The QCD phase diagram from Complex Langevin simulations

Manuel Scherzer (ITP Heidelberg)

work with Ion-Olimpiu Stamatescu and Denes Sexty

UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

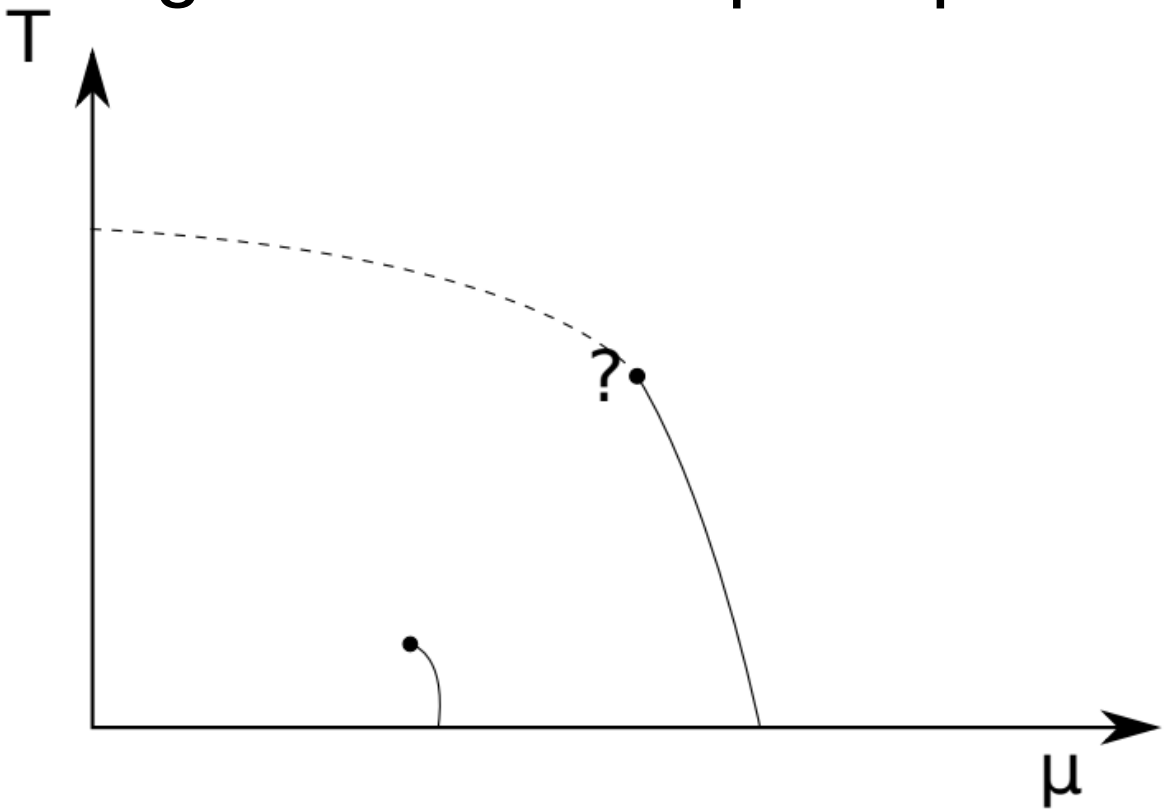
DFG Deutsche Forschungsgemeinschaft

Outline

- Intro to CLE in QCD at finite μ
- Viability of CLE for the Phase diagram
- Results for the phase transition

Motivation

Phase diagram from first principles



What is Langevin?

- Real Langevin: Stochastic differential equation with stationary solution $\rho(U) = e^{-S}$

- Update prescription:

$$U' = \exp\left(-\sum_j i\lambda_j \left[\epsilon D_j S_{eff}[U] + \sqrt{\epsilon}\eta_j\right]\right) U$$

What is Complex Langevin

- Complex Langevin: Analytic continuation of Real Langevin $SU(3) \to SL(3,\mathbb{C})$
- Sample a real probability distribution $P(U)$ with

$U \in SL(3,\mathbb{C})$ which fulfills

$$\langle \mathcal{O} \rangle_{\rho(U \in SU(3))} = \langle \mathcal{O} \rangle_{P(U \in SL(3,\mathbb{C}))}$$

i.e. the real process mimics the complex process

- Update prescription: same!

Our setup

- QCD with $N_f = 2$ Wilson fermions
- Parameters $\beta = 5.9$ $\kappa = 0.15$ $N_s \in \{8, 12, 20\}$

  vary N_t to vary temperature $a = 0.065 fm$
- Pion mass (depends on volume for small volumes)

$m_\pi \approx 1.3 GeV$ for $N_s = 20$

Possible issues

- Wrong convergence
  - Boundary terms
  - Poles/non-holomorphicity (Fermion determinant)

    (RMT: 1309.4335, 1712.07514; QCD-Models: 1701:02322)

- How to control?

Wrong convergence, when?

- Non-vanishing boundary terms (0912.3360, 1101.3270, 1808.05187)
  cf. Talk by I.-O. Stamatescu on Wednesday

- Accompanied by large drifts in non-compact direction

- Measure: Unitarity norm $N_U = \sum U^\dagger U - 1$

Unitarity Norm and boundary terms

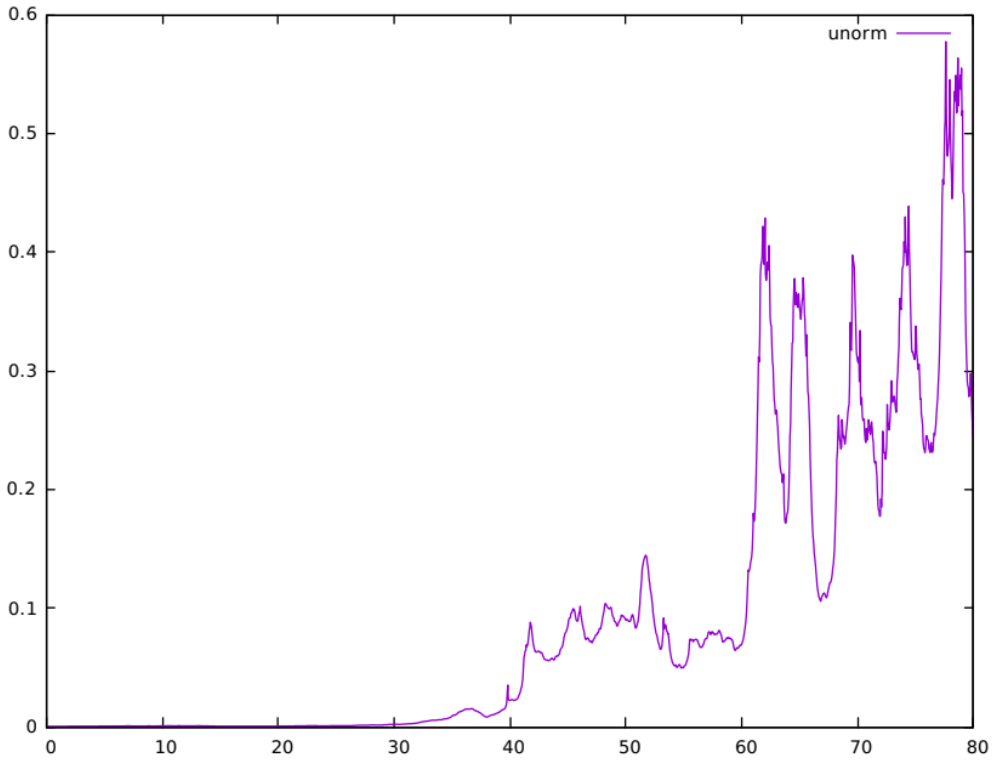
- Use gauge cooling (gauge transformation in direction towards the unitary manifold)
- Becomes too large at some point
- What is faster:

  Thermalization or Unorm?
- Cut off simulation

  if Unorm too large

  (we use ~0.1)
- Alternative: Dynamical stabilization,

  cf. Talk by F. Attanasio on Wednesday, 1808.04400

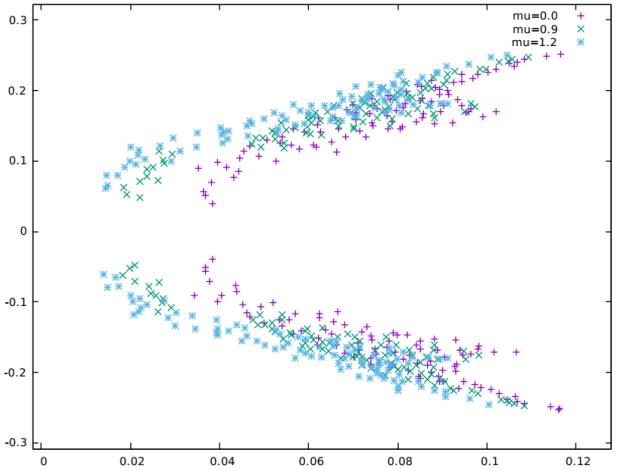


Possibly wrong convergence: Poles

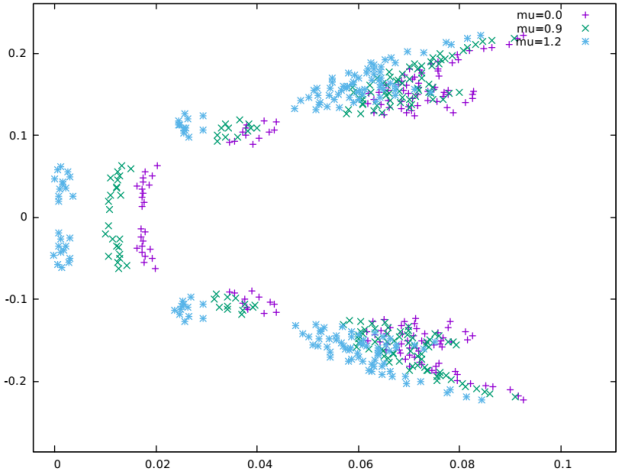
- Zero Eigenvalues of Fermion matrix?

  (note: low-T comparison during thermalization, since large chem. Pot. did not thermalize within reasonable time)

$N_t = 12$

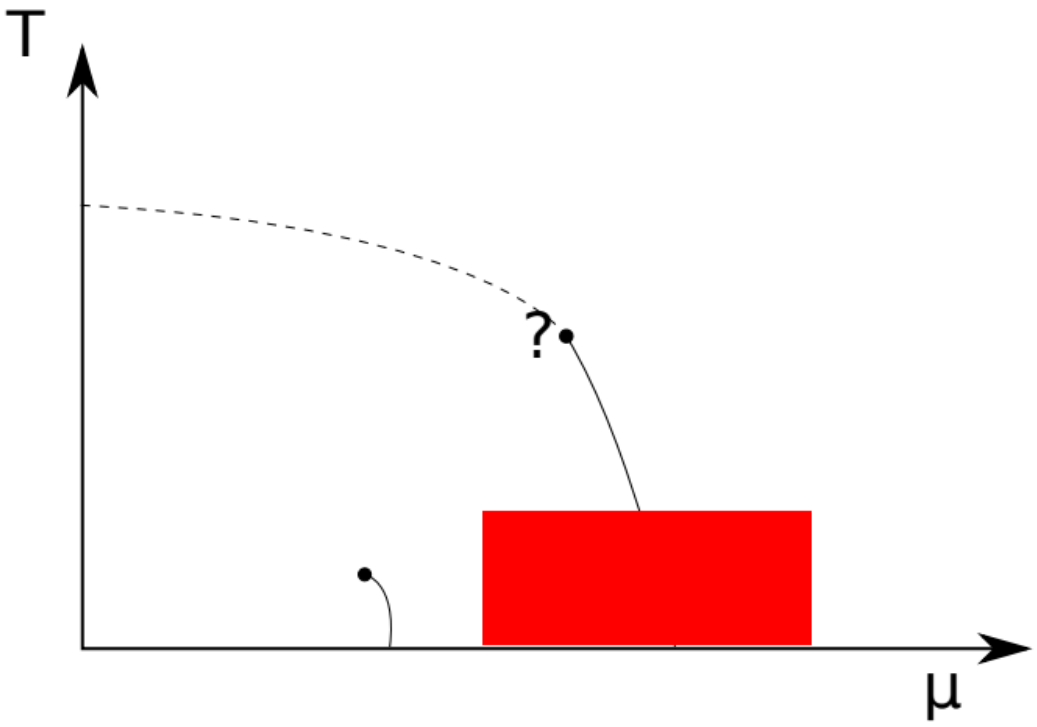


$N_t = 20$



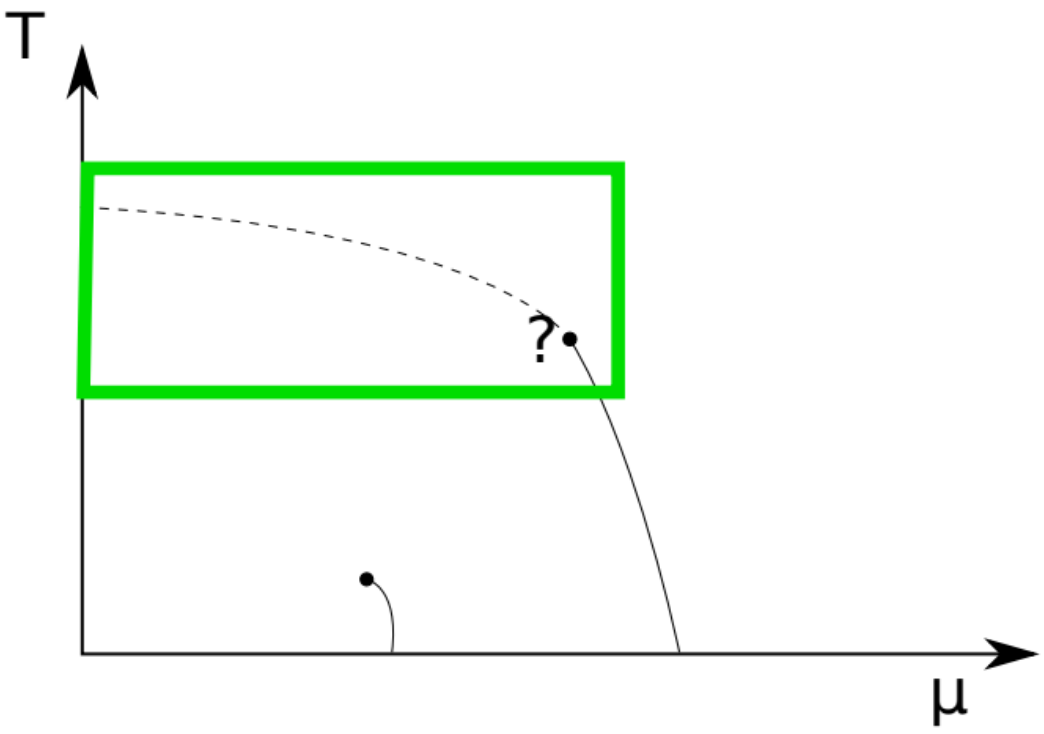
Possible phase diagram

- Poles exclude low T-high mu region
- Need to get poles away, deformation technique? 1805.03964
- Not good in RMT 1712.07514

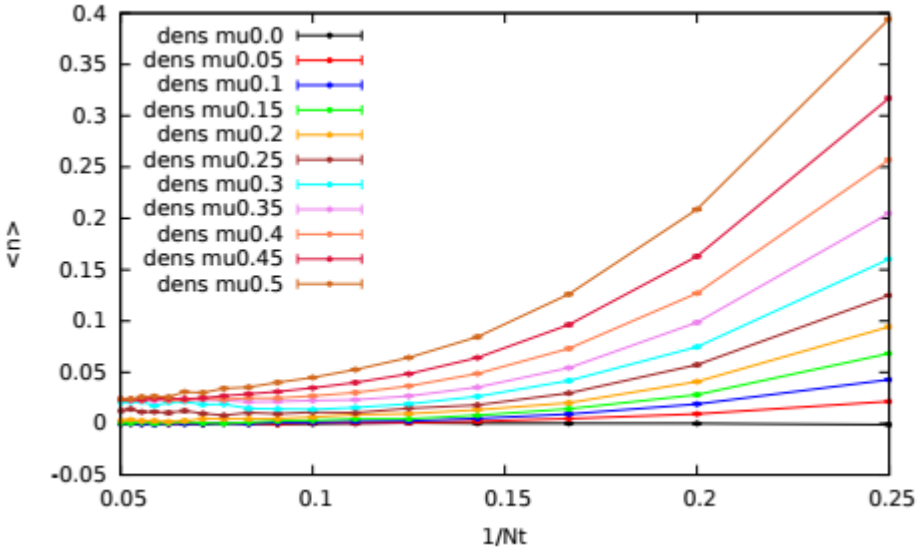


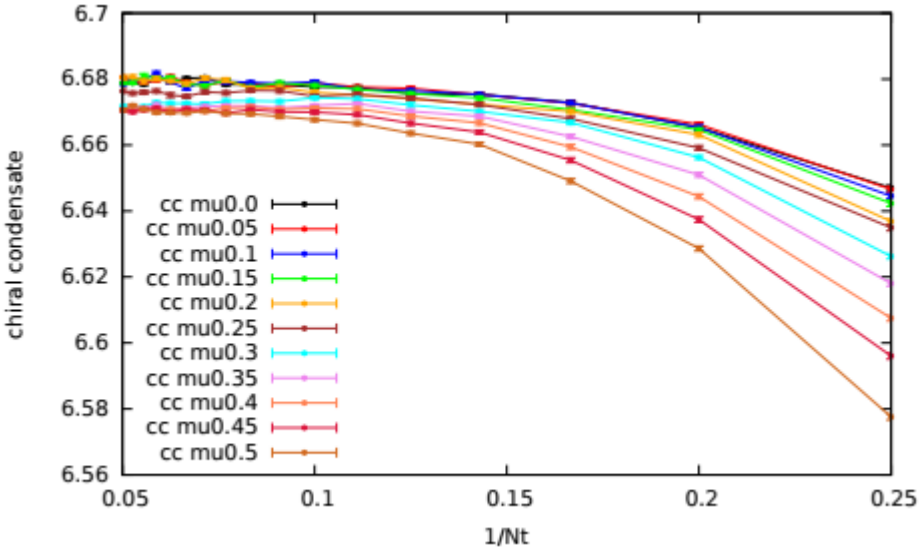
Current work

- How far can we resolve the transition?

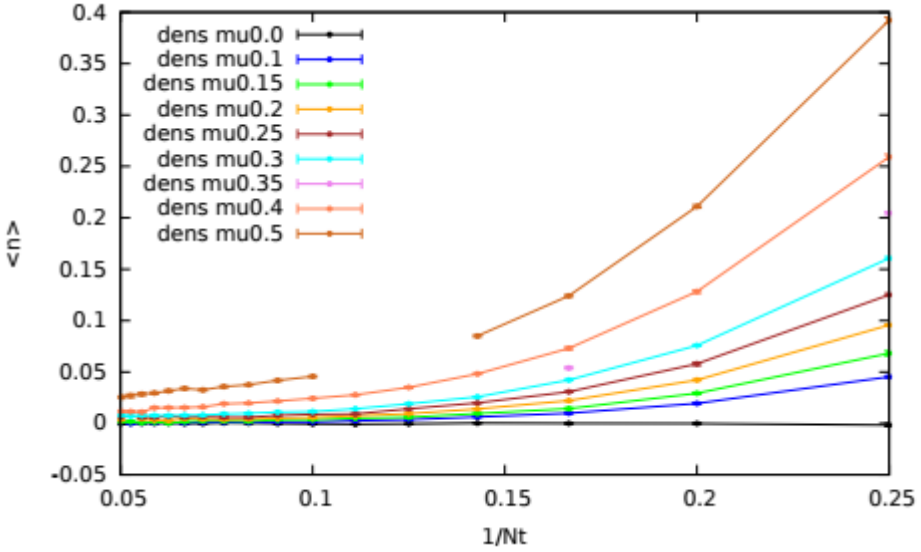


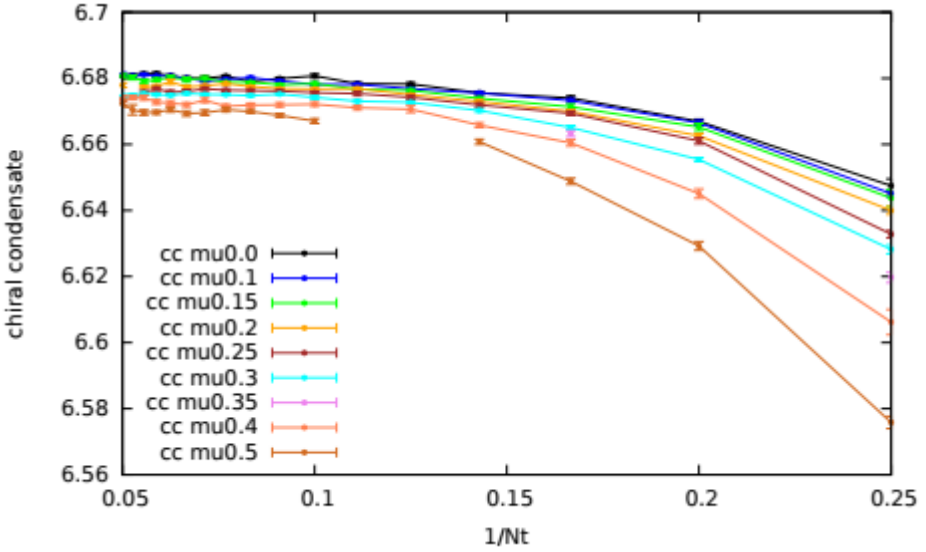
Ns=8, Transition for different mu



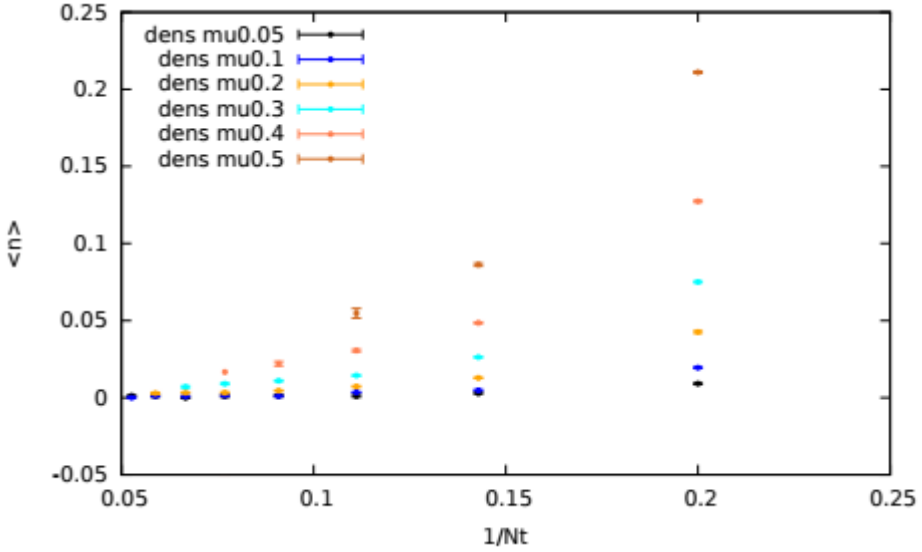


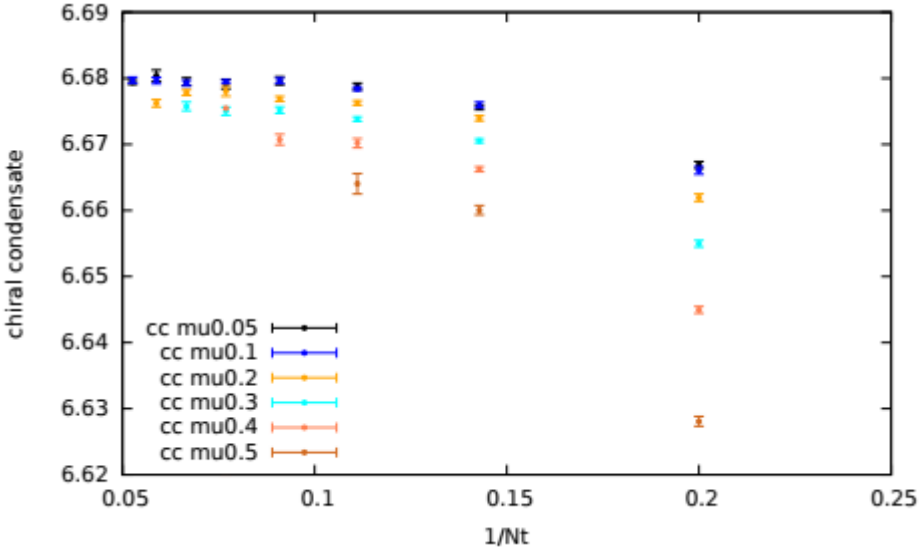
Ns=12, Transition for different mu





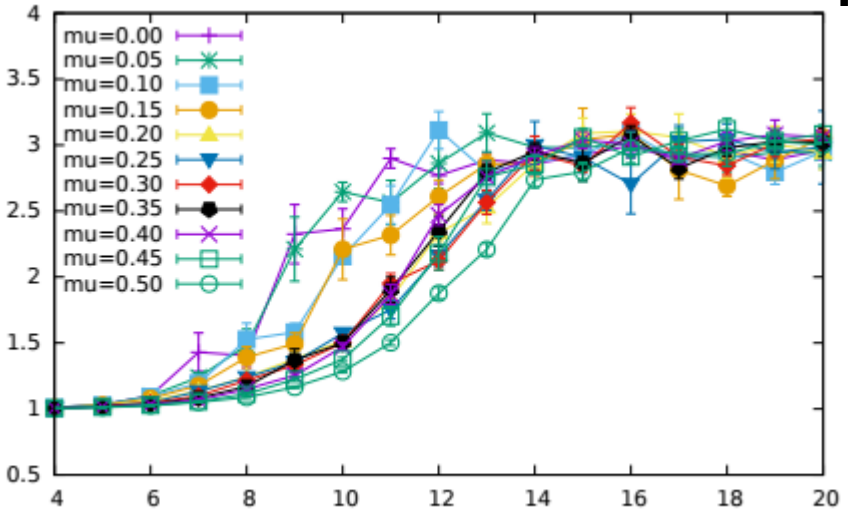
Ns=20, Transition for different mu

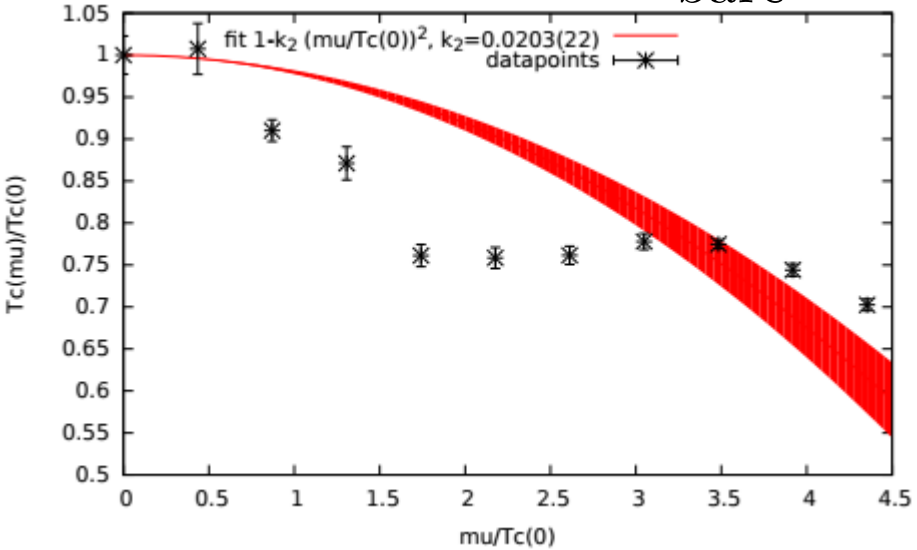




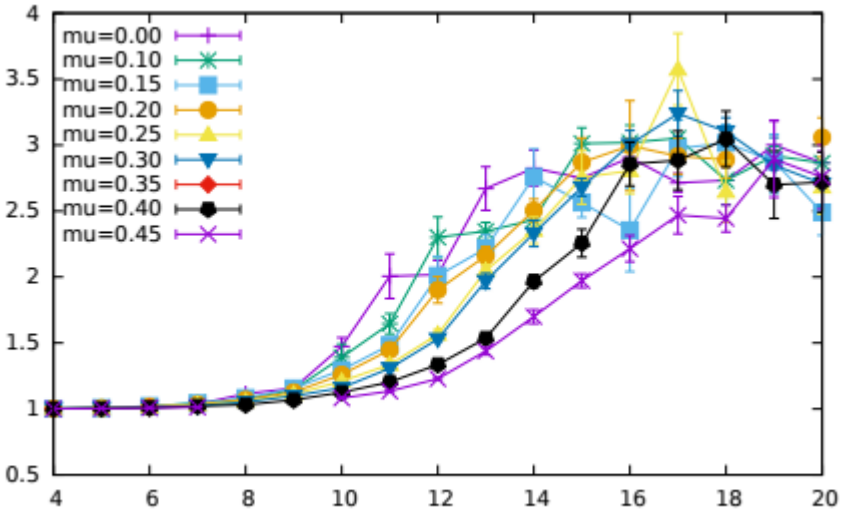
Ns=8 Curvature of transition line method 1

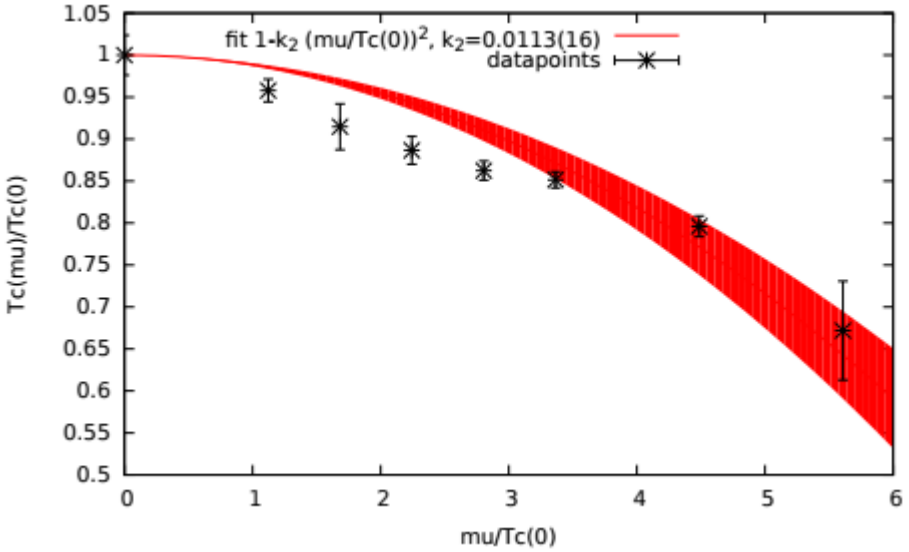
- Use Binder cumulant $\langle x^4 \rangle / \langle x^2 \rangle^2$ of (symmetrized)Polyakov loop
- Advantage: temperature dependence of renormalization drops out $P = e^{-c(a)N_t} P_{\mathrm{bare}}$





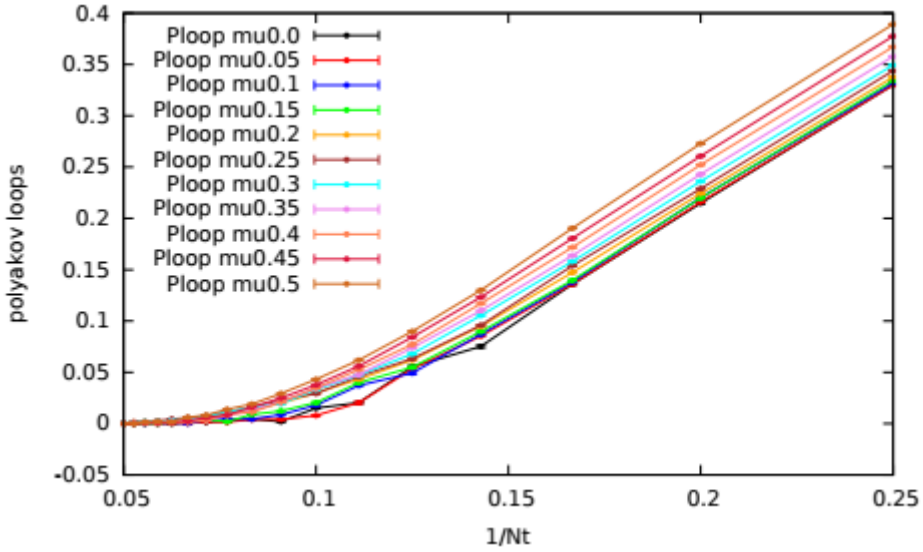
Ns=12 Curvature of transition line method 1

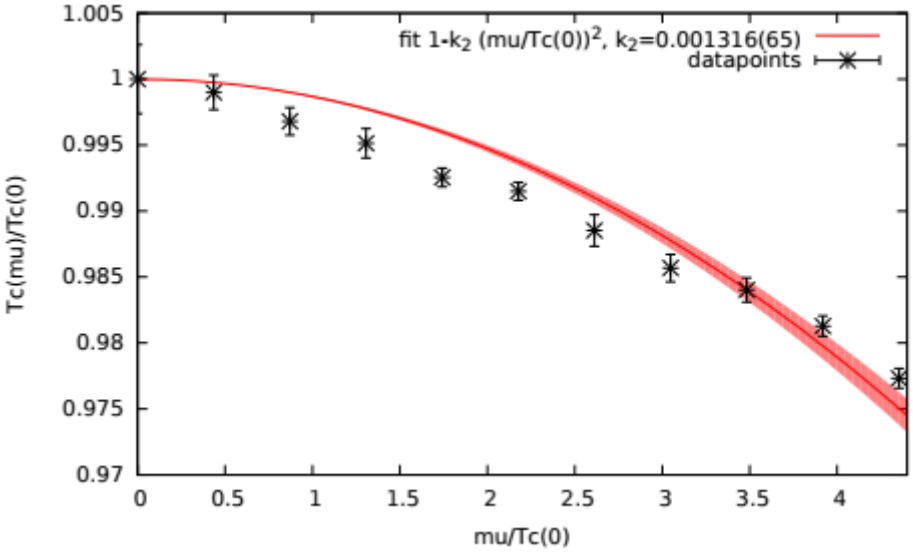




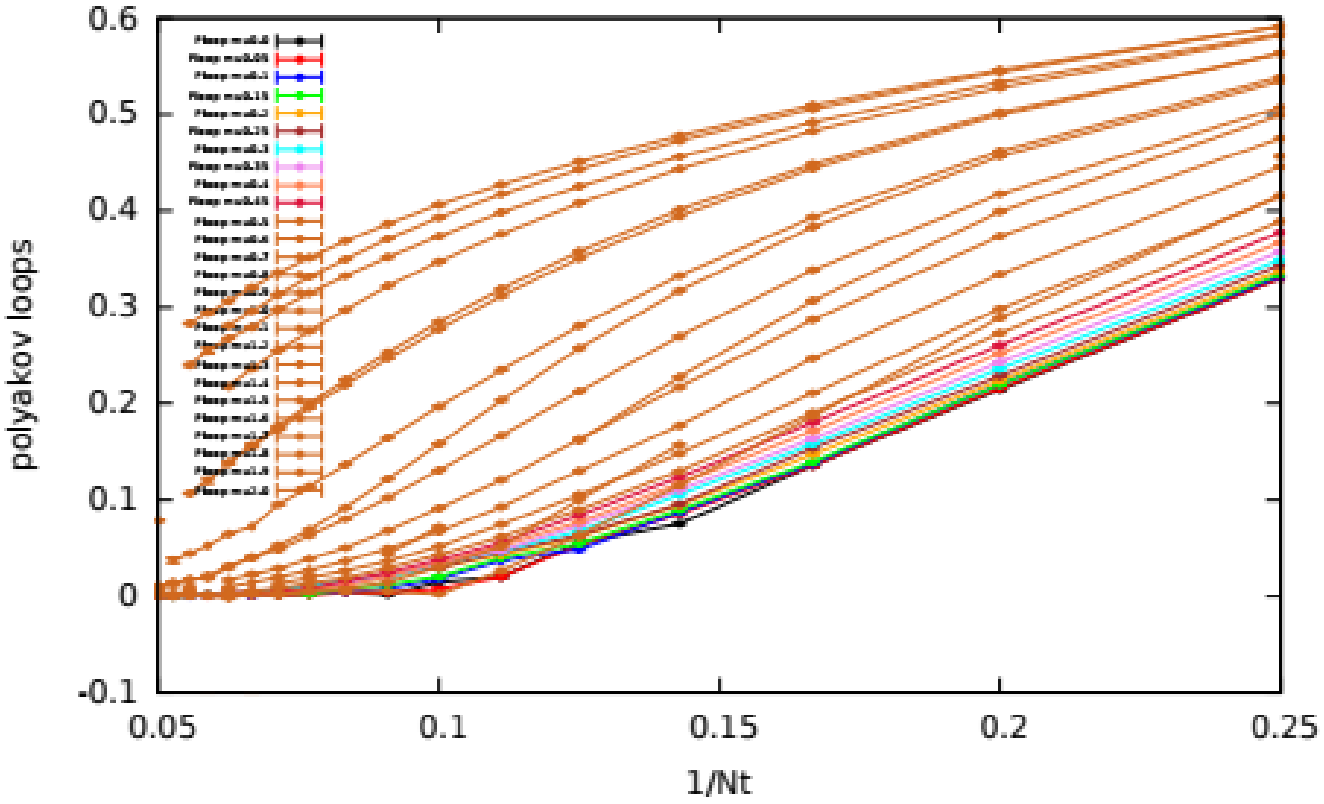
Curvature of transition line method 2

- Find transition at $\mu = 0$ from the Binder cumulant; estimate transition temperature at $\mu > 0$ from shift in the Polyakov loop





Reliable?



Summary and Outlook

- Determination of transition temperature in large range
- Outlook: Higher statistics especially for larger lattices
- Open Question: How to properly deal with zero Eigenvalues?

Thank you.