The QCD phase diagram from Complex Langevin simulations

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work with Ion-Olimpiu Stamatescu and Denes Sexty
Outline

• Intro to CLE in QCD at finite $\mu$

• Viability of CLE for the Phase diagram

• Results for the phase transition
Motivation

Phase diagram from first principles
What is Langevin?

- Real Langevin: Stochastic differential equation with stationary solution $\rho(U) = e^{-S}$

- Update prescription:

$$U' = \exp\left( - \sum_j i\lambda_j \left[ \epsilon D_j S_{eff}[U] + \sqrt{\epsilon} \eta_j \right] \right) U$$
What is Complex Langevin

- Complex Langevin: Analytic continuation of Real Langevin $SU(3) \rightarrow SL(3, \mathbb{C})$
- Sample a real probability distribution $P(U)$ with $U \in SL(3, \mathbb{C})$ which fulfills

$$\langle \mathcal{O} \rangle_{\rho(U \in SU(3))} = \langle \mathcal{O} \rangle_{P(U \in SL(3, \mathbb{C}))}$$

i.e. the real process mimics the complex process
- Update prescription: same!
Our setup

- QCD with $N_f = 2$ Wilson fermions
- Parameters $\beta = 5.9 \quad \kappa = 0.15 \quad N_s \in \{8, 12, 20\}$
  vary $N_t$ to vary temperature $a = 0.065 \, fm$
- Pion mass (depends on volume for small volumes)

$$m_\pi \approx 1.3 GeV \quad \text{for} \quad N_s = 20$$
Possible issues

- Wrong convergence
  - Boundary terms
  - Poles/non-holomorphicticity (Fermion determinant)

(RMT: 1309.4335, 1712.07514; QCD-Models: 1701:02322)

- How to control?
Wrong convergence, when?

- **Non-vanishing boundary terms** (0912.3360, 1101.3270, 1808.05187)
  cf. Talk by I.-O. Stamatescu on Wednesday

- Accompanied by large drifts in non-compact direction

- Measure: Unitarity norm \( N_U = \sum U^\dagger U - 1 \)
Unitarity Norm and boundary terms

- Use gauge cooling (gauge transformation in direction towards the unitary manifold)
- Becomes too large at some point
- What is faster: Thermalization or Unorm?
- Cut off simulation if Unorm too large (we use ~0.1)
- Alternative: Dynamical stabilization, cf. Talk by F. Attanasio on Wednesday, 1808.04400
Possibly wrong convergence: Poles

- Zero Eigenvalues of Fermion matrix?

(note: low-T comparison during thermalization, since large chem. Pot. did not thermalize within reasonable time)

\[ N_t = 12 \quad \text{and} \quad N_t = 20 \]
Possible phase diagram

- Poles exclude low T-high mu region
- Need to get poles away, deformation technique? 1805.03964
- Not good in RMT 1712.07514
Current work

- How far can we resolve the transition?
Ns=8, Transition for different mu
Ns=12, Transition for different mu
Ns=20, Transition for different $\mu$
Ns=8 Curvature of transition line method 1

- Use Binder cumulant $\frac{\langle x^4 \rangle}{\langle x^2 \rangle^2}$ of (symmetrized)Polyakov loop

- Advantage: temperature dependence of renormalization drops out $P = e^{-c(a)N_t} P_{\text{bare}}$
Ns=12 Curvature of transition line method 1
Curvature of transition line method 2

- Find transition at $\mu = 0$ from the Binder cumulant; estimate transition temperature at $\mu > 0$ from shift in the Polyakov loop
Reliable?
Summary and Outlook

- Determination of transition temperature in large range

- Outlook: Higher statistics especially for larger lattices

- Open Question: How to properly deal with zero Eigenvalues?
Thank you.