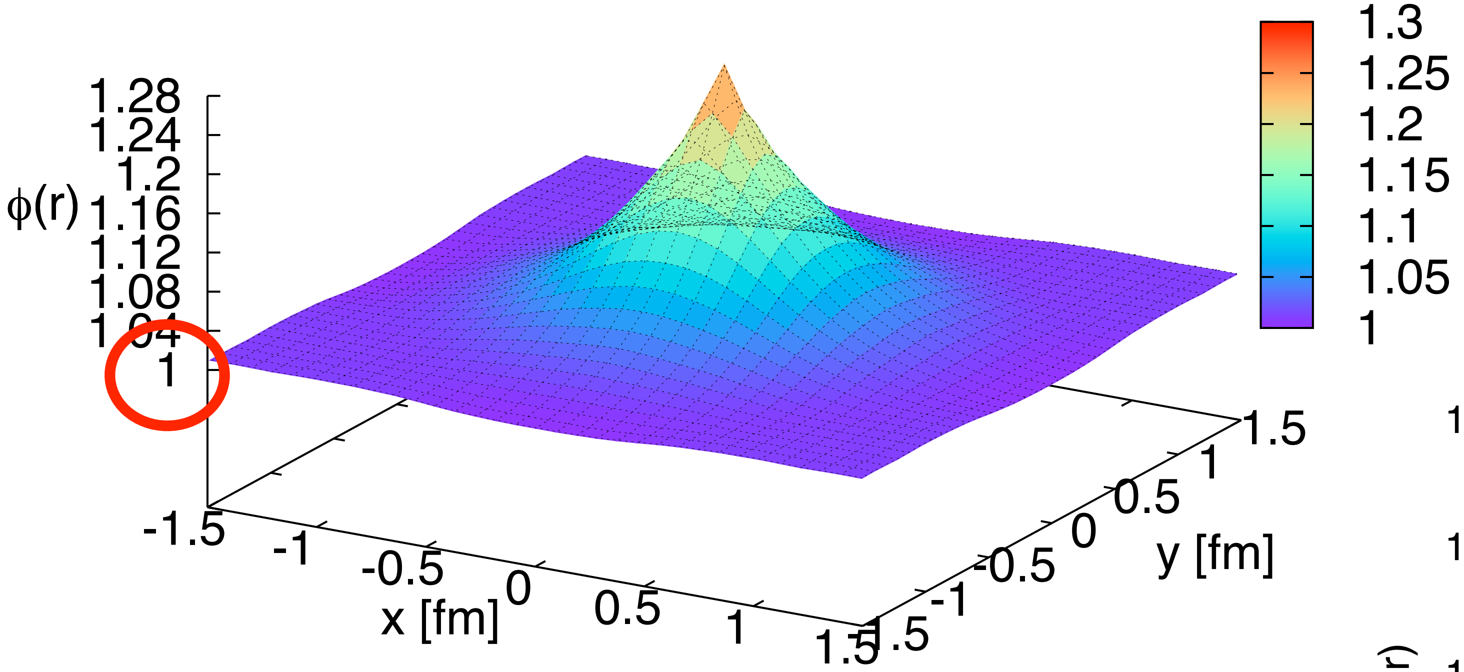
relative coordinate part

$$\vec{r} = \vec{x} - \vec{y}$$

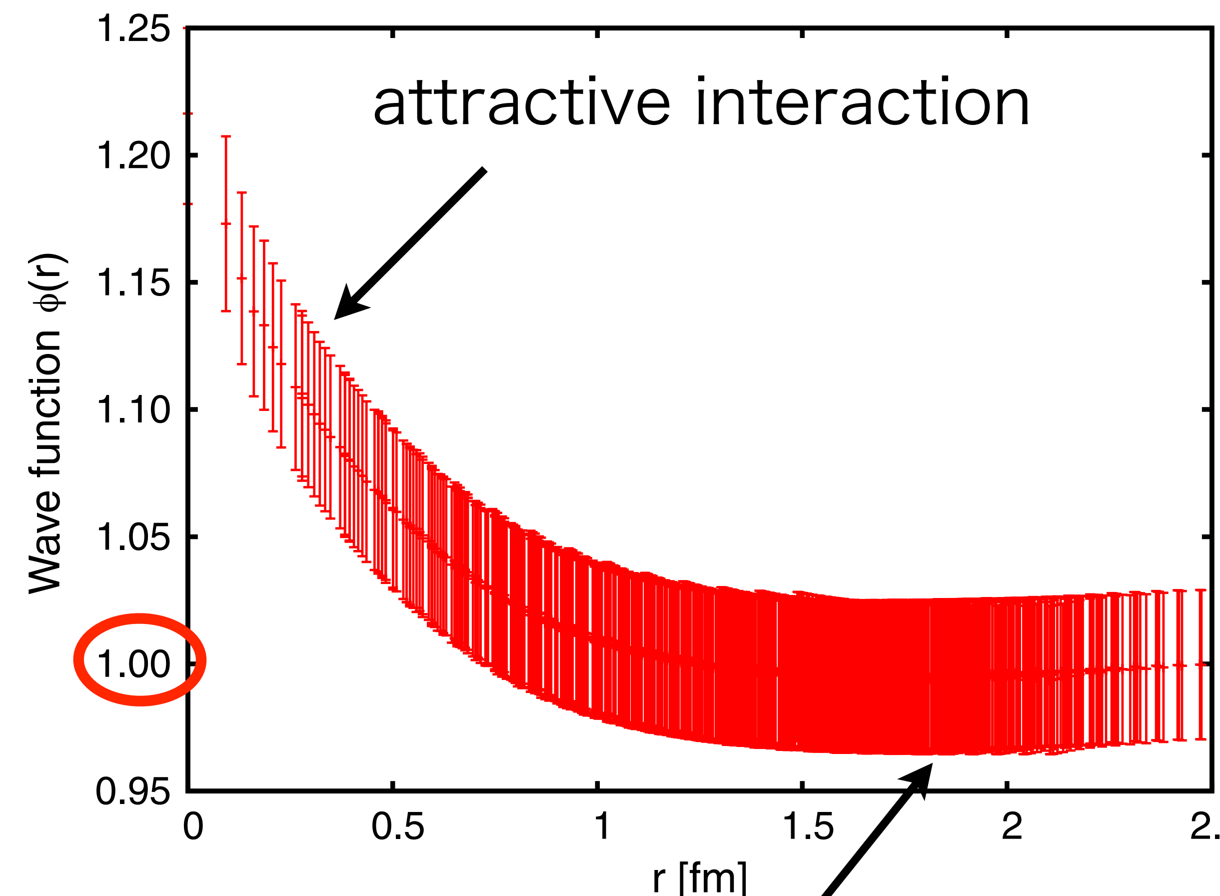
charmonium-hadron BS wave function

T. Kawanai, SS, Phys. Rev. D82 (2010) 091501

s-wave relative wave function



attractive interaction



Q: localized or extended ?

A: extended !

asymptotically plane wave at long distances

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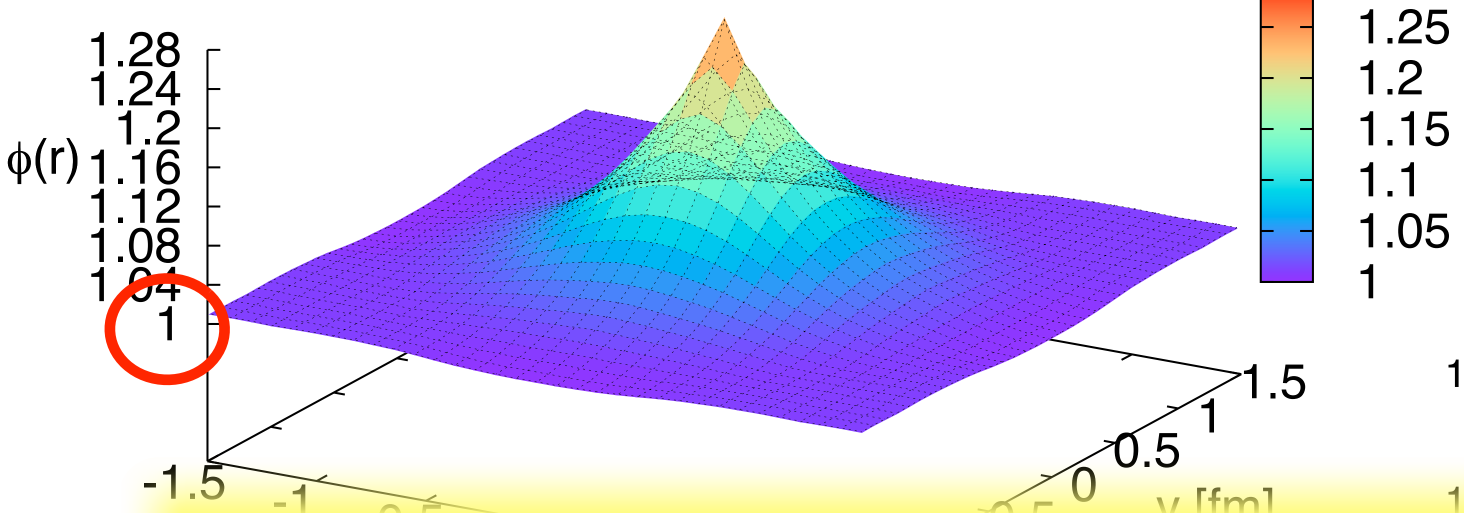
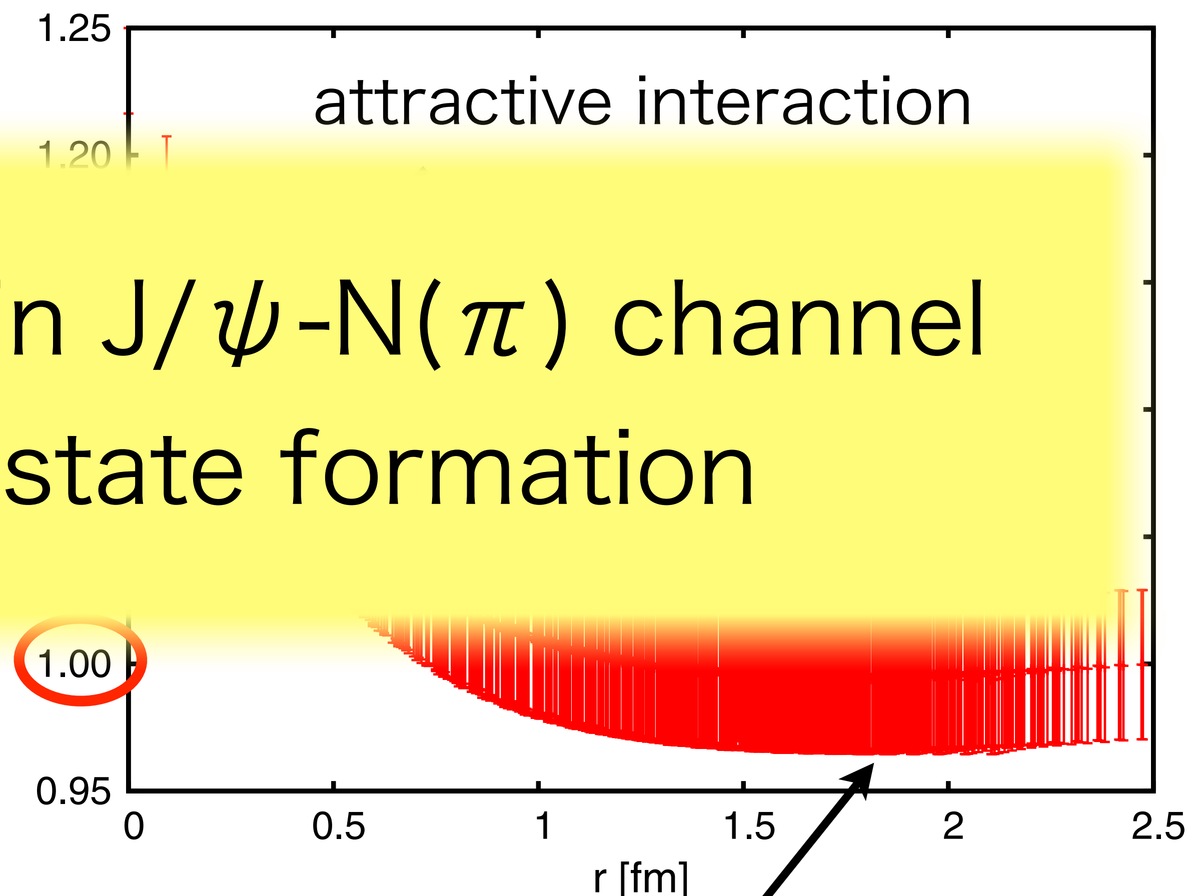
s-wave relative wave function





Negative ΔE observed in J/ψ-N(π) channel
does not means bound state formation

Q: localized or extended ?

A: extended !

asymptotically plane wave at long distances

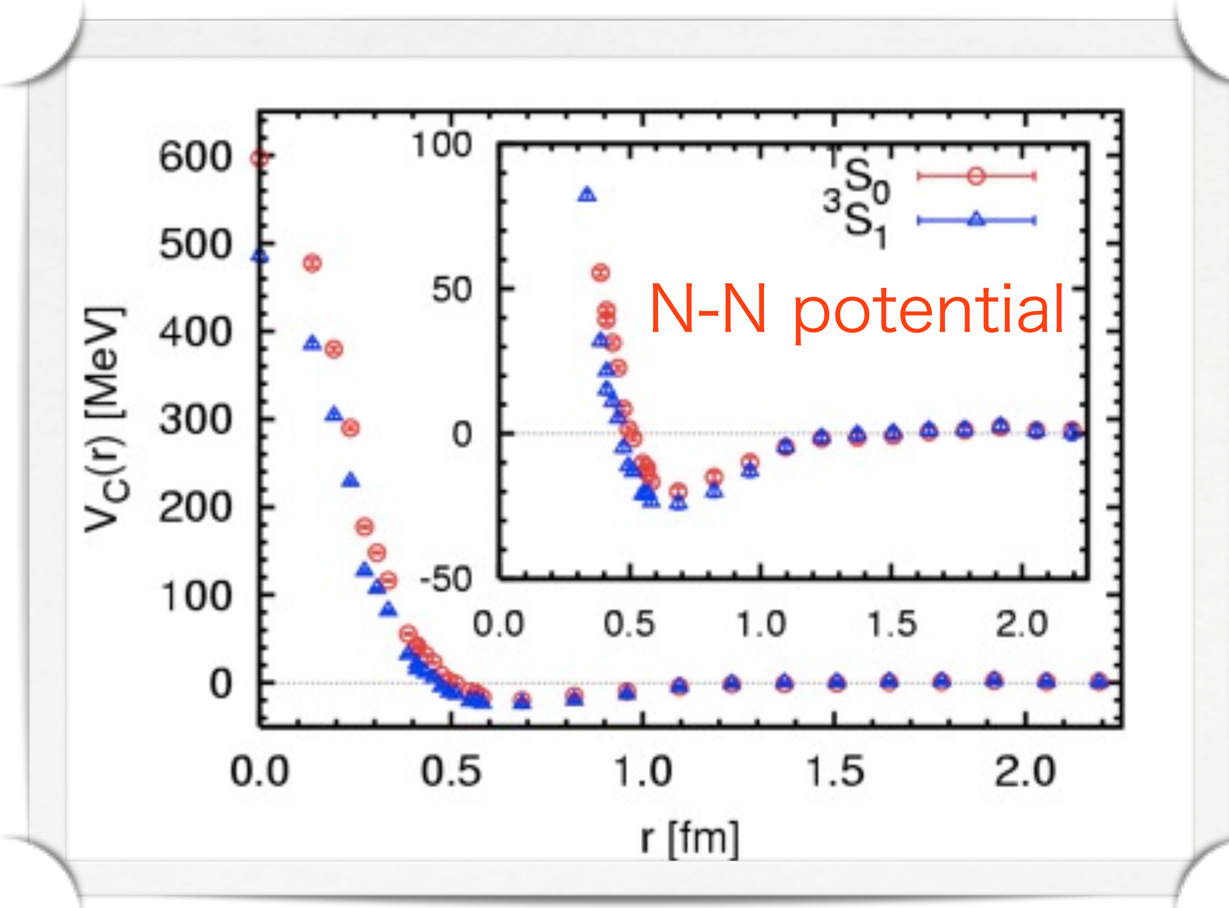
From BS-wave func. to Potential

✳ A potential can be determined from the BS-wave function through the stationary Schrödinger equation:

$$E\varphi_{\rm BS}(r) + \frac{\nabla^2}{2M_{\rm red}}\varphi_{\rm BS}(r) = V(r)\varphi_{\rm BS}(r)$$

✓ INPUT : $E,\ M_{\rm red},\ \varphi_{\rm BS}(r)$

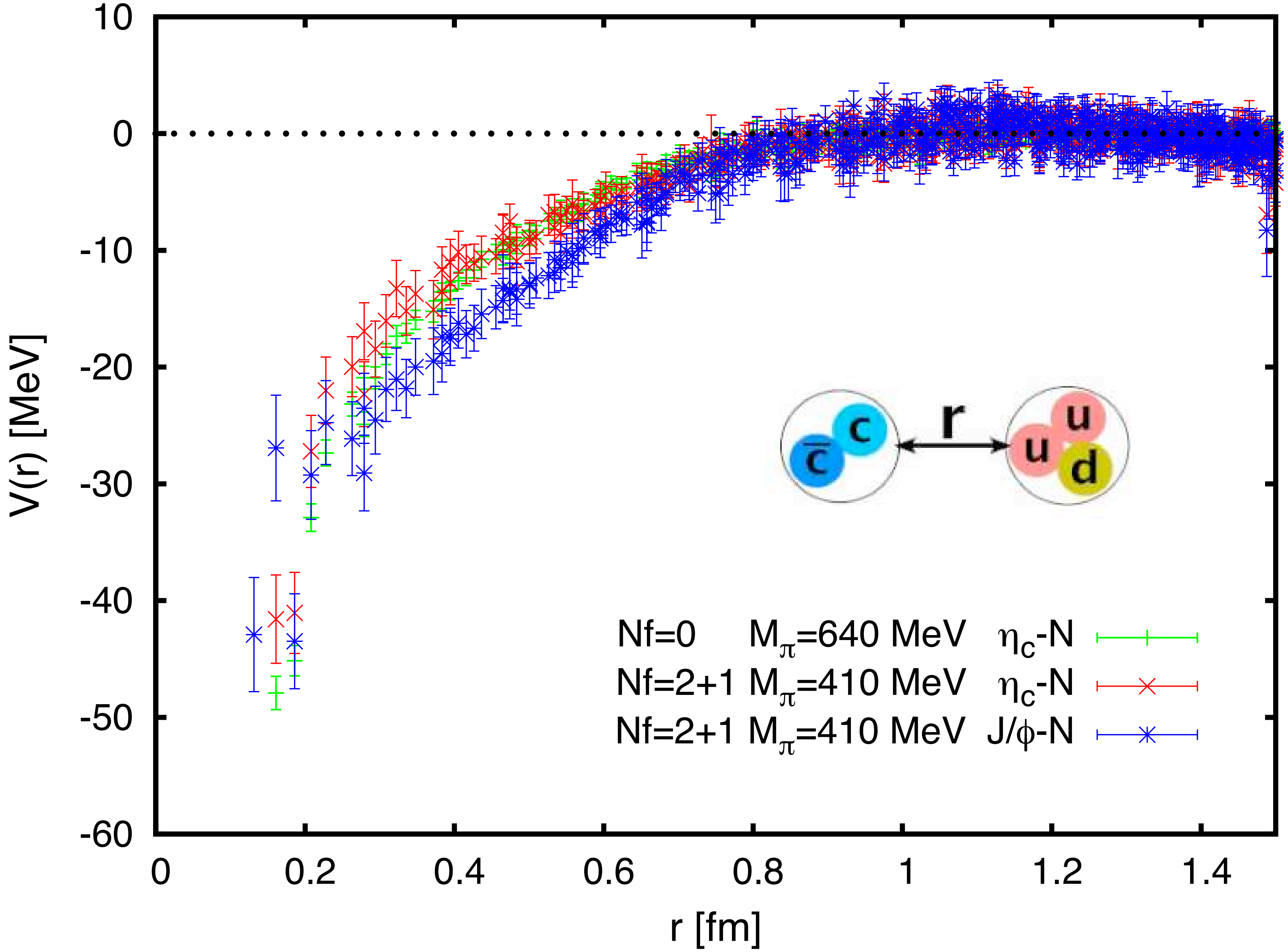
✓ OUTPUT : $V(r)$



Ishii-Aoki-Hatsuda, PRL99 (07) 022001

charmonium-nucleon potential

T. Kawanai, SS, Phys. Rev. D82 (2010) 091501
T. Kawanai, SS, arXiv:1011.1322, Proc. of Baryon 2010



✓ weakly attractive at short distances

✓ exponentially screened at large distances

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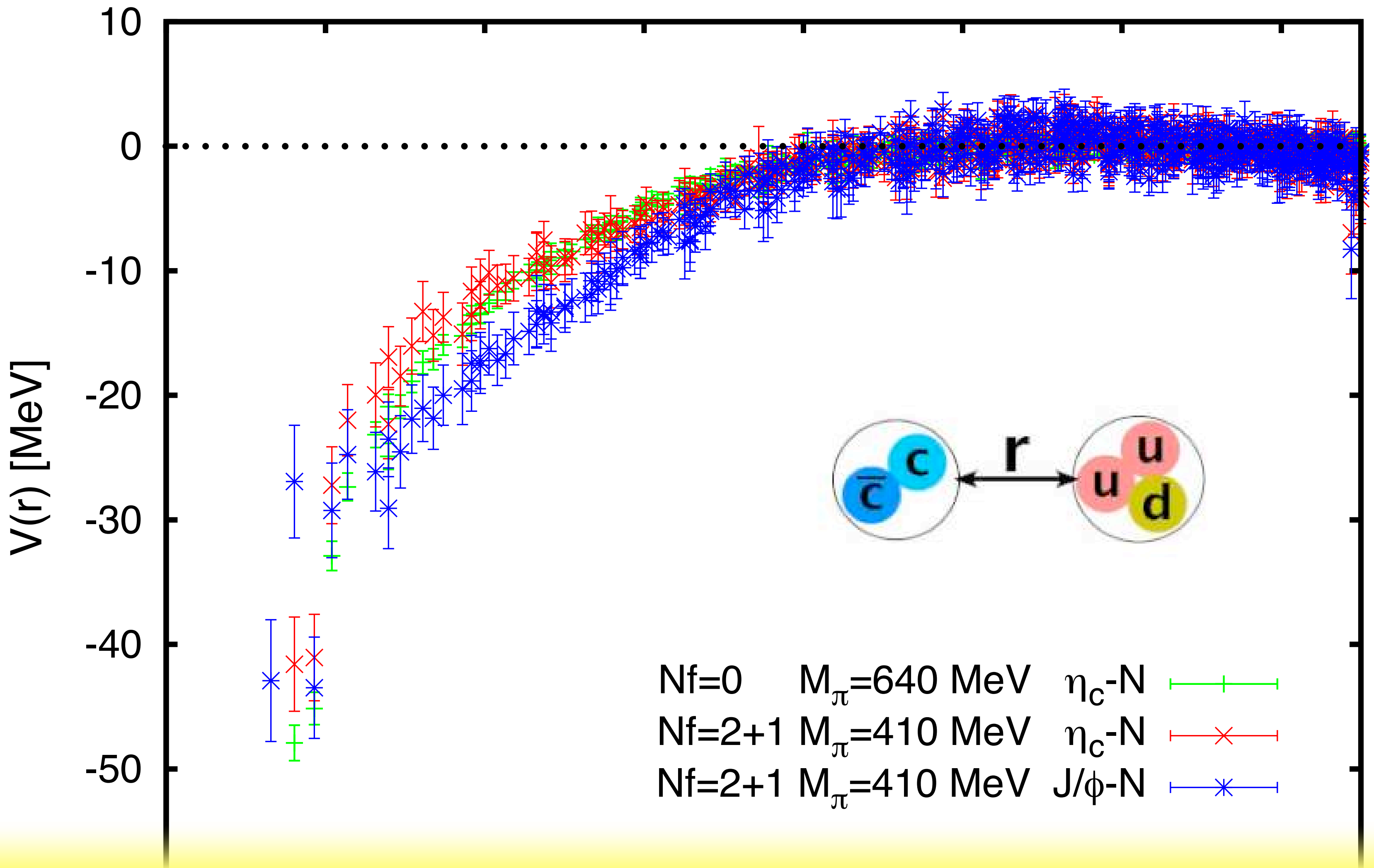
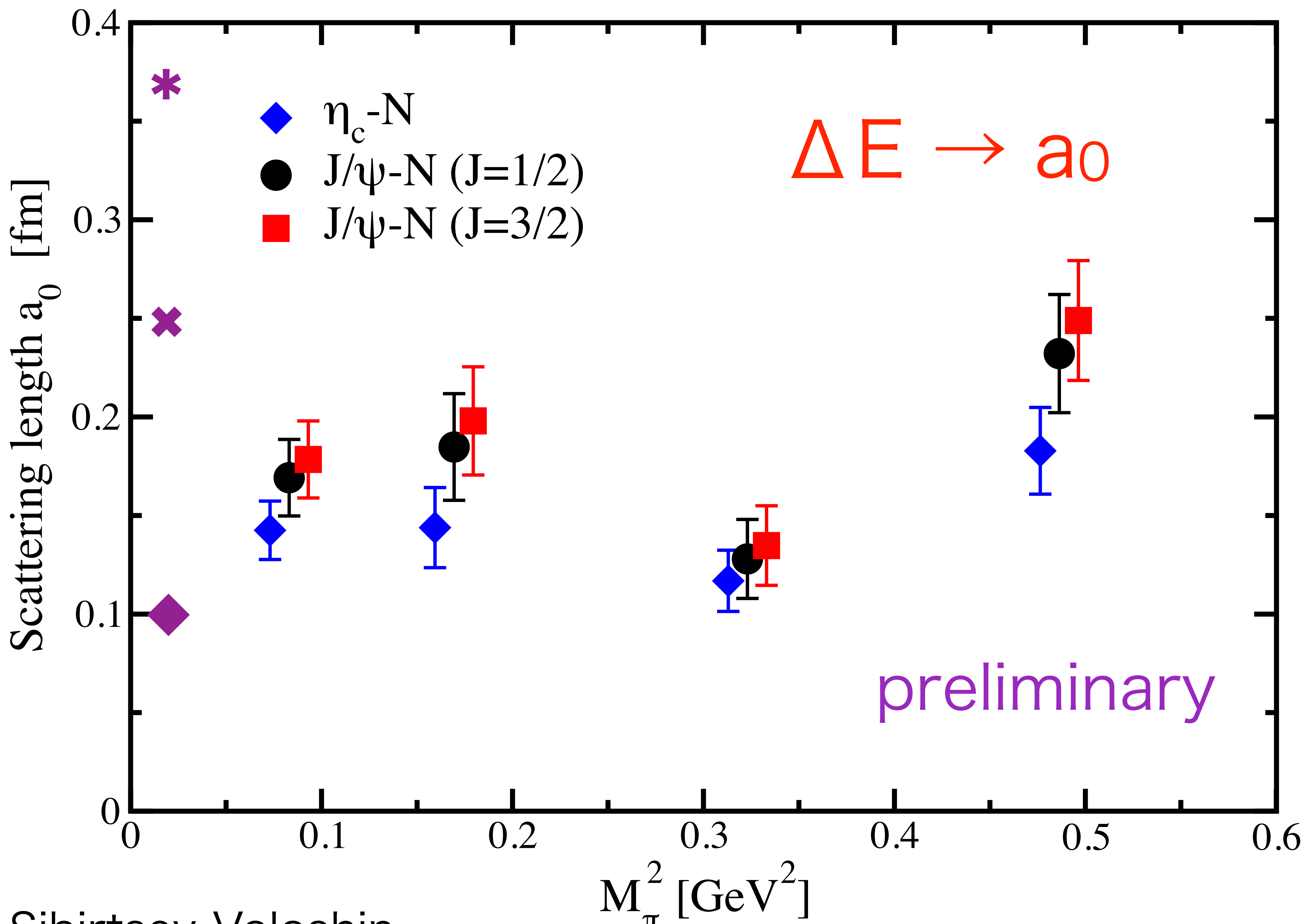


long-range screening of the color van der Waals force

✓ weakly attractive at short distances

✓ exponentially screened at large distances

scattering length a0

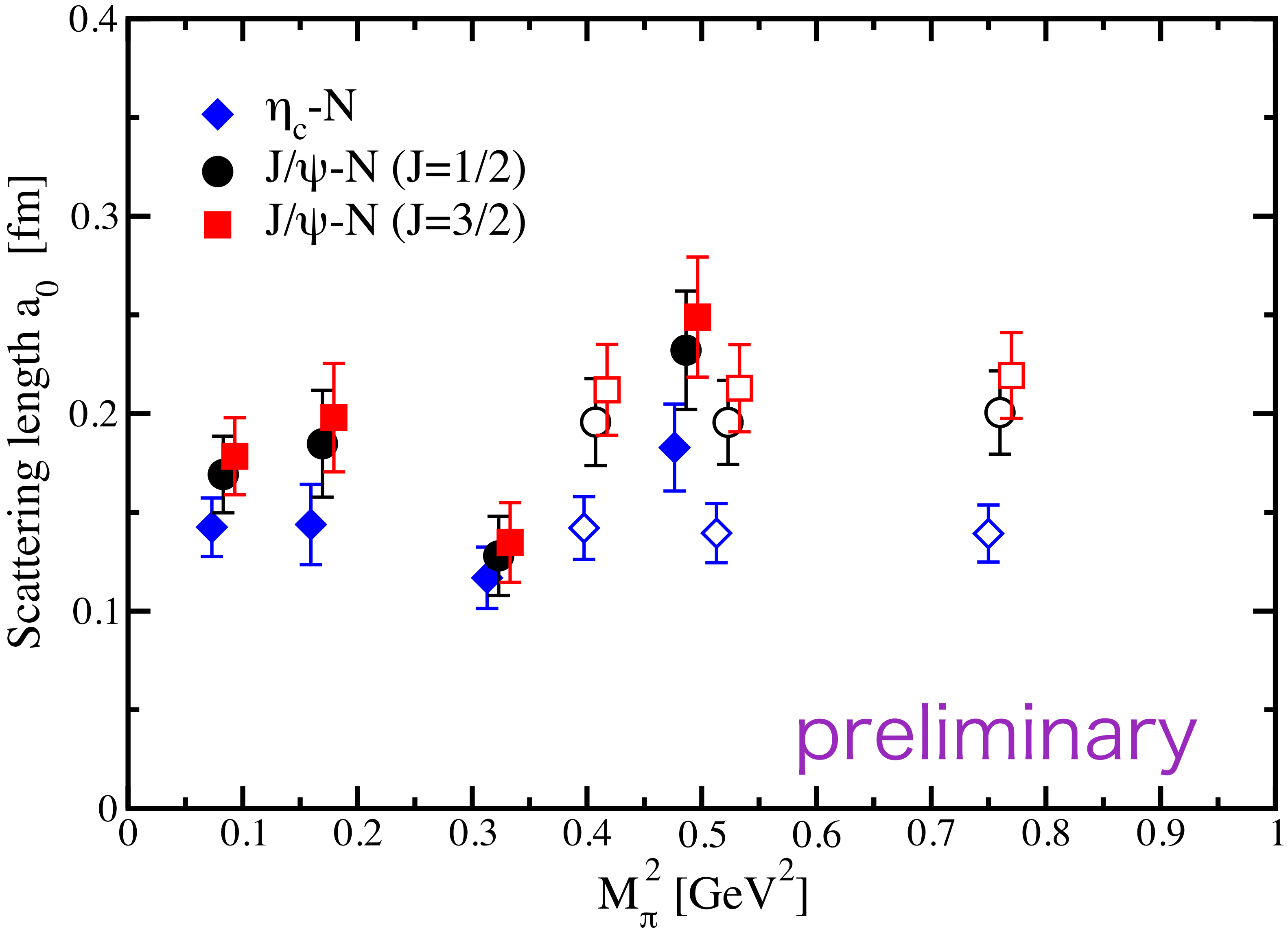


✱ Sibirtsev-Voloshin

✖ Brodsky-Miller

◆ QCDSR (Hayashigaki)

$$a_{\eta_c N} \approx 0.15 \text{ fm} < a_{J/\psi N} \approx 0.17 \text{ fm}$$



◆●■ **Full QCD**

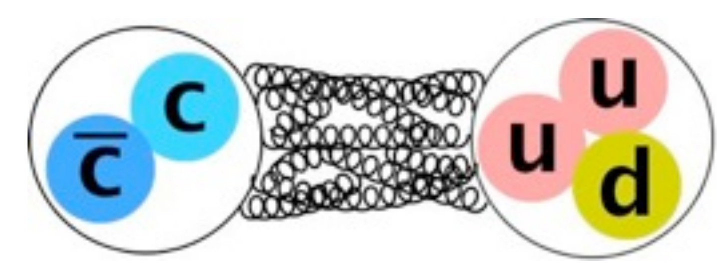
◇○□ **Quench QCD**

Kawanai and Sasaki,
arXiv:1011.1322 (Quench)

✓No strong N_f dependence

✓No large quark mass dependence

➡ multi-gluon exchange dominance



Twisted boundary condition

P.F. Bedaque, PLB593 (04) 82

- Generalized spatial boundary condition (b.c.)

$$\psi(x+L) = e^{i\phi}\psi(x)$$

$\phi = 0$: periodic boundary condition (PBC)
$\phi = \pi$: anti-periodic boundary condition (APBC)

- All momenta are quantized in finite volume as

$$p = \frac{2\pi}{L}\left(n + \frac{\phi}{2\pi}\right) \text{ with integer } n$$

accessible to any small momentum with the angle ϕ

Finite size effect on two-particle total energy

$$E_{2-\mathrm{body}} = M_1 + M_2 + \Delta E \neq M_1 + M_2$$

Scattering length

$$M_{\mathrm{red}} = M_1 M_2/(M_1+M_2)$$

$$\Delta E = \boxed{-\frac{2\pi a_0}{M_{\mathrm{red}} L^3}}\left[1 + c_1 \frac{a_0}{L} + c_2\left(\frac{a_0}{L}\right)^2\right] + \mathcal{O}(L^{-6})$$

Universal

Volume

depends on volume-shape
and boundary condition

L^3 box+Periodic BC :
Lüscher (86)

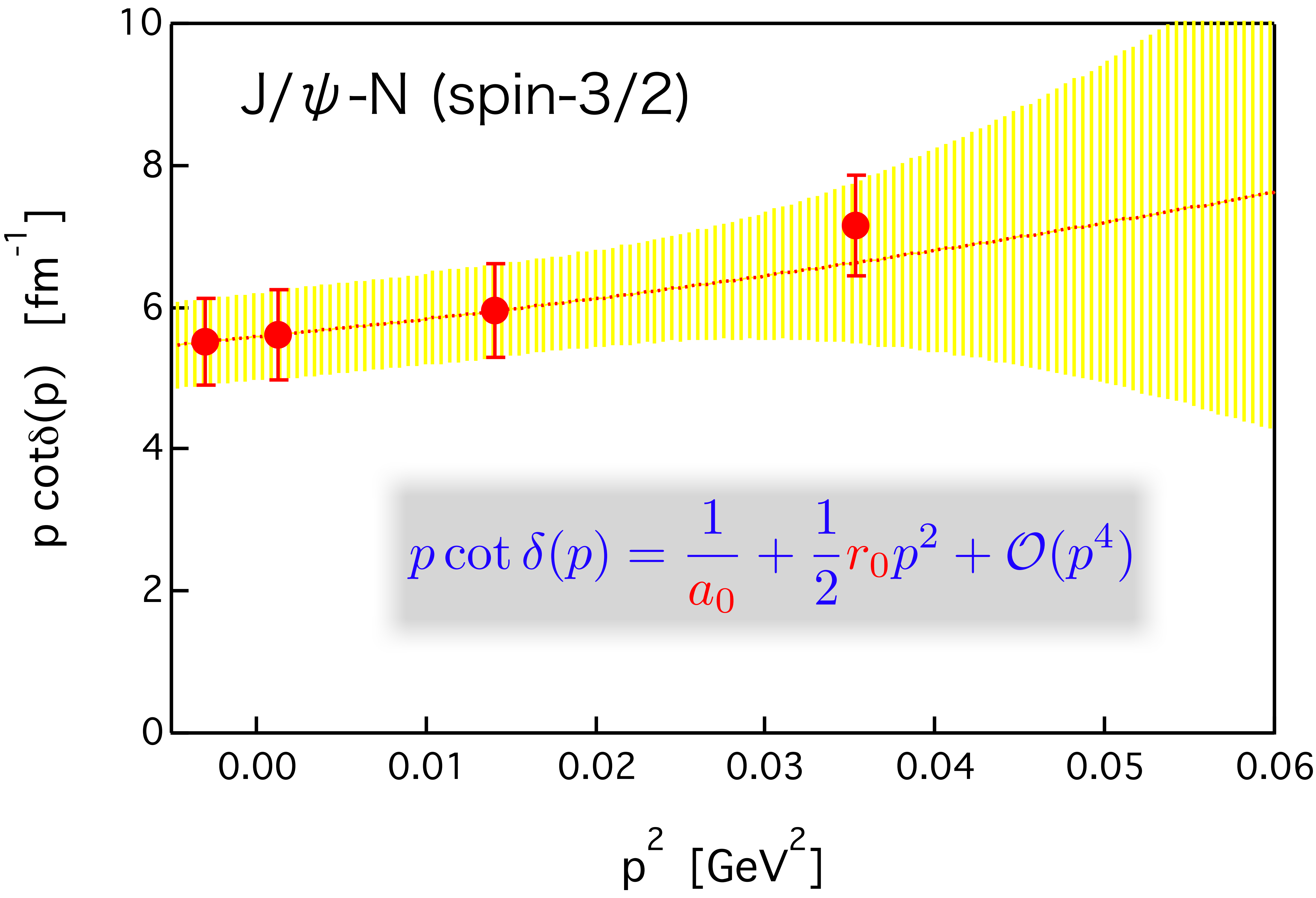
Twisted BC : Bedaque (04)

$$E_{2-\mathrm{body}} = E_1(\theta) + E_2(\theta) + \Delta E_\theta$$

$$\Delta E_\theta = -\frac{2\pi b}{M_{\mathrm{red}} L^3}\left[1 + c_1(\theta)\frac{b}{L} + c_2(\theta)\left(\frac{b}{L}\right)^2\right] + \mathcal{O}(L^{-6})$$

$$b = \left.(p\cot\delta(p))^{-1}\right|_{p^2=\theta^2} \approx a_0 + \mathcal{O}(\theta^2)$$

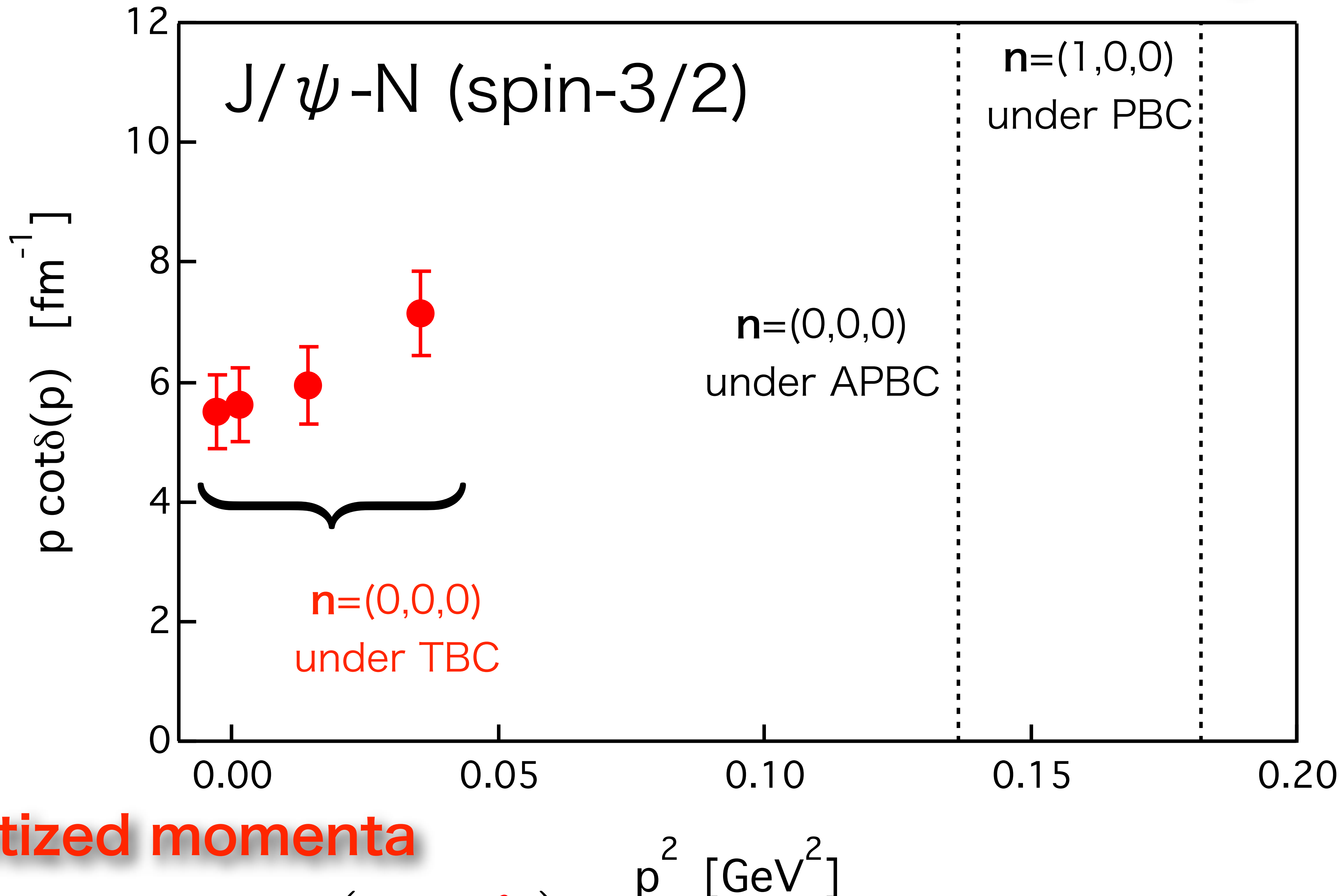
Effective range expansion



Twisted boundary condition

$$\psi(x+L) = e^{i\theta}\psi(x)$$

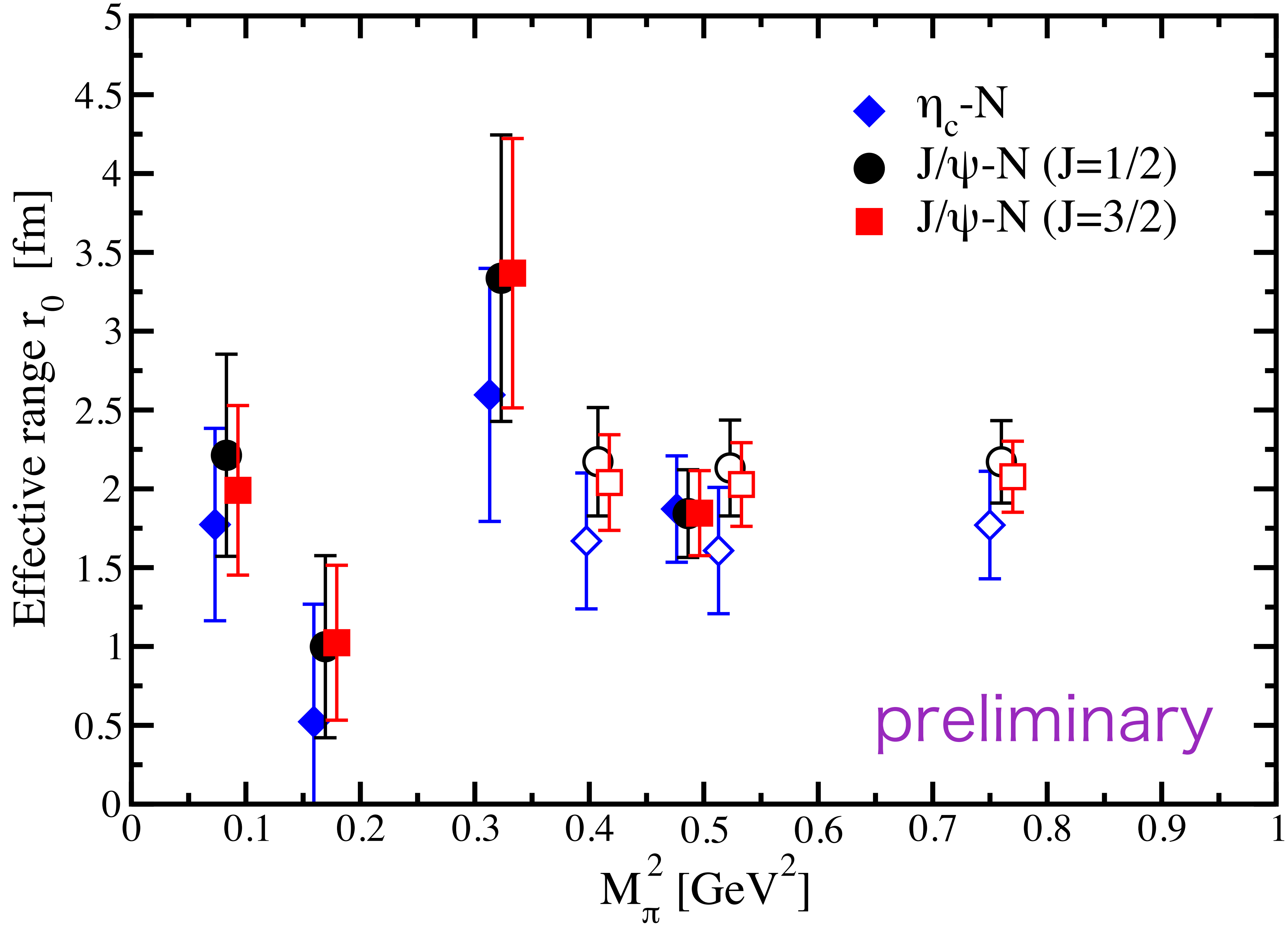
Θ=0 : Periodic BC
Θ=π : Anti-periodic BC



Quantized momenta

$$p = \frac{2\pi}{L}\left(n + \frac{\theta}{2\pi}\right)$$

effective range r_0

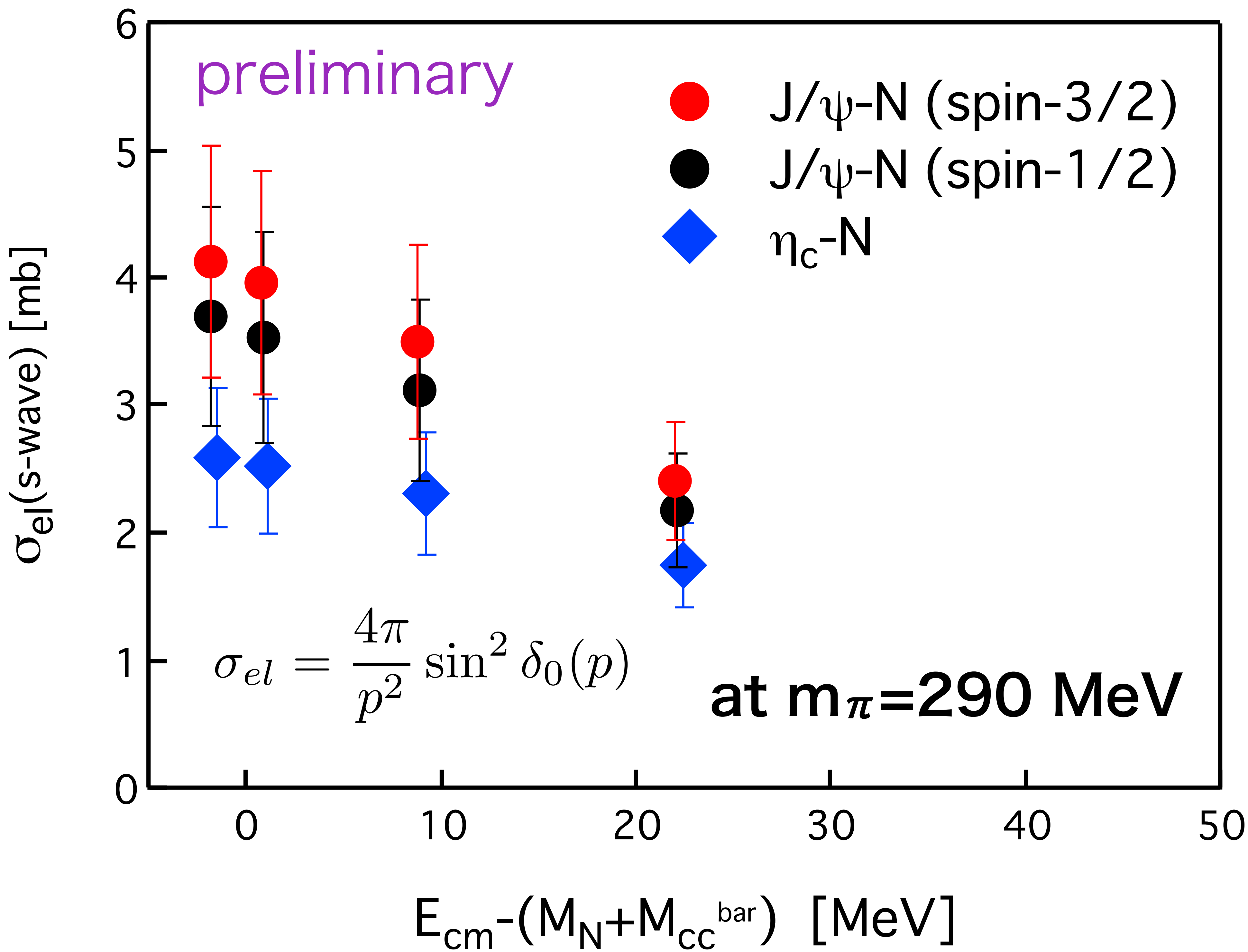


◆●■ **Full QCD**

◇○□ **Quench QCD**

Kawanai and Sasaki, arXiv:1011.1322 (Quench)

total elastic cross section of charmonium-nucleon



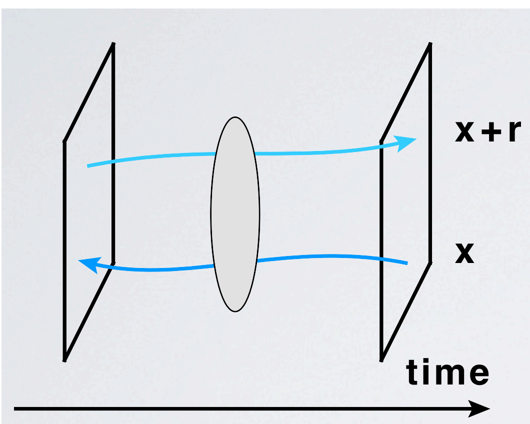
Structure of charmonia

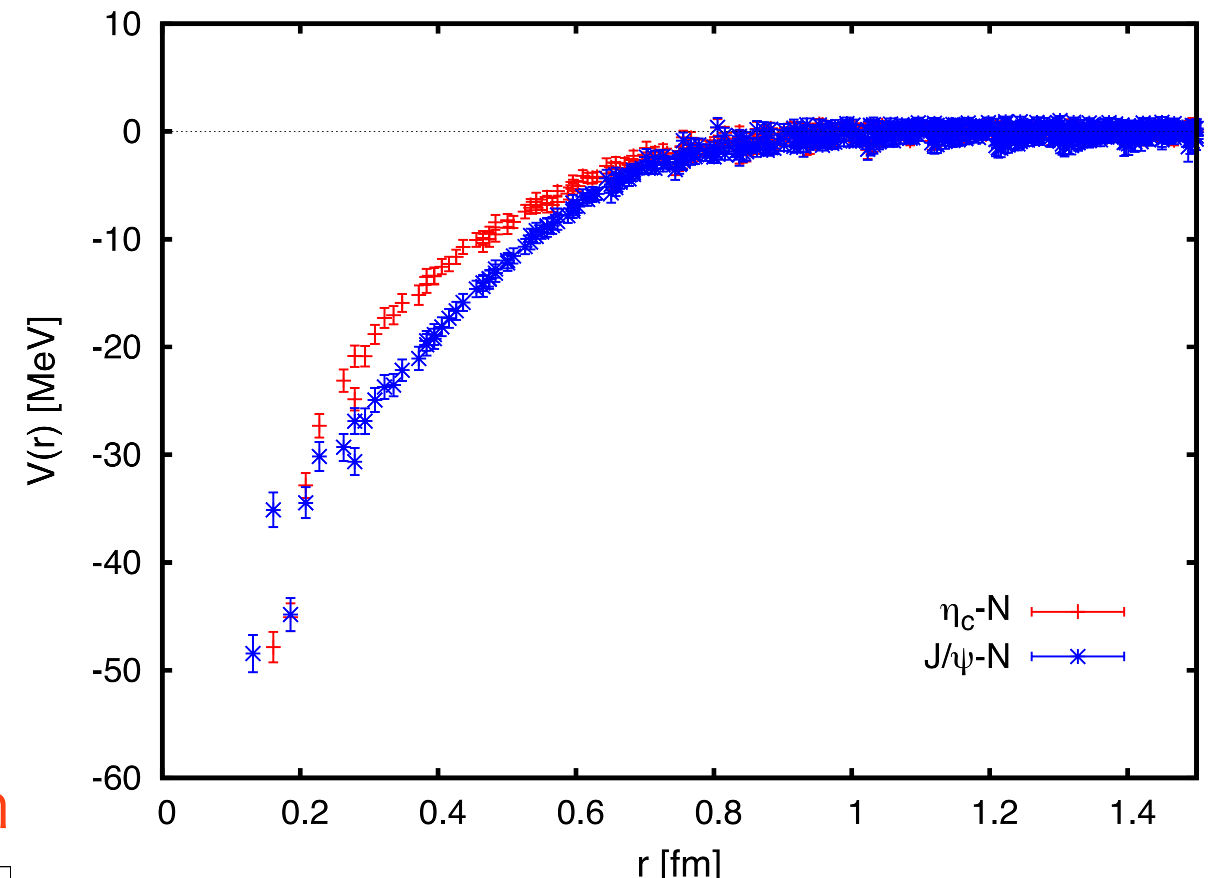
Why J/ψ-N interaction is stronger?

Equal-time Coulomb gauge BS amplitude

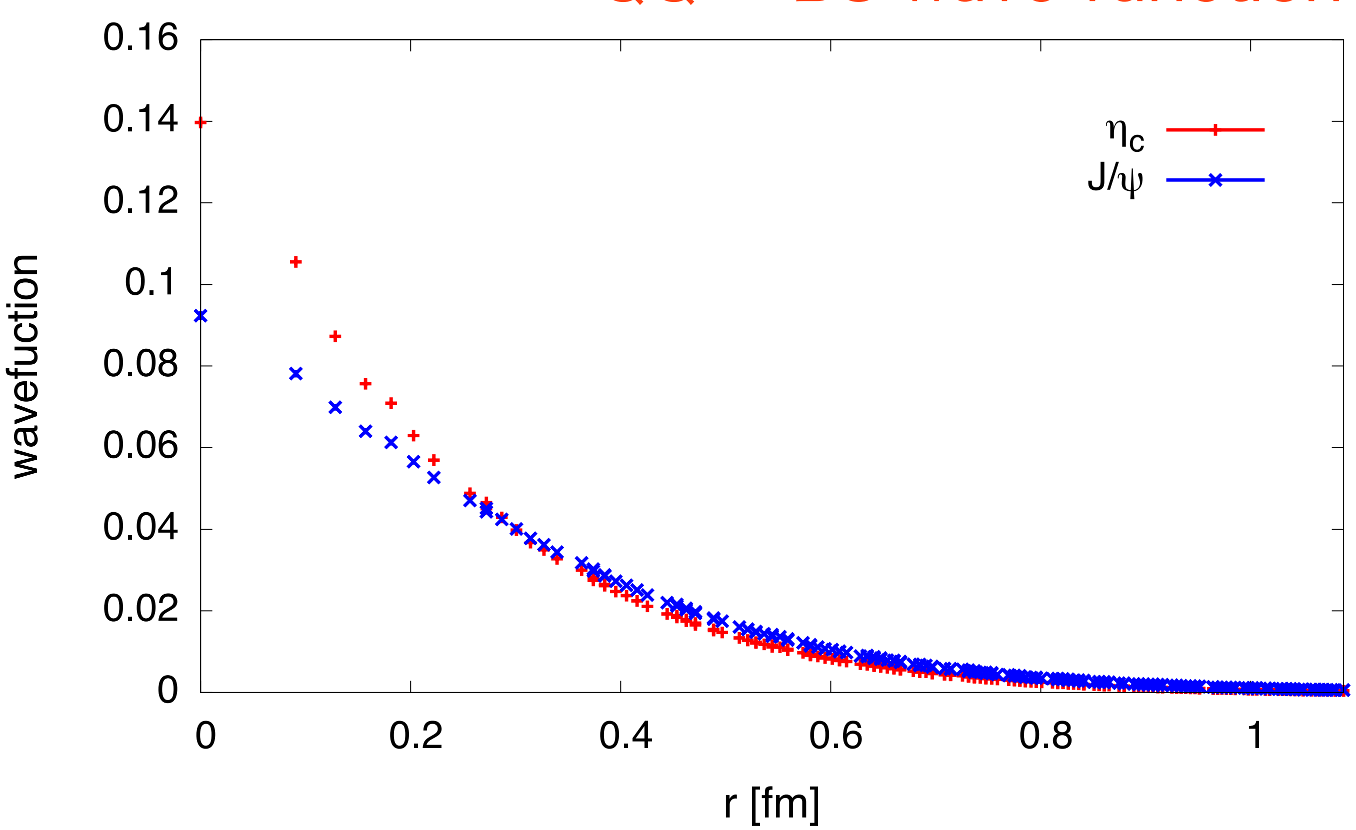
$$\phi^{\Gamma}_{Q\bar{Q}}(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|Q\bar{Q}\rangle$$

$$\text{for } \Gamma = \gamma_5, \gamma_i$$





QQ^{bar} BS wave function

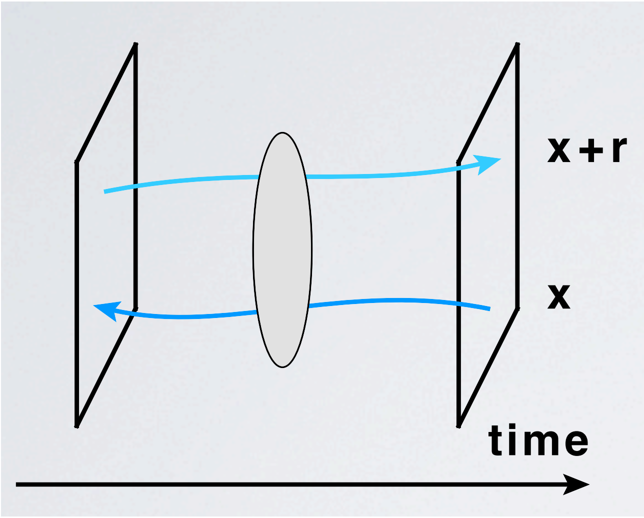


similar to dipole-dipole interaction?

structure of charmonia

Potential from BS wave func.

- Equal-time BS wave function $\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|Q\bar{Q}\rangle$



$$\begin{aligned} G_{4\mathrm{pt}} &= \sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{Q}(\mathbf{x},t)\Gamma Q(\mathbf{x}+r,t)(\bar{Q}(\mathbf{x}',t_{\mathrm{src}})\Gamma Q(\mathbf{y}',t_{\mathrm{src}}))^\dagger|0\rangle \\ &= \sum_{\mathbf{x}}\sum_{n} A_n \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^\Gamma(t-t_{\mathrm{src}})} \\ &\xrightarrow{t\gg t_{\mathrm{src}}} A_0 \phi_\Gamma(\mathbf{r}) e^{-M_0^\Gamma(t-t_{\mathrm{src}})} \end{aligned}$$

- Schrödinger eq. with non-local potential

$$-\frac{\nabla^2}{2\mu}\phi_\Gamma(\mathbf{r}) + \int dr' U(\mathbf{r},\mathbf{r}')\phi_\Gamma(\mathbf{r}') = E_\Gamma \phi_\Gamma(\mathbf{r})$$

- Velocity expansion

$$U(\mathbf{r}',\mathbf{r}) = \left\{V(r) + V_{\mathrm{S}}(r)\mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}} + V_{\mathrm{T}}(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\right\}\delta(\mathbf{r}'-\mathbf{r})$$

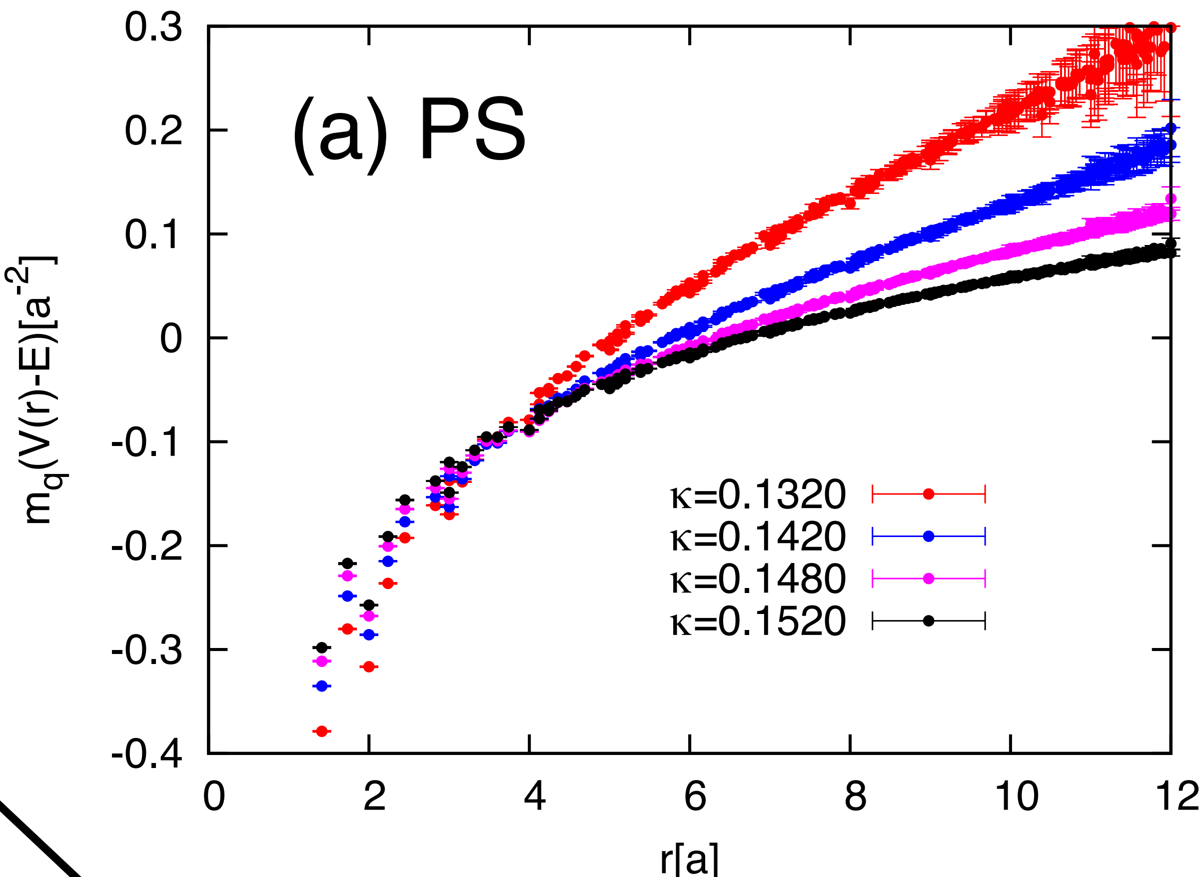
S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89

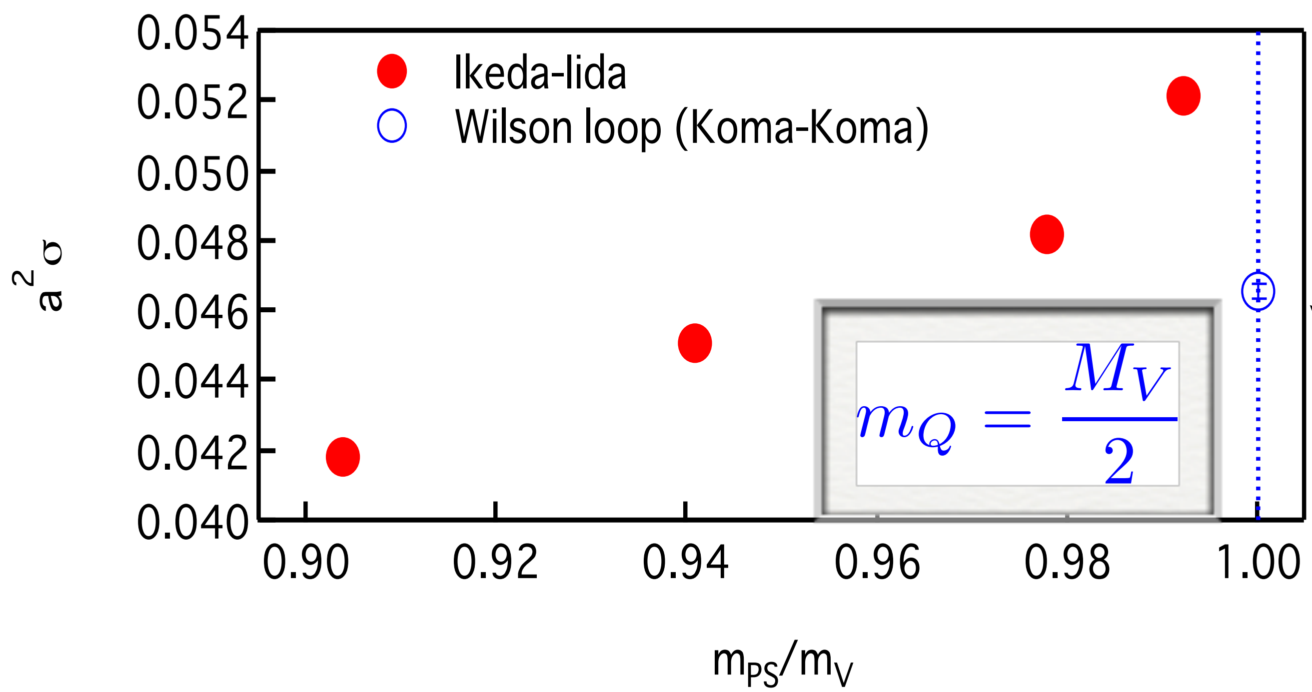
$Q\bar{Q}$ potential from BS wave func.

- Ikeda-Iida, arXiv:1011.2866 & 1102.2097

Cornell-like behavior!

$$\frac{\nabla^2 \phi_{Q\bar{Q}}(r)}{\phi_{Q\bar{Q}}(r)} = m_Q \left[V(r) - E \right]$$





Wilson loops

Inconsistent with the Wilson loops in the $m_Q \to \infty$ limit

Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

$$V_\Gamma(r) = V_{Q\bar{Q}}(r) + V_{\rm spin}(r)\vec{S}_Q \cdot \vec{S}_{\bar{Q}} \qquad \text{for} \;\; \Gamma = \gamma_5, \gamma_i$$

Propose a self-consistent determination of the quark mass

$$V_{\rm spin}(r) - \Delta E_{\rm hyp} = \frac{1}{m_Q}\left(\frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} - \frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)}\right)$$

Under a simple, but reasonable assumption of $\lim_{r\to\infty} V_{\rm spin}(r) = 0$

$$\boxed{m_Q = \lim_{r\to\infty} \frac{1}{\Delta E_{\rm hyp}}\left(\frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} - \frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)}\right)}$$

Interquark potential at finite quark mass

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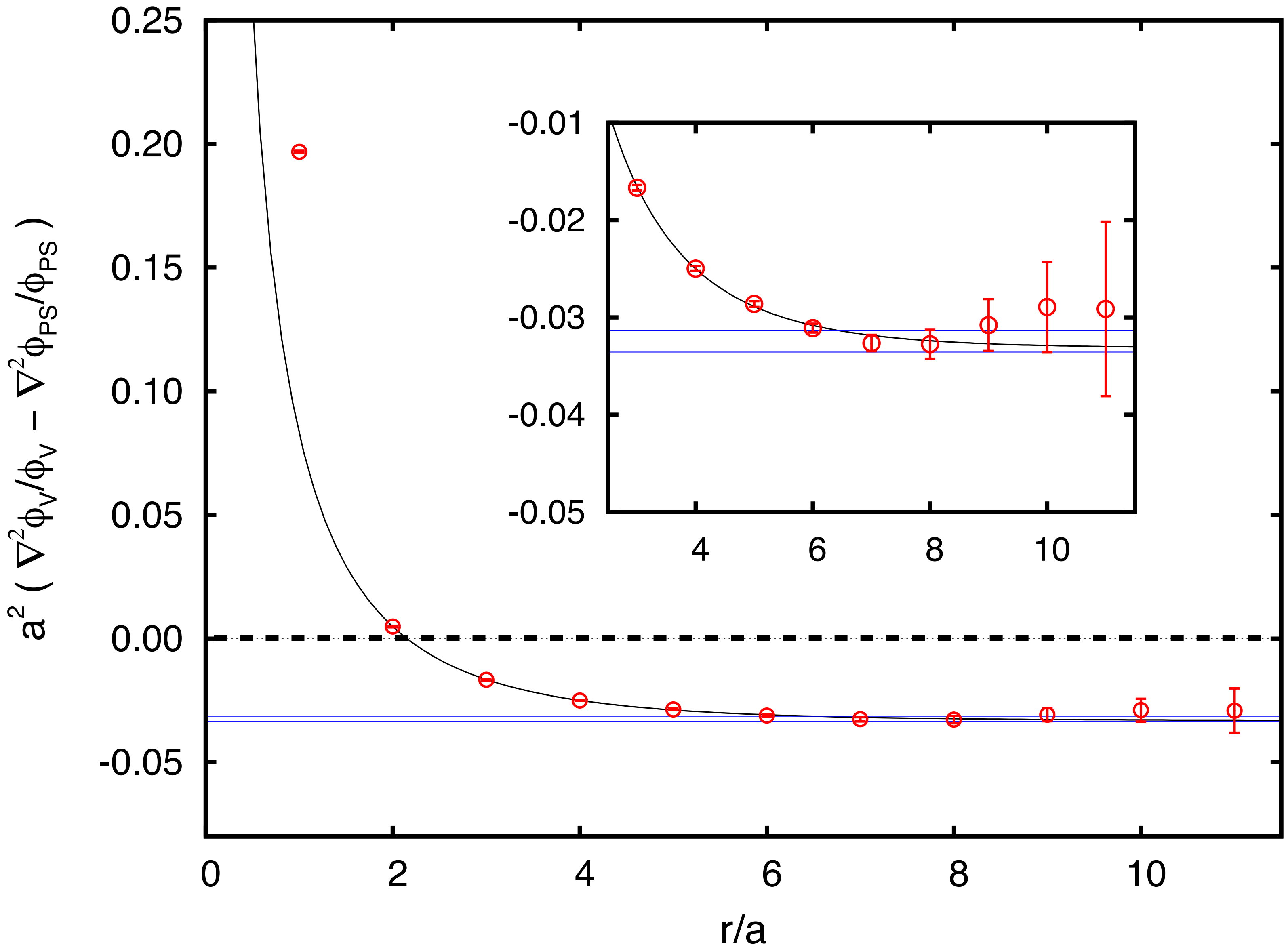
$$m_Q = \lim_{r\to\infty} \frac{1}{\Delta E_{\rm hyp}} \left(\frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} - \frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} \right)$$



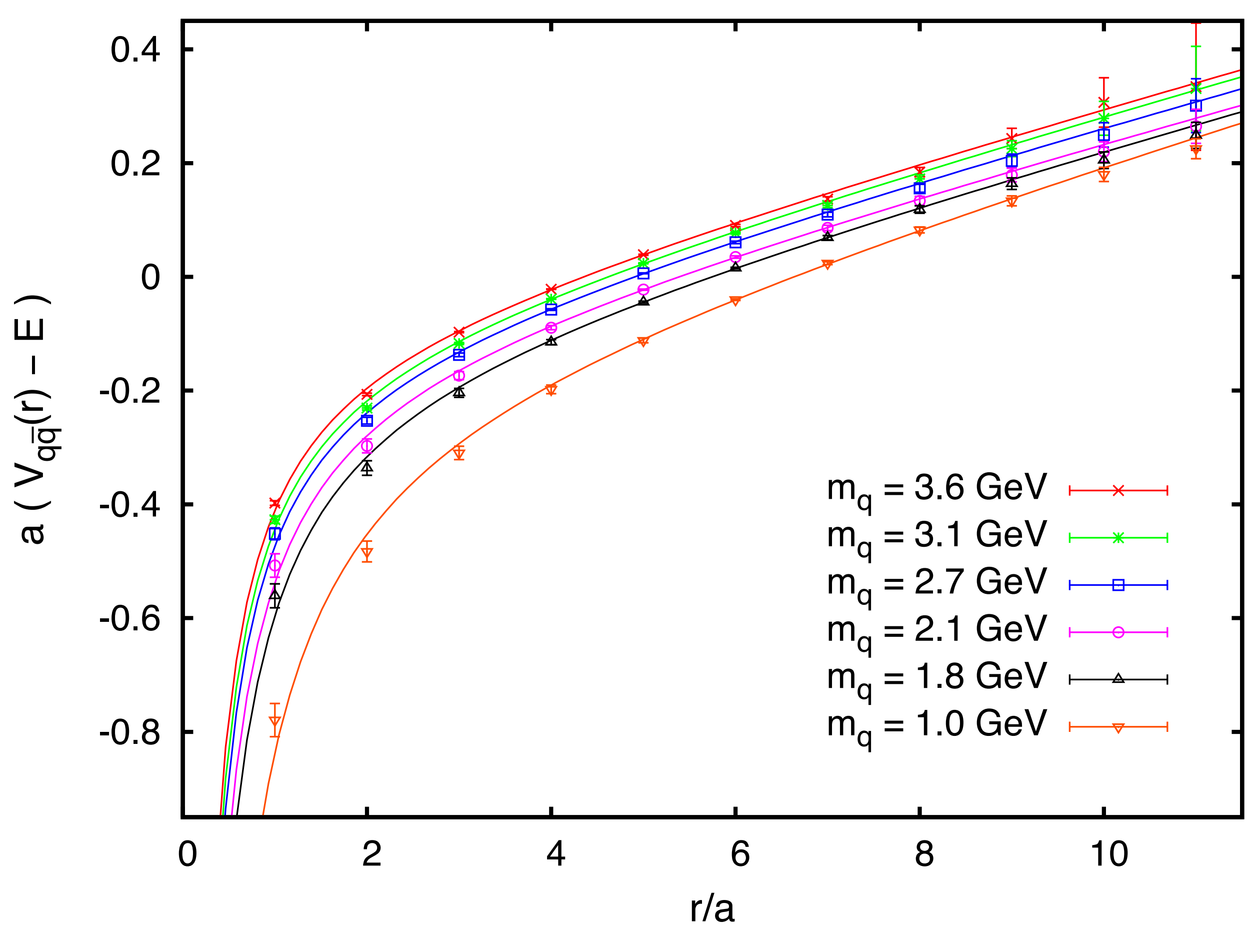
$$\lim_{r\to\infty} V_{\rm spin}(r) = 0$$

$$-m_Q \Delta E_{\rm hyp}$$

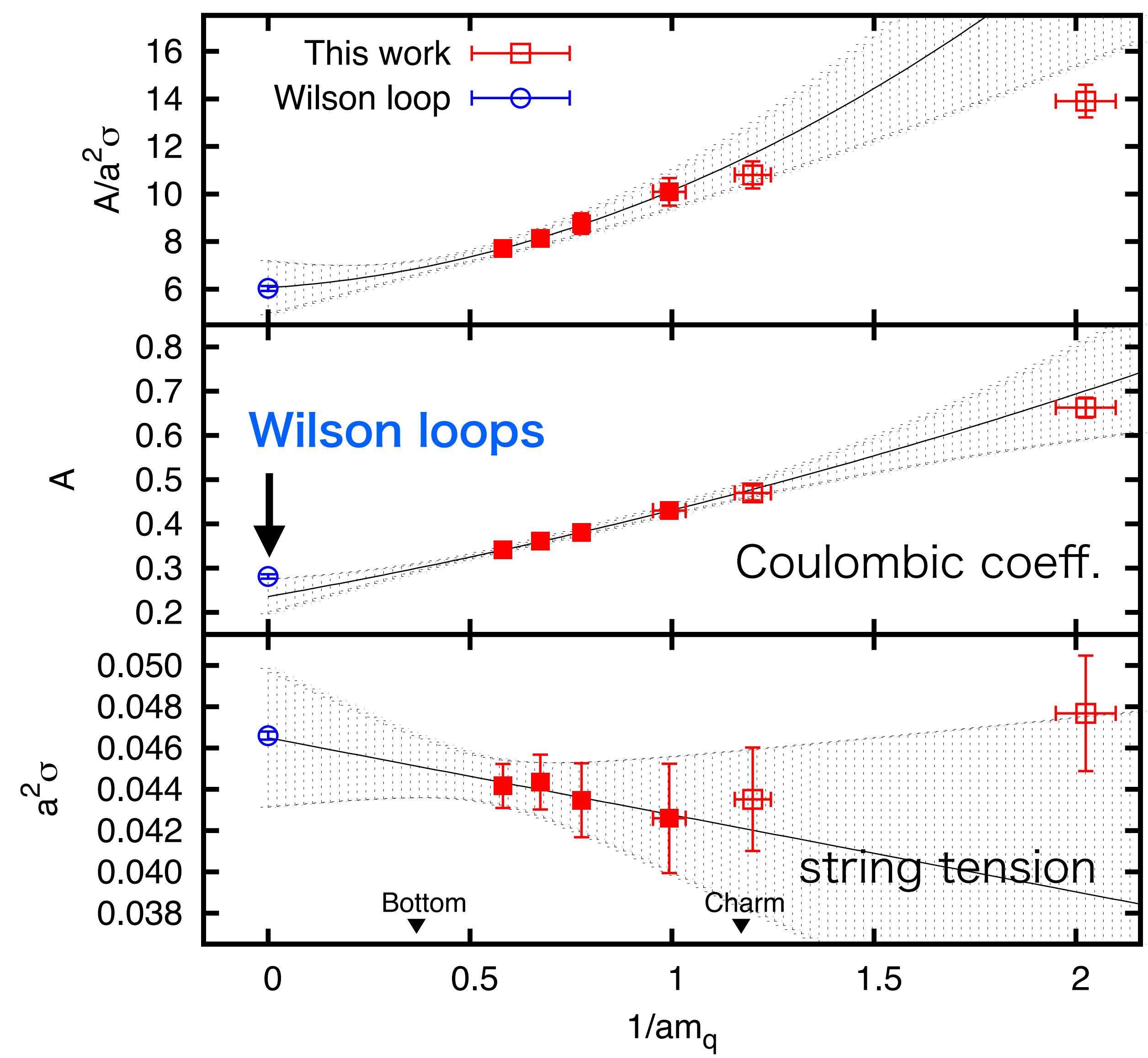
Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

$$V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0$$



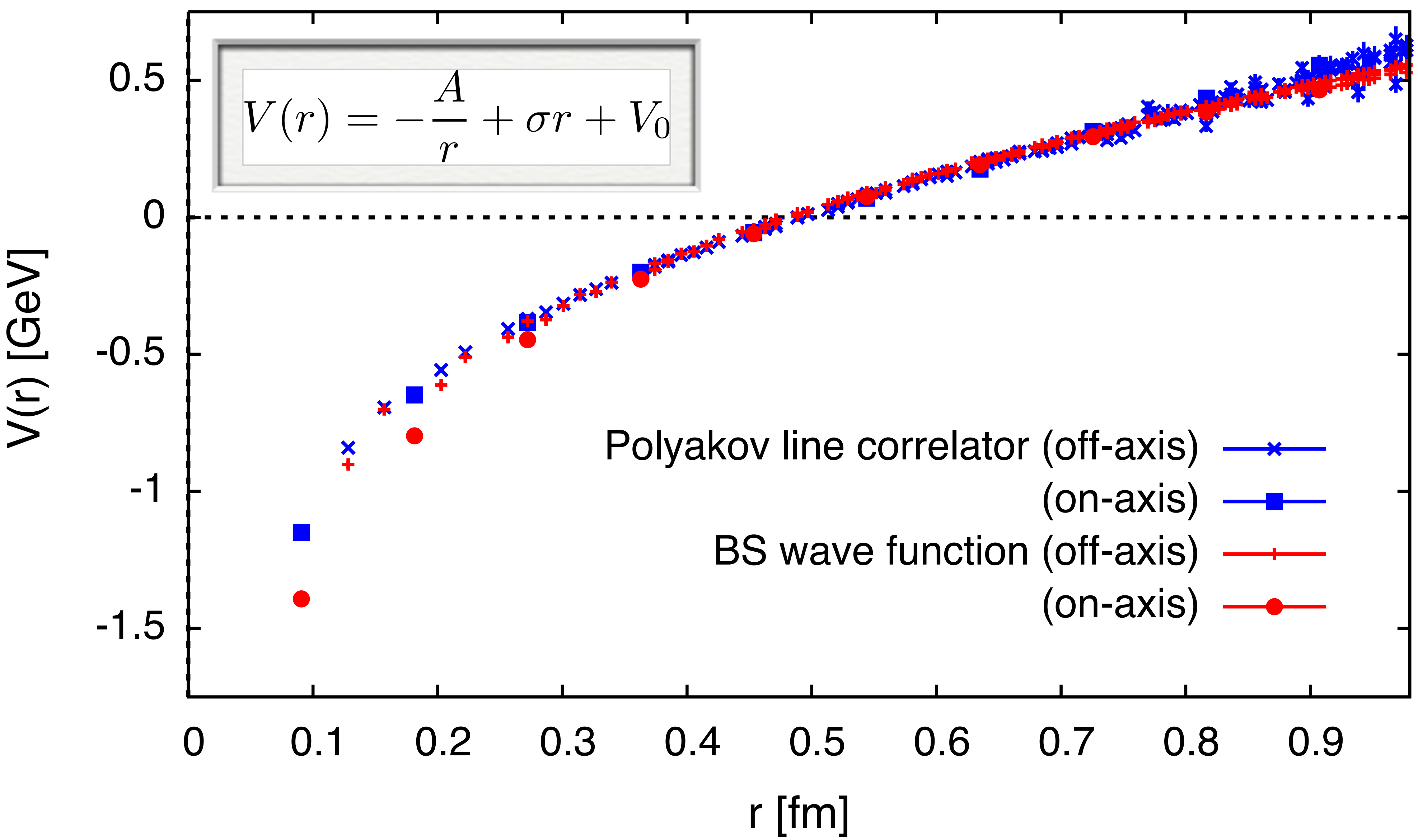
Quench + RHQ



Consistent with the Wilson loops in the $m_q \to \infty$ limit

Charmonium potential from full QCD

- Kawanai-Sasaki (2011) in preparation
  - ✳ PACS-CS configurations at m_π=156 MeV



$A_{c\bar{c}} = 0.813(22)$

$\sqrt{\sigma_{c\bar{c}}} = 0.394(7)$ GeV

$A_{\infty} = 0.403(10)$

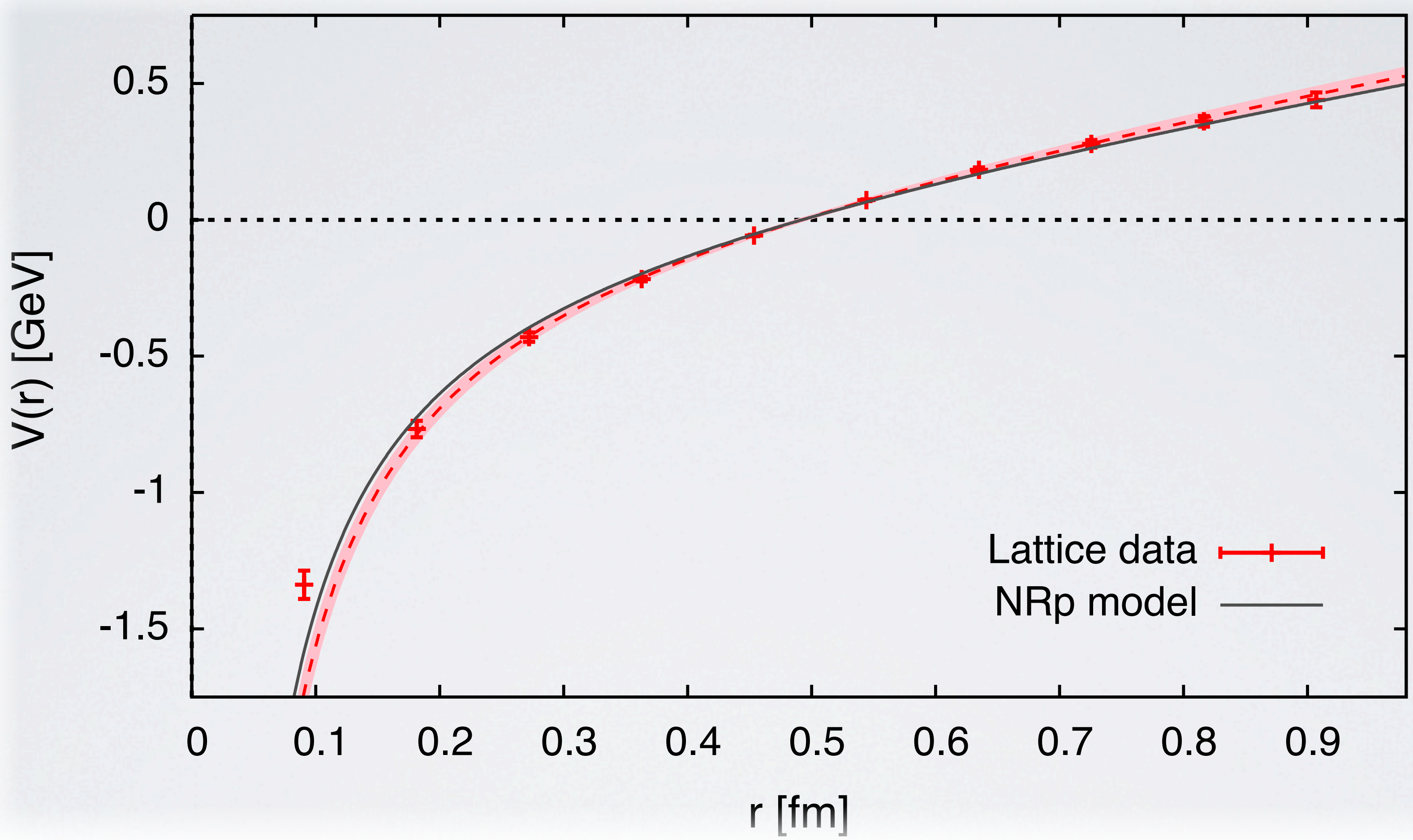
$\sqrt{\sigma_{\infty}} = 0.462(2)$ GeV

Polyakov loop correlator

Charmonium potential from full QCD

- Kawanai-Sasaki (2011) in preparation
    - ✳ PACS-CS configurations at m_π=156 MeV

The spin-independent part of the central potential resembles the NRp model



$$A_{c\bar{c}} = 0.813(22)$$

$$\sqrt{\sigma_{c\bar{c}}} = 0.394(7)\ \mathrm{GeV}$$

lattice results

$$A_{\mathrm{NRp}} = 0.7281$$

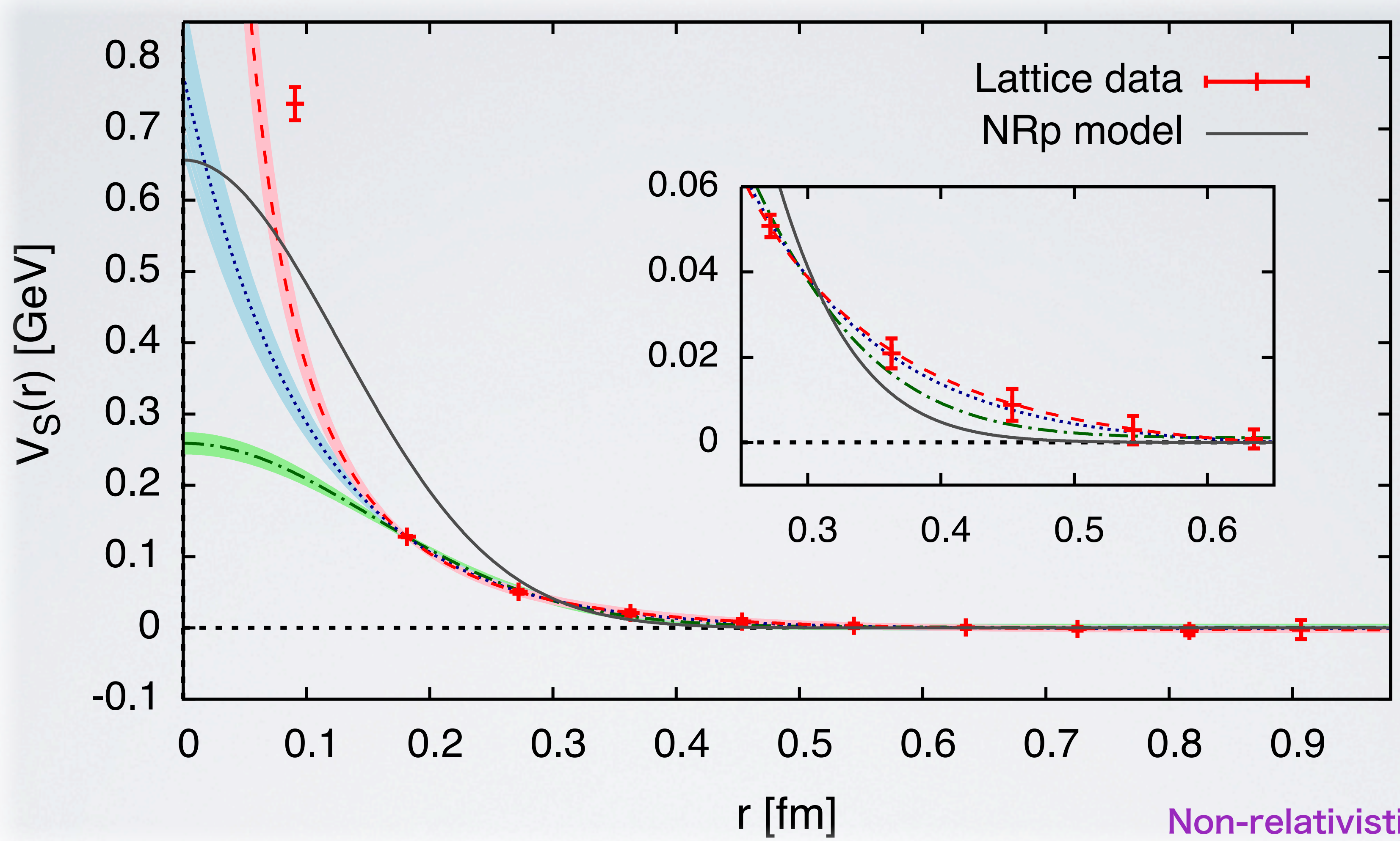
$$\sqrt{\sigma_{\mathrm{NRp}}} = 0.3775\ \mathrm{GeV}$$

Non-relativistic potential model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

Charmonium potential from full QCD

- Kawanai-Sasaki (2011) in preparation
  - ✴ PACS-CS configurations at m_π=156 MeV

A difference appears in the spin-spin potential



$$V_S(r) = \begin{cases} \alpha \exp(-\beta r)/r & : \text{Yukawa form} \\ \alpha \exp(-\beta r) & : \text{Exponential form} \\ \alpha \exp(-\beta r^2) & : \text{Gaussian form.} \end{cases}$$

functional form	α	β	χ^2/dof
Yukawa-type	0.287(8)	0.894(32) GeV	7.28
Exponential-type	0.825(19) GeV	1.982(24) GeV	1.46
Gaussian-type	0.313(26) GeV	1.02(95) GeV^2	25.75

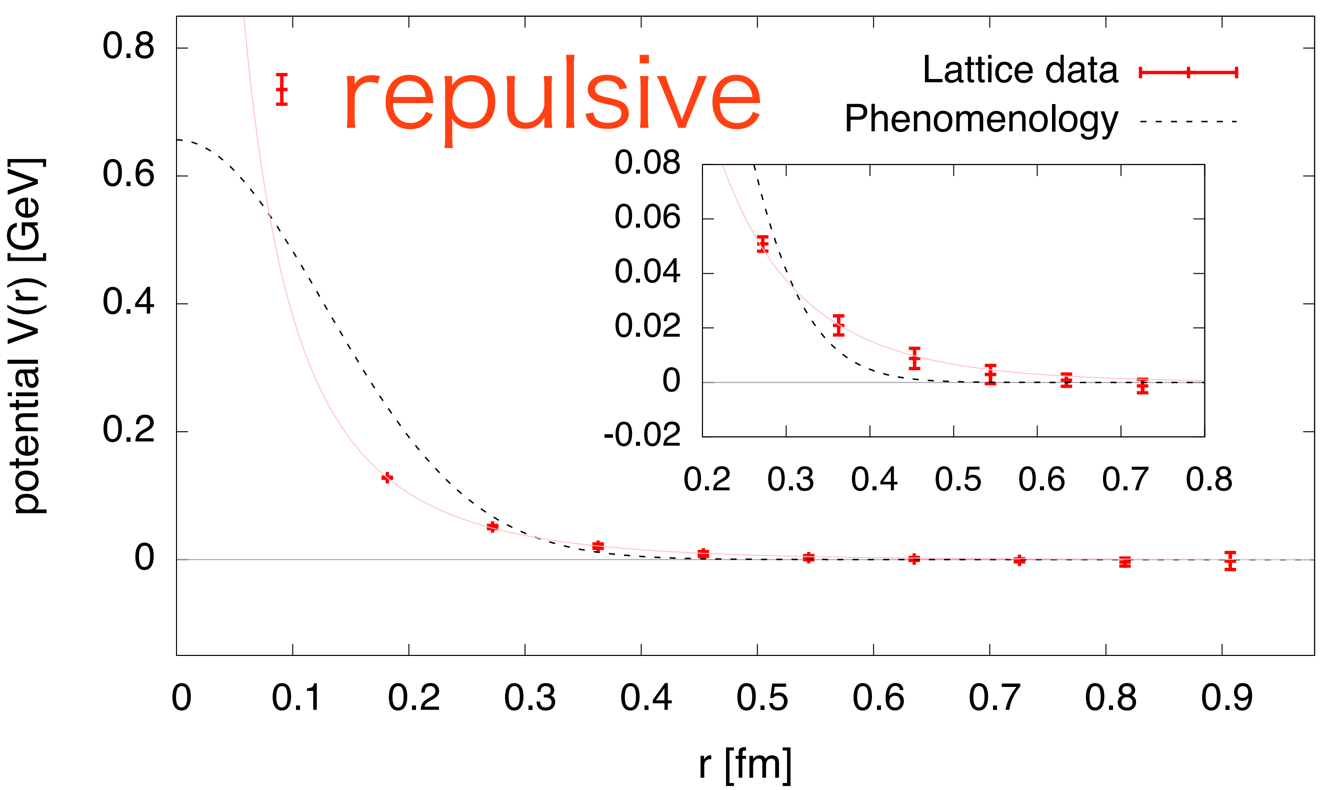
Non-relativistic potential model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

Comment on spin-spin potential

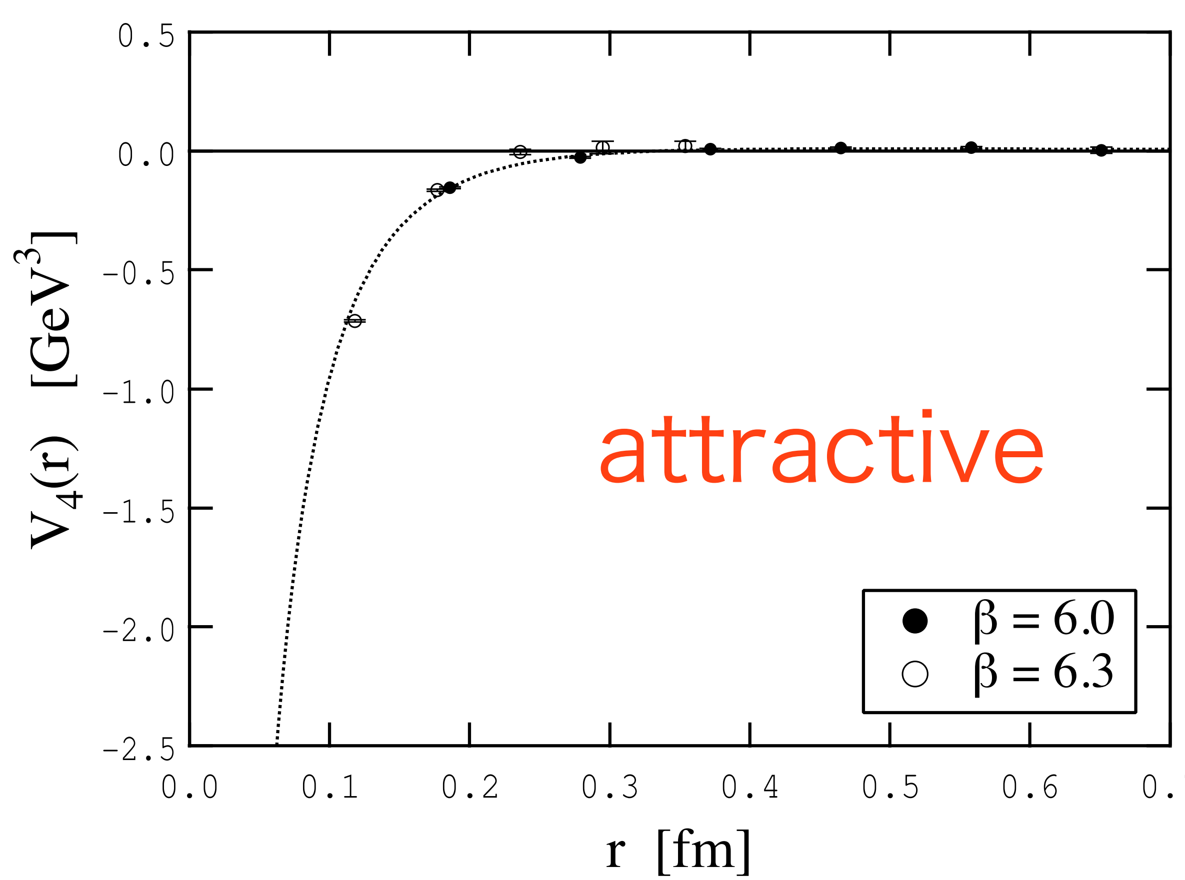
$$V(r) = V_{c\bar{c}}(r) + \mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} V_{\mathrm{spin}}(r)$$

$$V_{\mathrm{spin}}(r) \propto \nabla^2 V_{c\bar{c}}(r)$$

Our approach



Wilson loop approach



Note: $M(0^-) < M(1^-)$

Y. Koma and M. Koma, NPB769 (2007) 79

Summary

✳ Charmonium-hadron interaction

- Weakly attractive
  - ✓ Not strong enough to form hadronic-molecular states
  - ✓ Long-range screening of the color van der Waals force
- J/ψ-N scattering length ~ 0.17 fm (σ_{el}~3-4 mb)
  - ✓ Larger than QCDSR estimate, but smaller than model estimates
  - ✓ More detailed information on charmonium-hadron interaction is also available under twisted boundary conditions
    - Elastic scattering amplitude at low energies

Summary

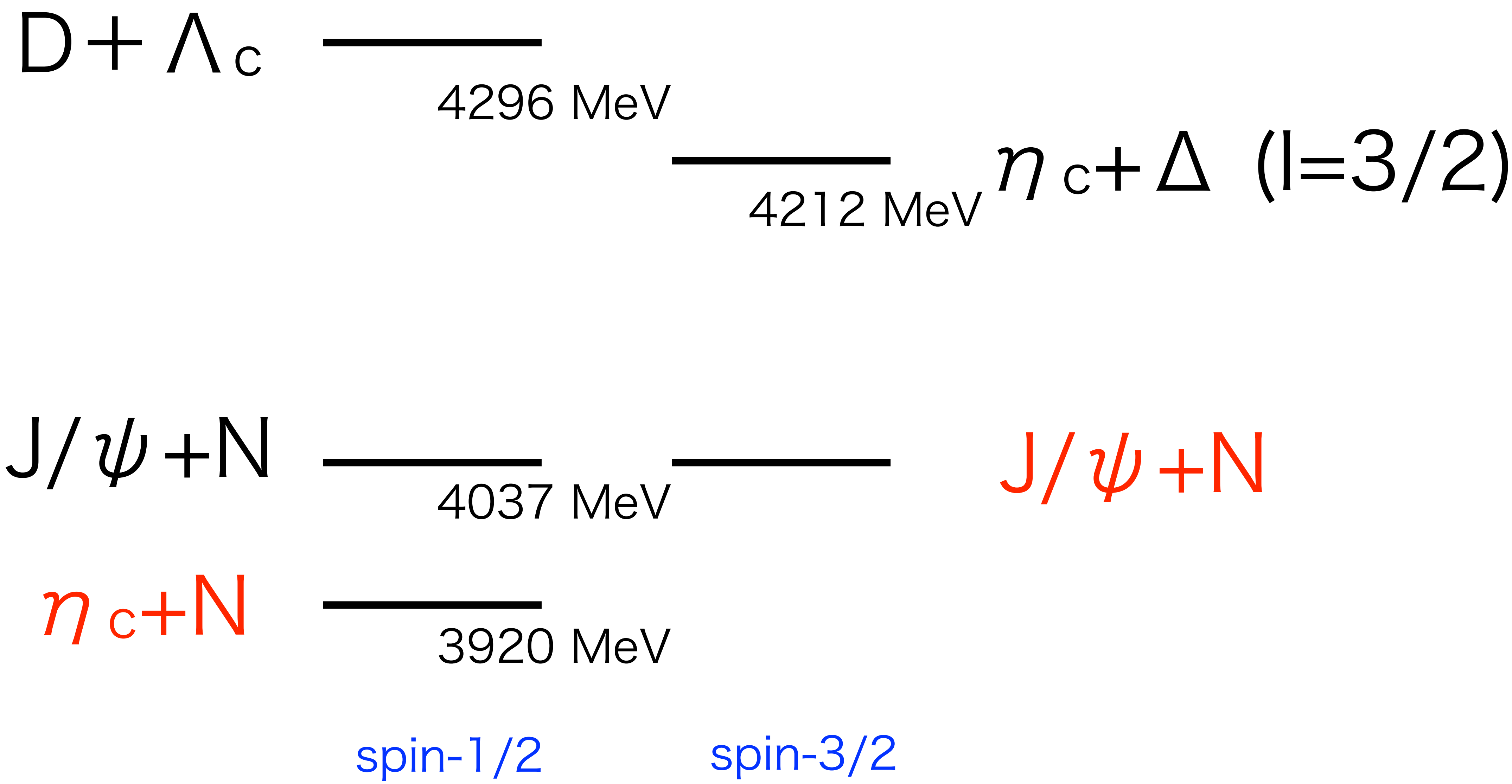
✴ Structure of charmonia

- BS wave function for $Q\bar{Q}$ system
  - ✓ Size of the J/ψ state is larger than that of the η_c
    - J/ψ-N interaction is stronger than the η_c-N case
- New method to calculate $Q\bar{Q}$ potential
  - ✓ $Q\bar{Q}$ potential is obtained at finite quark mass
  - ✓ Spin-independent $Q\bar{Q}$ potential smoothly approaches the Wilson loop result in the heavy quark limit
  - ✓ Spin-spin potential, which is determined in dynamical simulations for the first time, properly exhibits the short range repulsive interaction


Grazie

Backup Slides

Possible contamination from channel mixings



In the case of J/ψ-N, we cannot exclude the possible contamination of the η_c-N state to the spin-1/2 J/ψ-N state

How to treat heavy quarks

- Heavy quark mass introduces discretization errors of **$O((ma)^n)$**

✓ **At charm quark, it becomes severe:**

$m_c \sim 1.5$ GeV and $1/a \sim 2$ GeV, then $m_c a \sim O(1)$

- Relativistic heavy quark (RHQ) approach:

A.X. El-Khadra, A.S. Kronfeld, P.B. Mackenzie (1997)

✓ **All $O((ma)^n)$ and $O(a\Lambda)$ errors are removed by the appropriate choice of six canonical parameters $\{m_0, \zeta, r_t, r_s, C_B, C_E\}$**

$$S_{\mathrm{lat}} = \sum_{n,n'} \bar{\psi}_n \mathcal{K}_{n,n'} \psi_{n'}$$

explicit breaking of axis-interchange symmetry

$$\mathcal{K} = m_0 + \gamma_0 D_0 + \zeta \gamma_i D_i - \frac{r_t}{2} D_0^2 - \frac{r_s}{2} D_i^2 + C_B \frac{i}{4} \sigma_{ij} F_{ij} + C_E \frac{i}{2} \sigma_{0i} F_{0i}$$

✓ **We follow the Tsukuba procedure to determine parameters**

S. Aoki, Y. Kuramashi, S.-I. Tominaga (1999)

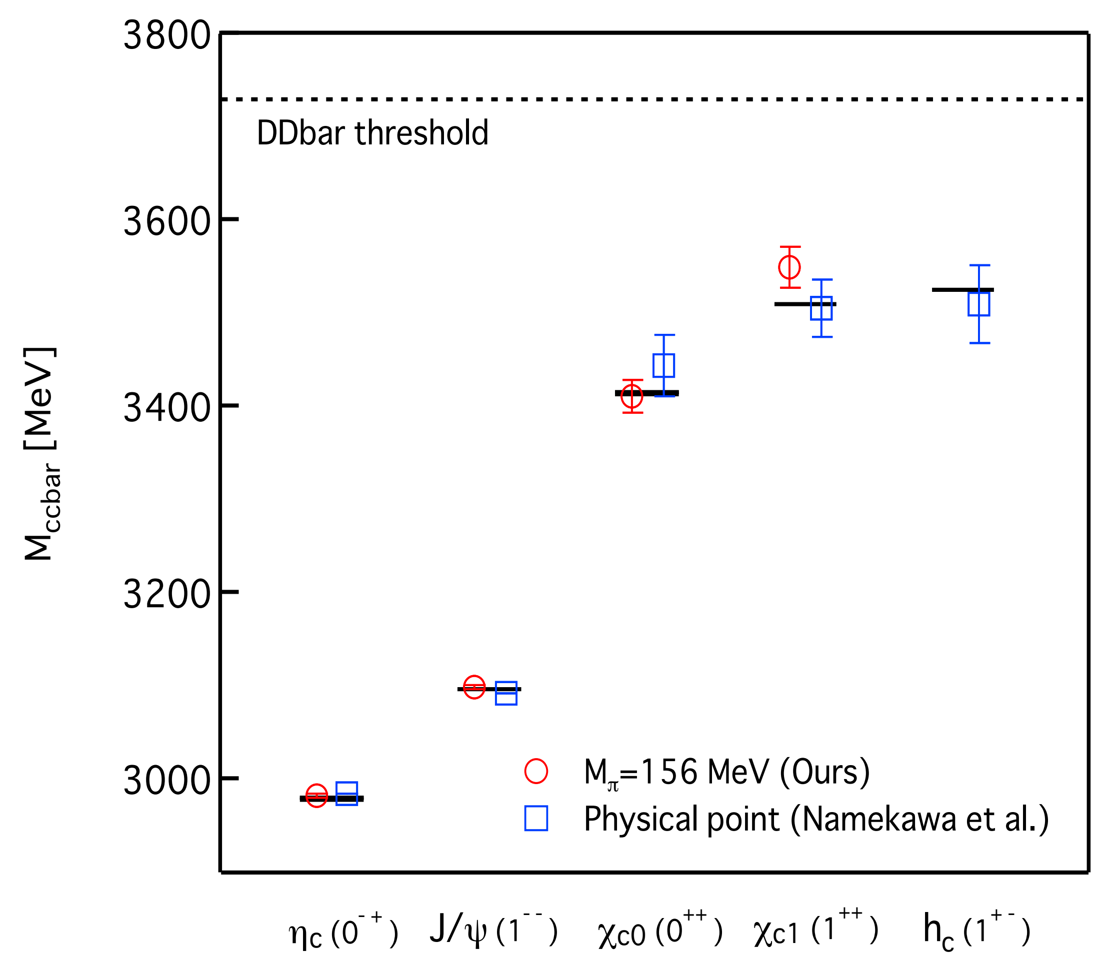
Tuning RHQ parameters

- RHQ action (Tsukuba-type) with 5 parameters
  - ✳ PACS-CS configurations at m_π=156 MeV
  - ✳ Relativistic Heavy Quark (RHQ) action for charm
    - ✓ 32^3 x 64 lattice
    - ✓ a = 0.0907(13) fm
    - ✓ La ~ 2.9 fm
    - ✓ 198 configs

➡ $\frac{1}{4}\left(M_{\eta_c} + 3M_{J/\psi}\right) = 3.069(2)\ \mathrm{GeV}$

➡ $\Delta M_{\mathrm{hyp}} = 111(2)\mathrm{MeV}$

✓ $c^2_{\mathrm{eff}} = 1.04(5)$

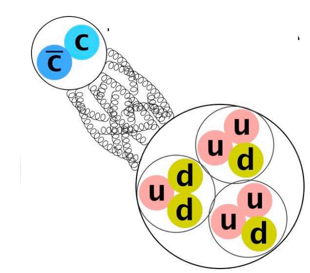


Namekawa et al., (PACS-CS), arXiv:1104.4600

$c\bar{c}$-nucleus as ideal mesic nuclei

✤ There is no Pauli exclusion effect and no nuclear absorption for the $c\bar{c}$ state in the nucleus. → No complicated many-body effect

✓ If the attraction between the $c\bar{c}$ and nucleon is sufficiently strong, the $c\bar{c}$ could be bound to nucleus for large A.



- Charmonium is bound to nuclei for A≥3
- Charmonium binding energy in nuclear matter is O(10)MeV

Brodsky et al. PRL64 (90) 1011, Luke et al. PLB288 (92) 355

✓ Precise information on the charmonium-nucleon interaction is indispensable for exploring nuclear-bound charmonium state.

Wilson-loop approach may spoil δ-type repulsive interaction

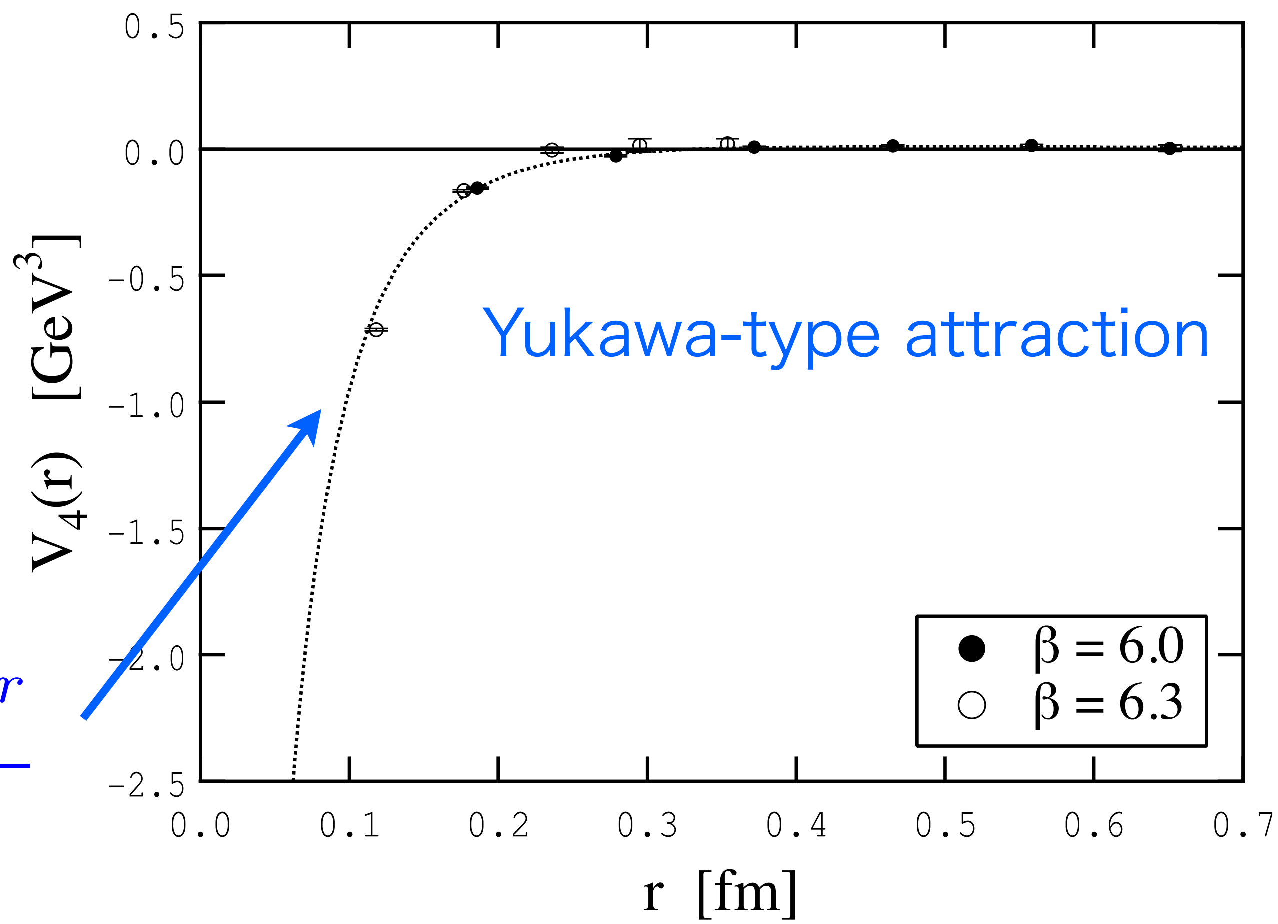
$$V_{\rm spin}(r) \propto \nabla^2 V_{c\bar{c}}(r)$$

$$V_{c\bar{c}}(r) = \begin{cases} -\dfrac{1}{r} & \text{Coulomb} \\ -\dfrac{e^{-\alpha r}}{r} & \text{Yukawa} \end{cases}$$

$$\nabla^2 \left(\frac{e^{-\alpha r}}{r} \right) = -4\pi\delta(r) + \alpha^2 \frac{e^{-\alpha r}}{r}$$

$$\nabla^2 \left(\frac{1}{r} \right) = -4\pi\delta(r)$$

origin of repulsive interaction



$$\nabla^2 V_{c\bar{c}} \to V''_{c\bar{c}} + \frac{2}{r} V'_{c\bar{c}}$$ in Wilson-loop approach