Spinning gauged boson stars in anti-de Sitter spacetime

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Research Training Group Graduiertenkolleg Models of Gravity

Outline

- Boson stars and Q-balls
- The Model
- Probe limit
- Gravitating spinning bosons
- Summary and further remarks

Boson Stars

Boson Star (BS) - a complex scalar field with harmonic time dependence coupled to gravity. It is regular everywhere and has finite energy.

$$\phi(t, r, \theta, \varphi) = \phi_0(r, \theta, \varphi)e^{i\omega t}$$

- Invariance of Einstein-Klein-Gordon Lagrangian under global $U(1)$ transformation $\phi \to \phi e^{i\varphi} \implies$ a conserved current J_a.
- The spatial integral of the time comoponent of this current N - the total number of bosonic particles.

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Rotating BS: $\phi(t, r, \theta, \varphi) = \phi_0(r, \theta)e^{i(k\varphi - \omega t)}$, where $k = 0, \pm 1, \pm 2, \ldots$
Quanization condition for the angular momentum J: $J = kN$

Q-balls

Q-ball - a stationary localized solution of a complex scalar field theory with a suitable self-interaction in flat space.

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Both **BS** and **Q-balls** are **non-topological solitons**

Model

Action:

$$I = \int_{\mathcal{M}} d^4x \sqrt{-g} \Big[\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \\ -\frac{1}{2} g^{\mu\nu} \left(D_\mu \psi^* D_\nu \psi + D_\nu \psi^* D_\mu \psi \right) - U(|\psi|) \Big],$$

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with

- ψ - complex scalar field
- A_μ - gauge potential
- $D_\mu\psi = \partial_\mu\psi + iqA_\mu\psi$ - covariant derivative of the scalar field
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ - $U(1)$ field strength tensor
- $g_{\mu\nu}$ - spacetime metric
- R - Ricci scalar
- Λ - cosmological constant

Field Equations

Field equations of the model take the form:

$$E_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} - 8\pi G T_{\mu\nu} = 0,$$
$$D_\mu D^\mu \psi = \tfrac{\partial U}{\partial |\psi|^2} \psi,$$
$$\nabla_\mu F^{\mu\nu} = iq\big[(D^\nu \psi^*)\psi - \psi^*(D^\nu \psi)\big] \equiv q j^\nu$$

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Energy-momentum tensor:

$$T_{\mu\nu} = F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + (D_\mu\psi^* D_\nu\psi + D_\nu\psi^* D_\mu\psi) -$$
$$-g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\left(D_\alpha\psi^* D_\beta\psi + D_\beta\psi^* D_\alpha\psi\right) + U(|\psi|)\right]$$

Metric and Ansatz

Matter fields:

$$\mathcal{A} = A_\mu dx^\mu = A_t(r,\theta)dt + A_\varphi(r,\theta)\sin\theta\left(d\varphi - \frac{W}{r}dt\right),$$
$$\psi(t,r,\theta,\varphi) = \phi(r,\theta)e^{i(n\varphi-\omega t)}$$

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Line element:

$$ds^2 = -F_0(r,\theta)(1+\tfrac{r^2}{\ell^2})dt^2 + F_1(r,\theta)\left(\frac{dr^2}{1+\frac{r^2}{\ell^2}} + r^2\, d\theta^2\right) +$$
$$+F_2(r,\theta)r^2\sin^2\theta\left(d\varphi - \frac{W(r,\theta)}{r}dt\right)^2$$

Boundary Conditions

Regularity at the origin ($r \to 0$):

$$\partial_r F_i|_{r=0} = W|_{r=0} = 0, \quad \phi|_{r=0} = 0, \quad \partial_r A_t|_{r=0} = A_\varphi|_{r=0} = 0$$

Spatial infinity ($\mathbf{r} \to \infty$):

$$F_i|_{r\to\infty} = 1, \quad W|_{r\to\infty} = 0, \quad \phi|_{r\to\infty} = A_t|_{r\to\infty} = A_\varphi|_{r\to\infty} = 0$$

Regularity on the z-axis ($\theta = 0$):

$$\partial_\theta F_i|_{\theta=0,\pi} = \partial_\theta W|_{\theta=0,\pi} = 0, \quad \phi|_{\theta=0,\pi} = \partial_\theta A_t|_{\theta=0,\pi} = A_\varphi|_{\theta=0,\pi} = 0$$

Equatorial palne ($\theta = \pi/2$):

$$\partial_\theta F_i|_{\theta=\pi/2} = \partial_\theta W|_{\theta=\pi/2} = 0 \ ,$$
$$\partial_\theta \phi|_{\theta=\pi/2} = \partial_\theta A_t|_{\theta=\pi/2} = \partial_\theta A_\varphi|_{\theta=\pi/2} = 0$$

Asymptotics

Matter fields:

$$\phi \sim \frac{c_1(\theta)}{r^\Delta} + \dots \ , \quad A_t \sim \frac{Q_e}{r} + \dots \ , \quad A_\varphi \sim \frac{\mu \sin\theta}{r} + \dots ,$$

where $\Delta = \frac{3}{2}\left(1 + \sqrt{1 + \frac{4}{9}M^2\ell^2}\right)$.

Metric functions:

$$F_0 = 1 + \frac{f_{03}(\theta)}{r^3} + \dots , \ F_1 = 1 + \frac{f_{13}(\theta)}{r^3} + \dots , \ F_2 = 1 + \frac{f_{23}(\theta)}{r^3} + \dots ,$$
$$W = \frac{w_2(\theta)}{r^2} + \dots$$

Global Charges

Energy:

$$E = \frac{1}{8G\ell^2}\int_0^{\pi} d\theta \sin\theta \left(5f_{13}(\theta) + 3f_{23}(\theta)\right)$$

Angular momentum:

$$J = -\frac{3}{8G}\int_0^{\pi} d\theta \sin^3\theta \ w_2(\theta)$$

Noether charge and **electric charge** are proportional to the total angular momentum:

$$J = nQ = \frac{Q_e n}{q}$$

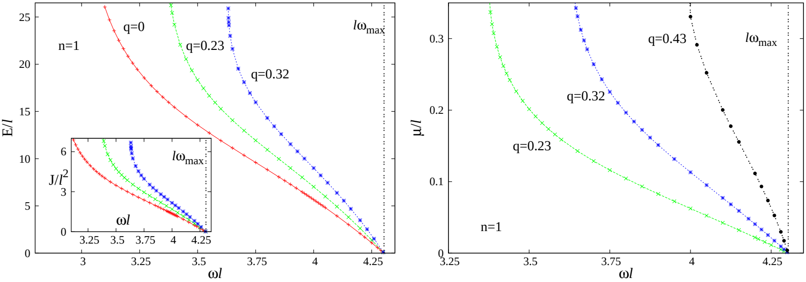
Probe Limit

Global AdS_4 metric:

$$ds^2 = \frac{dr^2}{1+\frac{r^2}{\ell^2}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - (1+\frac{r^2}{\ell^2})dt^2$$

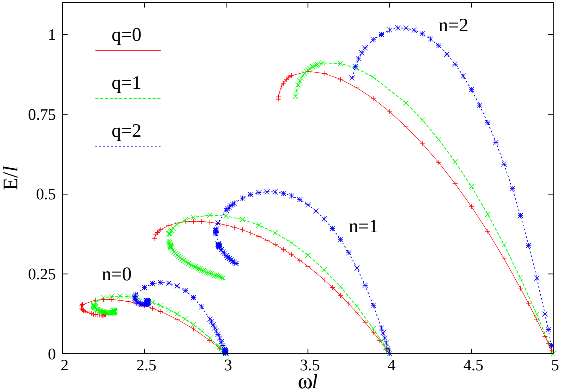
Self-interacting scalar potential:

$$U(\phi) = M^2\phi^2 - \lambda\phi^4 + \nu\phi^6$$



Gravitating boson stars

- no slowly rotating limit
- solutions exist for a limited range of frequencies
- $\omega_{max} = \frac{n+\Delta}{\ell}$
- global charges stay finite as $\omega \to \omega_{min}$



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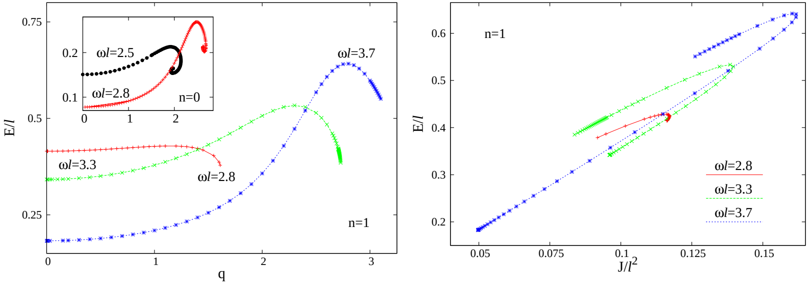
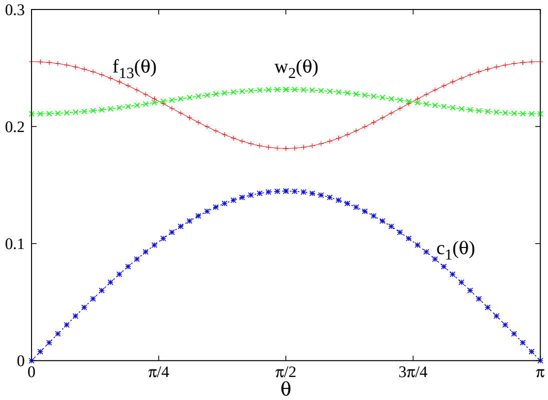


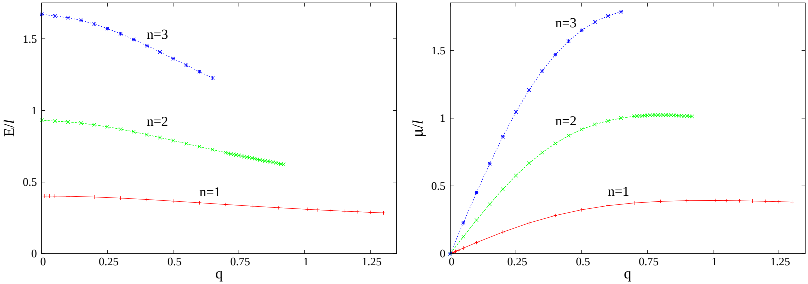
Gravitating boson stars

$$\phi \sim \frac{c_1(\theta)}{r^{\Delta}} + \ldots \ , \quad F_1 = 1 + \frac{f_{13}(\theta)}{r^3} + \ldots \ , \quad W = \frac{w_2(\theta)}{r^2} + \ldots$$



Static solutions

$$A_t = W = 0,\ U(|\psi|) = -\lambda|\psi|^4$$



Conclusions

- We constracted spining gauged boson stars in AdS_4 spacetime. Their basic properties are similar to those of the ungaged configurations.
- Solutions exist for value of gauge coupling constant $q < q_{max}$.
- This type of solutions can be considered as simple prototypes of more complicated spinning configurations, possibly with non-Abelian gauge fields.
- From the holographic point of view spinning gauged boson stars describe zero temperature states of a conformal field theory (CFT) defined in a a fixed background. The asymptotics of the temporal component of the gauge potential gives the charge density and the chemical potential of the CFT. Likewise, the field ψ is dual to an operator $\mathcal{O}$ in the CFT, with a scaling dimension Δ.

Thank You for Your Attention!