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How much can heavy fields be excited during inflation?

arxiv: 1405.????

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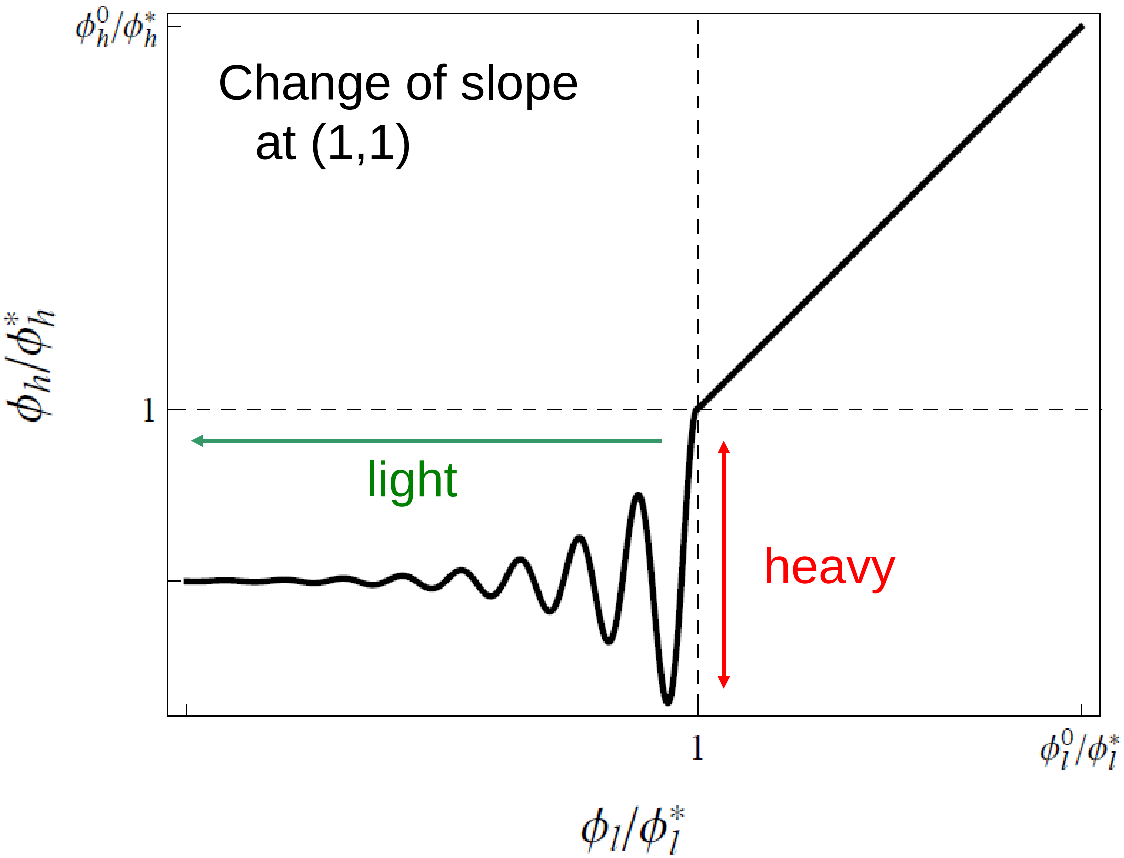
How much can we excite a heavy field during inflation without inducing observable repercussions?

Formulation of Problem:

Sudden turn in field space, sudden change of slope etc. leads to the excitation of heavy fields, and subsequent, transient oscillations:



How Big can these oscillations be to be consistent with current observations of the CMBR?

Mostly studied in the literature:

The effect of the subsequent oscillations onto perturbations (EFT by Achuccaro et al., Chen et al. (analytic), Langlois et al. (analytic and numeric), Yamaguchi et al. (numeric), …)

We are interested in the effect of the excitation mechanism.

Setup: 2 field toy-model

Consider sudden change in mass of one field (slow roll of the light field and inflation is uninterrupted!)

$$S = \int d^4 \sqrt{-g} \left(\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \sum_{i=1}^{2} \partial_\mu \phi_i \partial_\nu \phi_i + V(\phi_1, \phi_2) \right)$$

$$V(\phi_1, \phi_2) = \sum_{i=1}^{2} V_i(\phi_i)$$

$$V_i^-(\phi_{i-}) = \frac{1}{2} m_{i-}^2 \phi_{i-}^2 \quad , \quad t \leq t_* \quad ,$$

$$V_i^+(\phi_{i-}) = \begin{cases} \frac{1}{2} m_{1+}^2 (\phi_{1+} - \tilde{\phi}_{1+})^2 \\ \frac{1}{2} m_{2+}^2 \phi_{2+}^2 \end{cases} \quad , \quad t \geq t_*$$

$$m_l \equiv m_{1-} \equiv m_{2-} \equiv m_{2+} \ll m_{1+} \equiv m_h$$

Background Dynamics (solve EOM):

Before mass change: slow roll of both fields

$$\phi_l \approx \phi_l^* \left[1 + \sqrt{\frac{2}{3}} \left(\frac{M_{pl}}{\phi_l^*}\right) m_l \left(t_* - t\right)\right]$$

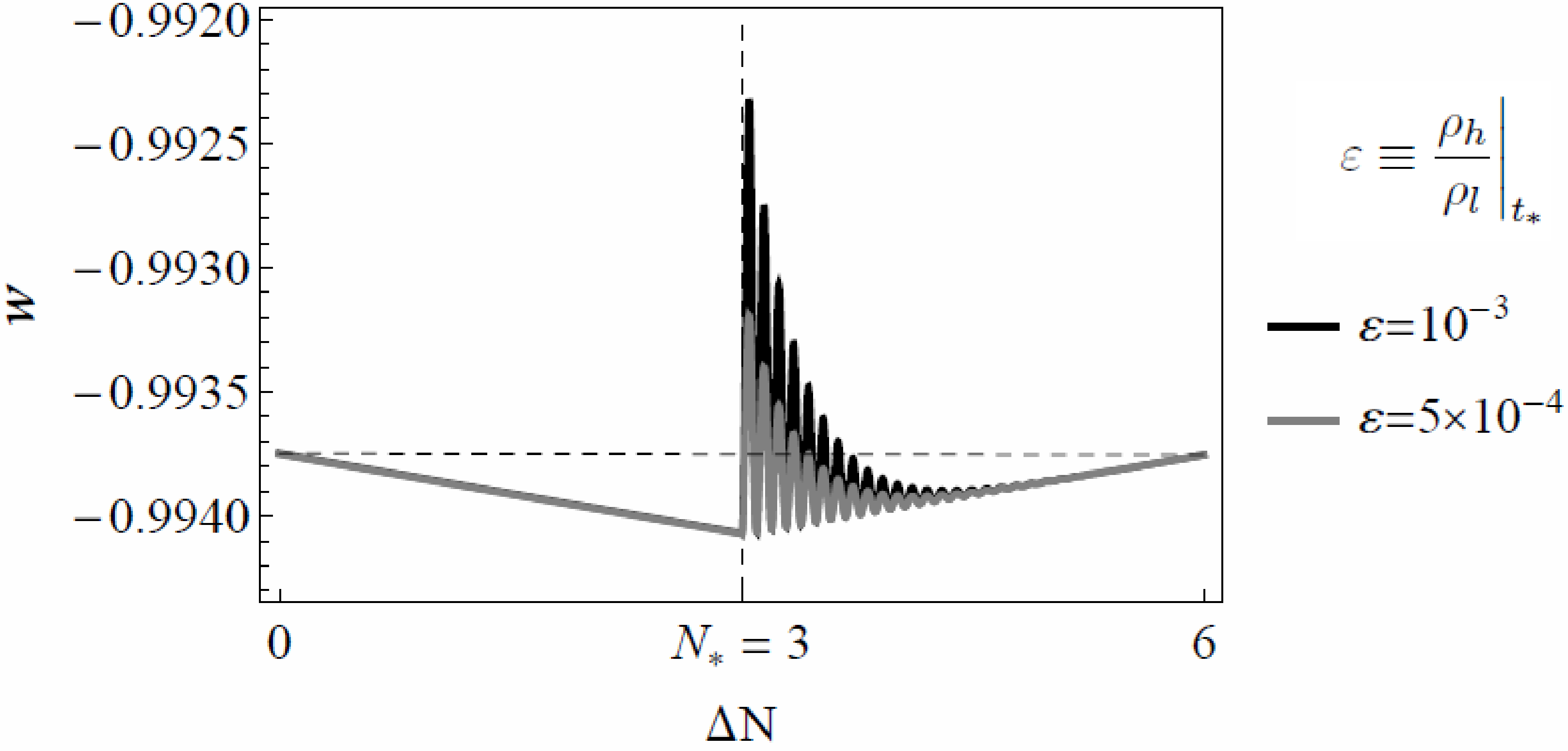
$$\phi_h \approx \phi_h^* \left[1 + \sqrt{\frac{2}{3}} \left(\frac{M_{pl}}{\phi_l^*}\right) m_l \left(t_* - t\right)\right]$$

After mass change: heavy field oscillates

$$\phi_l \simeq \phi_l^* \left[1 - \sqrt{\frac{2}{3}} \left(\frac{M_{pl}}{\phi_l^*}\right) m_l \left(t - t_*\right)\right]$$

$$\phi_h - \tilde{\phi}_h \simeq C \left(\frac{a}{a_*}\right)^{-3/2} \sin\left[m_h(t - t_*) + \gamma\right]$$

Equation of state parameter: $1 + w = \frac{\dot{\phi}_l^2 + \dot{\phi}_h^2}{3M_{pl}^2 H^2}$



Choose parameters s.t. inflation is not interrupted.

Two effects:

1. Transient, fast oscillations in *w* (visible above)
2. Small change in asymptotic value before and after transition (above ~ 10^{-6} not visible to the naked eye, but more important for perturbations.)

Perturbations:

We compute the power-spectrum two ways:

1. Sudden transition approximation where we ignore the intermediate transient oscillations in ``*w*'' and we ignore perturbations in the heavy field.

Simple effect: jump in ``*w*''

$$\frac{\Delta w}{1+w_-} \equiv \frac{w_- - w_+}{1+w_-} \approx \frac{\varepsilon}{1+\varepsilon} \approx \varepsilon$$

causes transient oscillations in Power-spectrum;
(known effect, see Joy, Sahni, Starobinski 08, multi field: Battefeld et al. 10, …)

2. Two field setup at the perturbed level; keep oscillations in ``*w*''.

Analytic treatment possible, but quite cumbersome. Results are consistent with the much simpler sudden transition approximation!
(Main new result of our article)

1. Sudden Transition approximation:

Jump in asymptotic value of ``*w'':*

$$\frac{\Delta w}{1+w_-} \equiv \frac{w_- - w_+}{1+w_-} \approx \frac{\varepsilon}{1+\varepsilon} \approx \varepsilon$$

EOM of Sasaki-Mukhanov variable of light field (truncate perturbations):

$$\ddot{Q}_l + 3H\dot{Q}_l + \frac{k^2}{a^2}Q_l + \left(V_l'' - \frac{M_{pl}^{-2}}{a^3}\left(\frac{a^3}{H}\dot{\phi}_l^2\right)^{\cdot} \right) Q_l = 0$$

Solutions:

$$Q_l^- = \frac{\sqrt{\pi}}{2} e^{i(\nu_l^- + 1/2)\pi/2} \sqrt{-\tau}\, \mathcal{H}^{(1)}_{\nu_l^-}(-k\tau) e_l(\vec{k}) ,$$

$$Q_l^+ = \frac{\sqrt{\pi}}{2} \sqrt{-\tau} \left[\alpha(k)\mathcal{H}^{(1)}_{\nu_l^+}(-k\tau) + \beta(k)\mathcal{H}^{(2)}_{\nu_l^+}(-k\tau) \right] e_l(\vec{k})$$

Deruelle-Mukhanov matching conditions
(see D.Battefeld, T.B., Firouzahi, Koschravi 10;
multi field: D.B., T.B., Giblin, Pease 10):

$$\left[\dot{Q}_l\right]^{\pm} = 0 \quad , \quad \left[\frac{Q_l}{1+\omega}\right]^{\pm}$$

D.Battefeld, T.B., Firouzahi, Koschravi 10
T.B., R.C.de Freitas, to appear;

Power-spectrum:

After computing the Boloiubov coefficients, one gets:

$$\frac{\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}^{SR}} = \cos^2\left[x + \left(\nu_l + \frac{1}{2}\right)\frac{\pi}{4}\right] + \left(\frac{1+w_+}{1+w_-}\right)^2 \sin^2\left[x + \left(\nu_l + \frac{1}{2}\right)\frac{\pi}{4}\right]$$
$$\approx 1 - 2\varepsilon \sin^2\left(-x + \frac{\pi}{2}\right),$$

Oscillations in the power-spectrum on CMBR scales are constrained to be less than 1% (see e.g. PLANCK 13 publications).

Thus:

$$\varepsilon = \frac{\rho_h}{\rho_l} \lesssim 0.01$$

For quadratic potentials, this entails a maximal oscillation amplitude of:

$$\Xi \simeq \frac{m_l}{m_h}\phi_l^* \sqrt{\varepsilon} \leq \sqrt{\epsilon_m} 1.5 M_{pl}$$

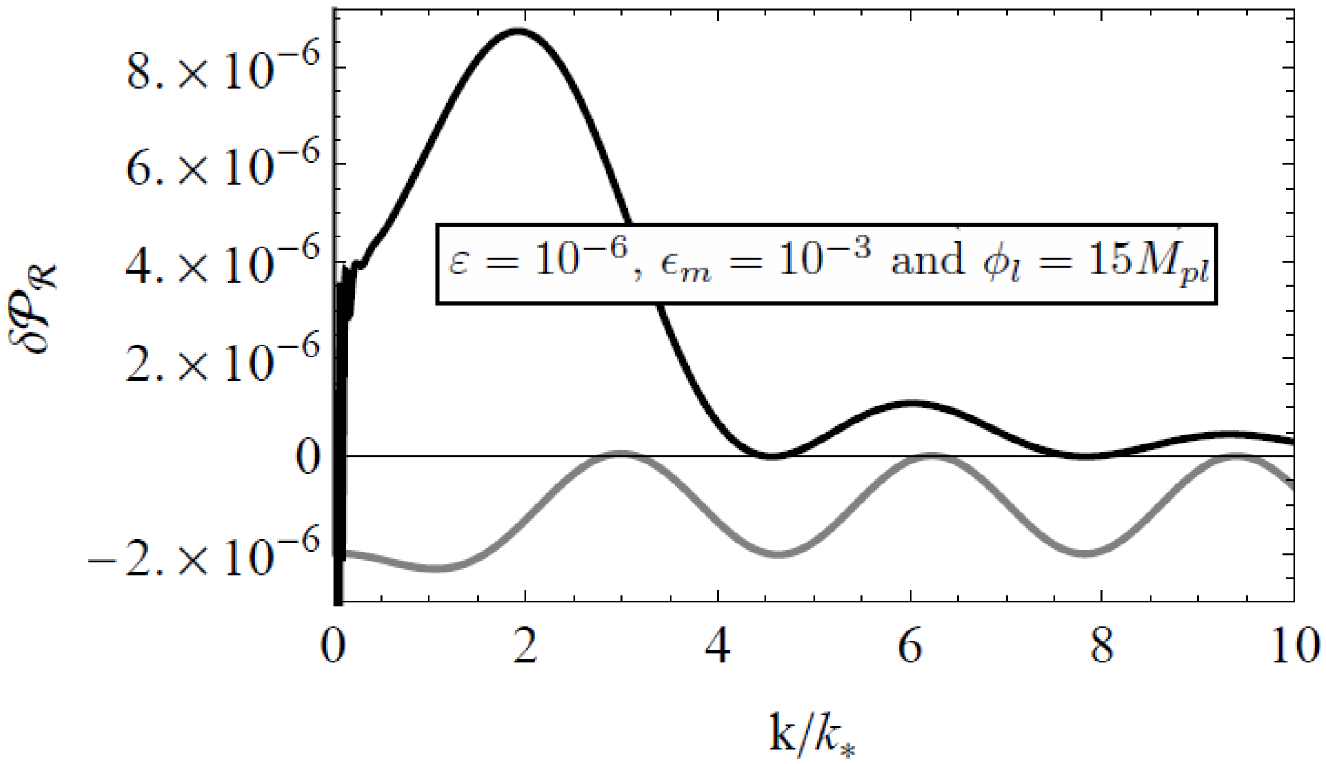
$$\epsilon_m \equiv \frac{m_l^2}{m_h^2}$$

(Consistent with necessary condition for truncating pert. to the light field, T.B., J. Niemeyer, D.Vlaykov 13)

2. Result of full two field setup at perturbed level:

Details of tedious computation in T.B., R.C.de Freitas to appear.

Black: two field analysis.
Grey: sudden transition approximation (we use different off-set for clarity).



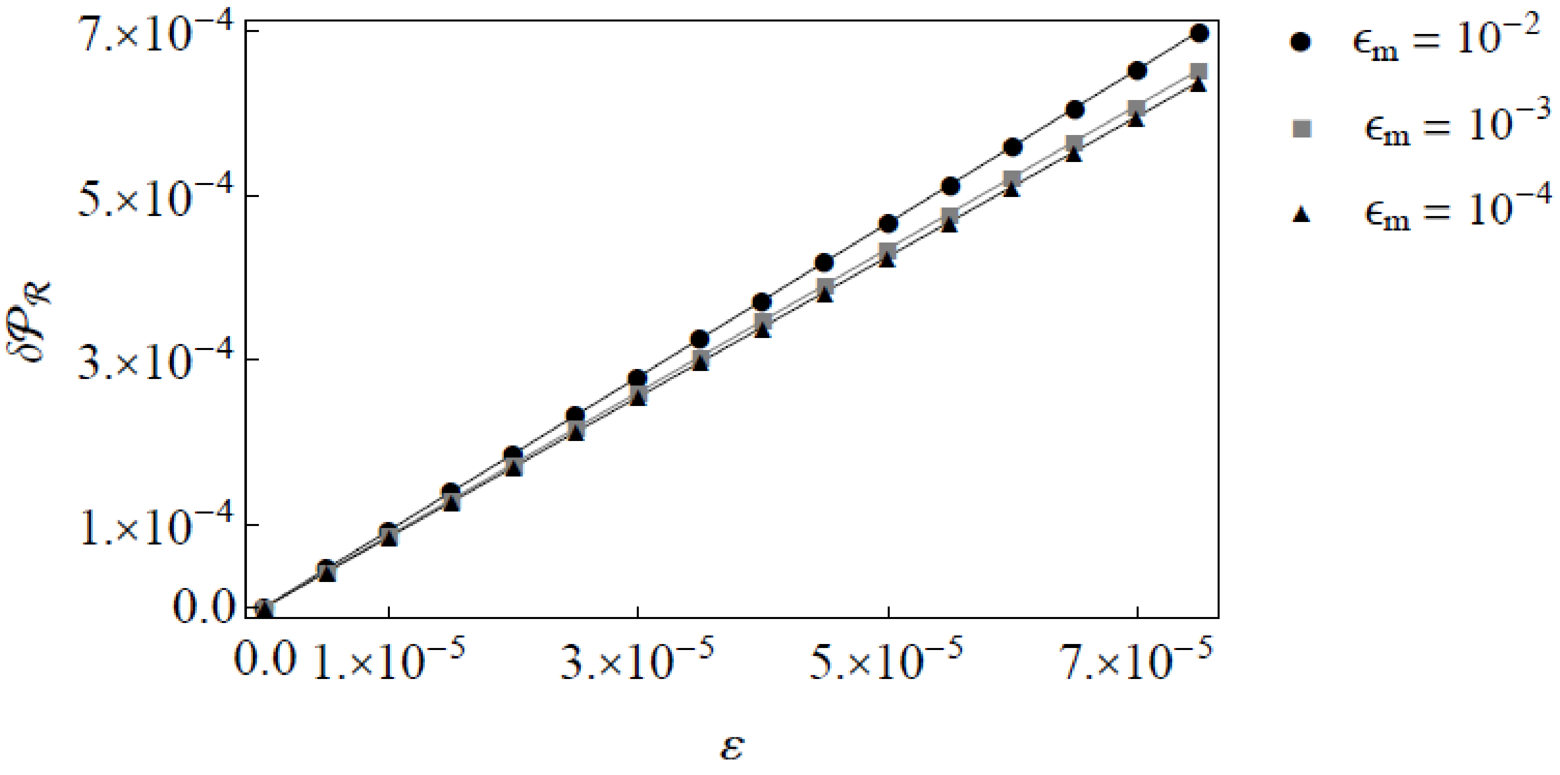
Damping in full result expected (cause: finite timescale of transition).
Approximation in 1. recovers well: initial amplitude and frequency (not the phase)

2. Result of full two field setup at perturbed level:

Plotted: Amplitude of first peak.

Small dependence on mass ratio (no dependence in sudden transition approx.);

Linear dependence on energy ratio (as expected).



Discussion:

The leading order effect onto the power-spectrum comes often from the excitation event (subsequent oscillations of the heavy field and resonance effects, as in Chen et al., are sub-leading!).

The simple, sudden transition approximation provide the order of magnitude of the effect.

This effect appears to be insensitive to the details of the excitation. It merely depends on how much energy is transferred.

The maximally allowed value of the heavy fields excitation is so small, that a truncation of fluctuations to the light field can be justified (see condition in Battefeld, Niemeyer, Vlaykov 13) and the sudden transition approximation can be used.

Take home messages:

$$\varepsilon = \frac{\rho_h}{\rho_l} \lesssim 0.01$$

The heavier a field, the less it can be excited.

Only a small fraction of the energy can be dumped into a heavy field on scales observable in the CMBR.

The leading order effect is expected to stem from the excitation event (the jump in the EOS parameter).

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