Viable Cosmological Solutions in Massive Bimetric Gravity

arXiv: 1312.3208 (F.K., A. Patil, L. Amendola)
1402.1988 (F.K., L. Amendola)

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Linear theory:

$$S_{FP} = \frac{1}{2}\int \mathrm{d}^D x \left[-\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + 2\partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - 2\partial_\mu h^{\mu\nu}\partial_\nu h \right.$$
$$\left. + \partial_\lambda h \partial^\lambda h - m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

Fierz, Pauli (1939)

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Main historical problems

- vDVZ discontinuity
- Boulware-Deser ghost in non-linear extensions

- Idea: Additional tensor field $f_{\mu\nu}$

$$S = -M_g^2 \int \mathrm{d}^4x \sqrt{-g} \left[R(g) + m^2 \left(F(g^{-1}f) + \tilde{F}(f^{-1}g) \right) \right]$$

Hassan, Rosen (arXiv 1103:6055, 1109:3515)

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Non-linear and ghost-free action in massive bigravity

$$S = -\frac{M_g^2}{2} \int \mathrm{d}^4x \sqrt{-\det g}\, R(g) - \frac{M_f^2}{2} \int \mathrm{d}^4x \sqrt{-\det f}\, R(f)$$
$$+ m^2 M_g^2 \int \mathrm{d}^4x \sqrt{-\det g} \sum_{n=0}^{4} \beta_n e_n(\sqrt{g^{-1}f})$$
$$+ \int \mathrm{d}^4x \sqrt{-\det g}\, \mathcal{L}_m(g, \phi)$$

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Assumptions and notations

- no radiation and curvature
- ratio of scale factors: $r \equiv b/a$
- $B_0 \equiv \beta_0 + 3\beta_1 r + 3\beta_2 r^2 + \beta_3 r^3$
- $B_1 \equiv \beta_1 + 3\beta_2 r + 3\beta_3 r^2 + \beta_4 r^3$
- $B_2 \equiv \beta_1 + 2\beta_2 r + \beta_3 r^2$
- prime denotes derivative wrt N
- express β_n and H^2 in units of H_0^2

$g_{\mu\nu}$-equations

$$3\mathcal{H}^2 = a^2\left(\rho_m + B_0\right)$$
$$2\mathcal{H}'\mathcal{H} + \mathcal{H}^2 = a^2\left(B_0 + B_2 r'\right)$$

$f_{\mu\nu}$-equations

$$\mathcal{H}^2 = \frac{a^2}{3r} B_1$$
$$(i\text{-}f_{\mu\nu}) = \left(1 + \frac{r}{3\mathcal{H}_b}\partial_t\right)(0\text{-}f_{\mu\nu})$$

Useful relations

$$\Omega_m = 1 - \frac{B_0}{B_1} r$$

$$\rho_m = \frac{B_1}{r} - B_0$$

$$r' = 3\,\frac{\rho_m}{\rho_{m,r}}$$

$$w_{eff} = -r\,\frac{B_0 + B_2 r'}{B_1} = -1 + \Omega_m - r r'\,\frac{B_2}{B_1}$$

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How does r evolve?

Extracting viable cosmological branches

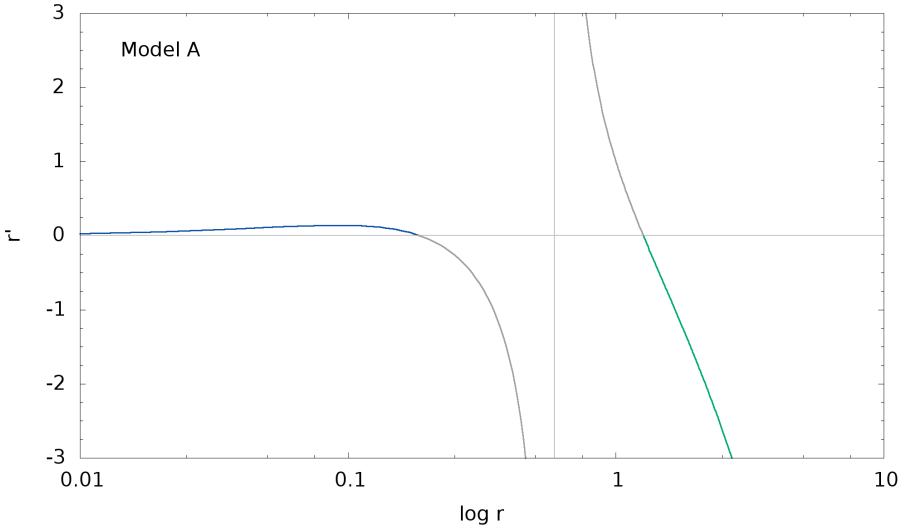
$$\frac{\beta_3}{3}r^4 + \left(\beta_2 - \frac{\beta_4}{3}\right)r^3 + (\beta_1 - \beta_3)\,r^2 + \left(\frac{\rho}{3} + \frac{\beta_0}{3} - \beta_2\right)r - \frac{\beta_1}{3} = 0$$

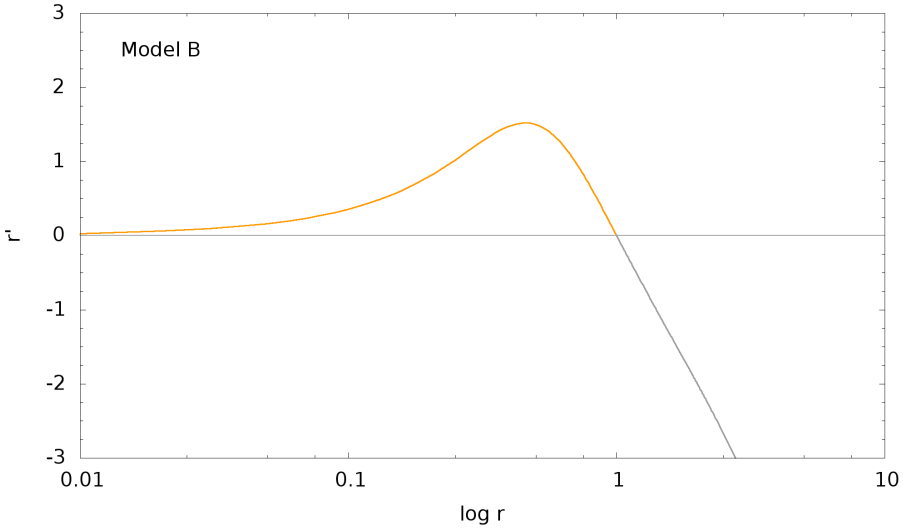
- up to 4 different branches

$\Rightarrow$ Which of them are cosmological viable?

Definition of viability

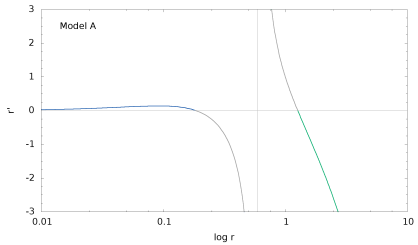
- ρ_m and ρ_{mg} not identically zero
- monotonic expansion, i.e. $\rho_m + \rho_{mg} > 0$
- matter domination in asymptotic past
- $r \geq 0$ and without singularities in r'

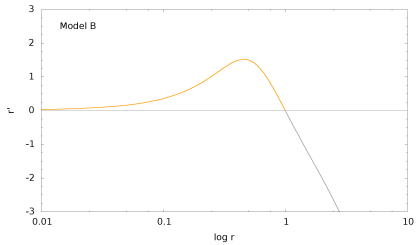


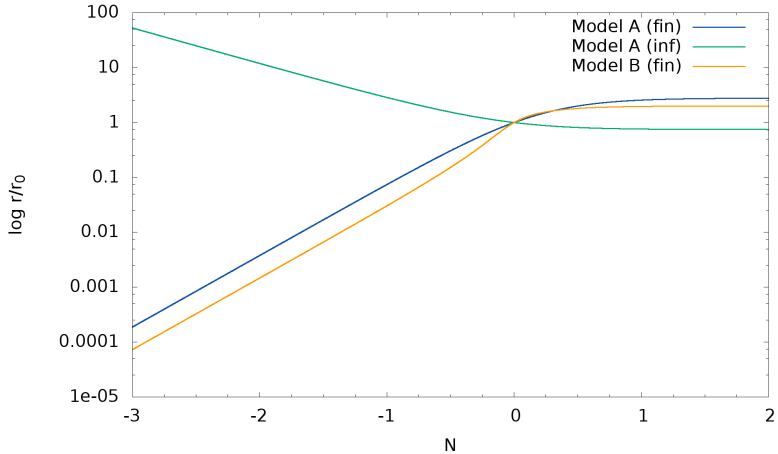


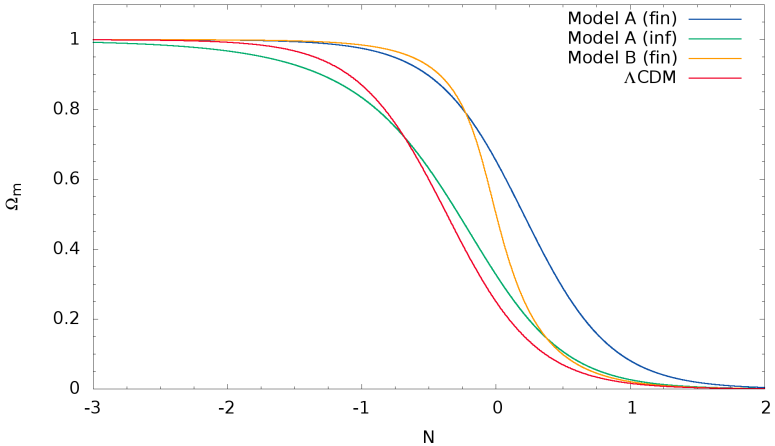
Model A: $\beta_i = (1, \frac{1}{5}, 0, 0, 1)$

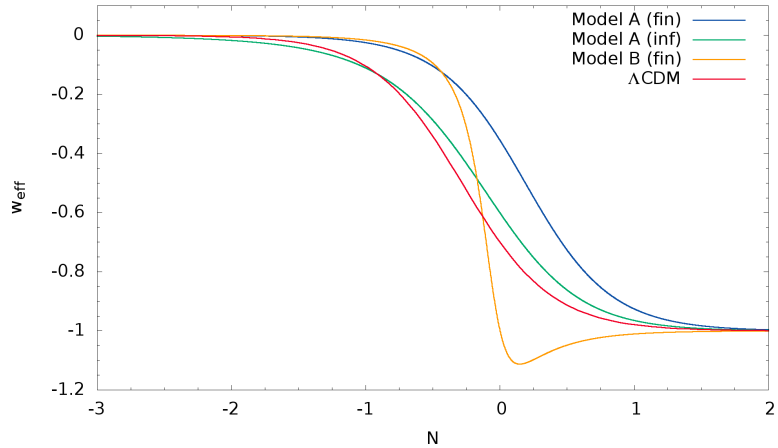
Model B: $\beta_i = \hat{\beta} = \frac{4}{9}$

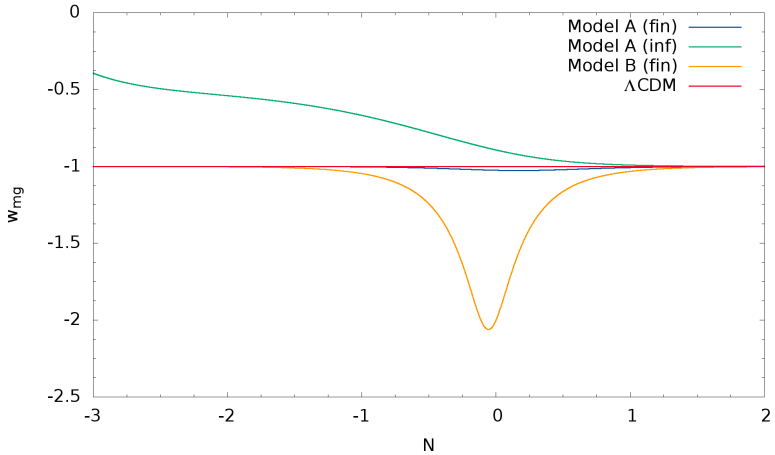

Model A
r'
log r


Model B
r'
log r


Model A (fin)
Model A (inf)
Model B (fin)
log r/r0
N


Model A (fin)
Model A (inf)
Model B (fin)
ΛCDM
Ωm
N


Model A (fin)
Model A (inf)
Model B (fin)
ΛCDM
weff
N


Model A (fin)
Model A (inf)
Model B (fin)
ΛCDM
wmg
N

Recipe to find viable branches

ToDo list:

- find all viable solutions r_0 that fulfills both Friedmann eqs. at $t = 0$
- $r_0 \in (0, r_{c_1})$ or $r_0 \in (r_{c_n}, \infty)$?
- check criteria of viability

Recipe to find viable branches

Simple theorem

Every branch $(0, r_c)$ is cosmological viable if

- r_c smallest strictly positiv root
- $\exists\, 0 < r_i < r_c$ with $r'(r_i) > 0$
- $\exists\, 0 < r_i < r_c$ with $\rho_m(r_i) > 0$
- r' finite $\forall\, r \in\ (0, r_c)$

Phantom EOS

$$w_{mg} = -1 - \frac{B_2 r r'}{\Omega_{mg} B_1} = -1 - \frac{B_2}{B_0} r'$$

Near (final) de Sitter state: $\Omega_m \simeq 0 \Rightarrow B_0 \simeq B_1/r > 0$

- finite branch: $B_2 > 0 \Leftrightarrow$ phantom EOS
- infinite branch: $\beta_1 < 0 \Leftrightarrow$ phantom EOS

Minimal Bimetric Models: only non-zero β_1

$$r' = \frac{3r\left(1-3r^2\right)}{1+3r^2}$$

Analytic solutions

$$r(a) = \frac{1}{6a^3}\left(-A \pm \sqrt{12a^6 + A^2}\right)$$

$$\Omega_m(a) = -\frac{A}{6a^6}\left(-A \mp \sqrt{12a^6 + A^2}\right)$$

$$w_{eff} = \pm\frac{A}{\sqrt{12a^6 + A^2}} - 1$$

$$w_{mg} = \mp\frac{A}{\sqrt{12a^6 + A^2}} - 1$$

with $A = \frac{3(\Omega_{m0}-1)^2 - \beta_1^2}{\beta_1(\Omega_{m0}-1)} = \frac{3}{\beta_1} - \beta_1$

Minimal Bimetric Models: only non-zero β_1

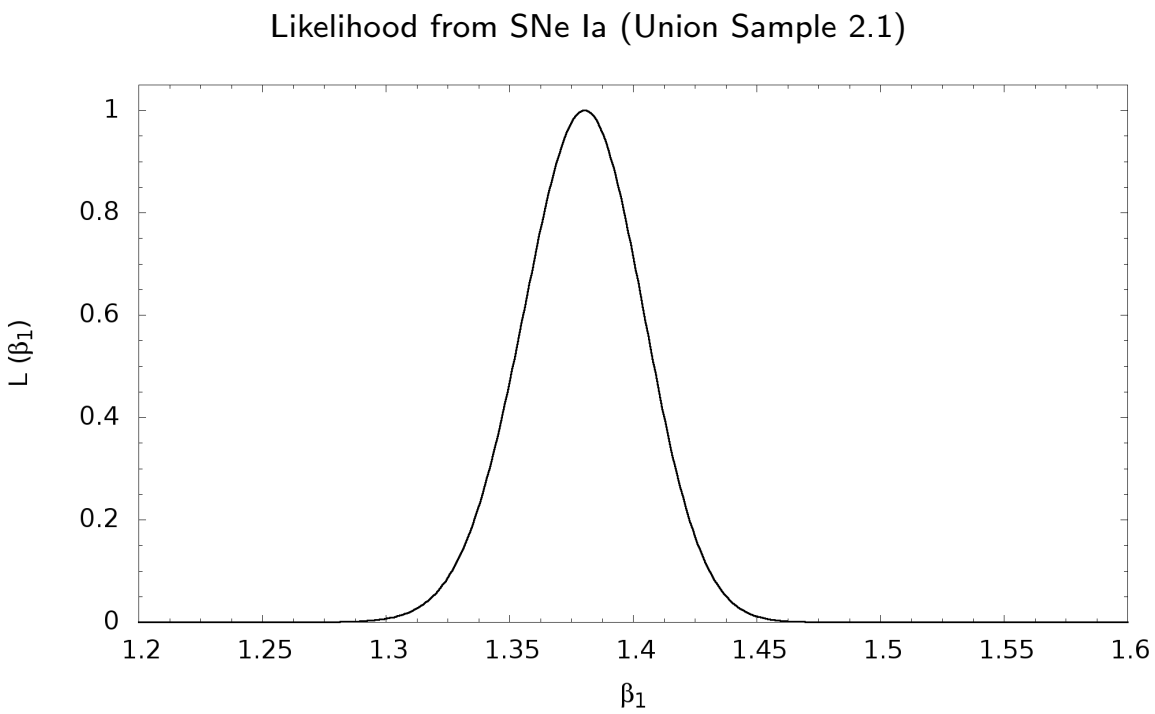
$$r' = \frac{3r\left(1-3r^2\right)}{1+3r^2}$$

Analytic solutions

$$w_{mg} = \frac{2}{\Omega_m - 2}$$

All other β_n-models (except ΛCDM) are not viable!

Minimal Bimetric Model (MBM)



	$\beta(\Omega_{m0})$	χ^2_{min}	β_1	Ω_{m0}	w_0	w_a
β_1	$\sqrt{3(1-\Omega_{m0})}$	578.3	$1.38^{+0.03}_{-0.03}$	$0.37^{+0.02}_{-0.02}$	$-1.22^{+0.02}_{-0.02}$	$-0.64^{+0.05}_{-0.04}$

Viable 2-parameter models with $\beta_0 = 0$

- free $\beta_1,\ \beta_2$ (finite branches)
- free $\beta_1,\ \beta_3$ (finite branches)
- free $\beta_1,\ \beta_4$ (finite + infinite branches)

Perturbation equations in MBM

$$g = g_0 + h \qquad\qquad f = f_0 + h_f$$

with perturbed part

$$\mathrm{d}s_h^2 = 2e^{ik\cdot r}a^2\left(-\Psi\mathrm{d}t^2 + \left(\Phi\,\delta_{ij} + k_i k_j E\right)\mathrm{d}x^i\mathrm{d}x^j\right)$$

$$\mathrm{d}s_{h_f}^2 = 2e^{ik\cdot r}b^2\left(-\frac{\mathcal{H}_b^2}{\mathcal{H}^2}\Psi_f\mathrm{d}t^2 + \left(\Phi_f\,\delta_{ij} + k_i k_j E_f\right)\mathrm{d}x^i\mathrm{d}x^j\right)$$

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and apply gauge transformation:

$$\begin{aligned}
\tilde{\Phi} &= \Phi - \mathcal{H}^2 E' \\
\tilde{\Psi} &= \Psi - (\mathcal{H}^2 + \mathcal{H}\mathcal{H}')E' - \mathcal{H}^2 E'' \\
\tilde{\Phi}_f &= \Phi_f - \frac{r\mathcal{H}^2 E_f'}{(r' + r)} \\
\tilde{\Psi}_f &= \Psi_f - \frac{\mathcal{H} r^2 (\mathcal{H} E_f')'}{(r' + r)^2} - \frac{\mathcal{H}^2 E_f' r(r^2 + 2r'^2 + 2rr' - rr'')}{(r' + r)^3}
\end{aligned}$$

Matter equations

$$\delta' + \theta\mathcal{H}^{-1} + 3\Phi' - 3\mathcal{H}^2 E'' - 6\mathcal{H}\mathcal{H}' E' + k^2 E' = 0$$
$$\theta' + \theta + k^2 E'\mathcal{H}' - k^2\Psi\mathcal{H}^{-1} + k^2\mathcal{H}\left(E'' + E'\right) = 0$$

QS limit

$$\delta'' + \left(1 + \frac{\mathcal{H}'}{\mathcal{H}}\right)\delta' + \frac{k^2\Psi}{\mathcal{H}^2} = 0$$

$g_{\mu\nu}$-equations (QS limit)

$$[00]\quad \Phi\left(1+\frac{2k^2}{3a^2r\beta_1}\right)-\Phi_f+\frac{1}{3}k^2\Delta E-\frac{\delta\rho}{3\beta_1 r}=0,$$

$$[0\,i]\quad \Phi'-\Psi+\frac{a^2\rho}{2\mathcal{H}k^2}\theta+\left(\mathcal{H}^2-\mathcal{H}\mathcal{H}'\right)E'=0$$

$$[i\,j]\quad \Phi+\Psi+a^2r\beta_1\Delta E=0$$

$$[i\;i]\quad \left(2+\frac{2k^2}{3a^2r\beta_1}\right)\Phi-2\Phi_f+\Psi\left(1+\frac{2k^2}{3a^2r\beta_1}\right)-\frac{6\left(2-3r^2\right)}{3+9r^2}\Psi_f+\frac{2k^2}{3}\Delta E=0$$

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$f_{\mu\nu}$-equations (QS limit)

$$[00]\quad \Phi + \left(-1 - \frac{2k^2 r}{3a^2\beta_1}\right)\Phi_f + \frac{k^2}{3}\Delta E = 0$$

$$[0\,i]\quad \Phi'_f + \frac{\left(-4+6r^2\right)}{1+3r^2}\Psi_f + \frac{3a^2\left(-1+3r^2\right)\beta_1}{4r(-2+3r^2)}E' + \frac{3a^2\left(1-3r^2\right)\beta_1}{4r(-2+3r^2)}\Delta E' = 0$$

$$[i\,j]\quad \Psi_f + \Phi_f + \frac{a^2\left(1+3r^2\right)\beta_1}{-4r+6r^3}\Delta E = 0$$

$$[i\;i]\quad \Phi + \frac{1}{2}\Psi + \frac{1}{3}k^2\Delta E$$
$$+ \left(-1 + \frac{2k^2 r\left(-2+3r^2\right)}{3a^2(1+3r^2)\beta_1}\right)\Phi_f + \left(1 - \frac{3}{1+3r^2} + \frac{2k^2 r\left(-2+3r^2\right)}{3a^2(1+3r^2)\beta_1}\right)\Psi_f = 0$$

$f_{\mu\nu}$-equations (QS limit)

$$[00]\quad \Phi + \left(-1 - \frac{2k^2 r}{3a^2\beta_1}\right)\Phi_f + \frac{k^2}{3}\Delta E = 0$$

$$[0\,i]\quad \cancel{\Phi_f' + \frac{\left(-4+6r^2\right)}{1+3r^2}\Psi_f + \frac{3a^2\left(-1+3r^2\right)\beta_1}{4r(-2+3r^2)}E' + \frac{3a^2\left(1-3r^2\right)\beta_1}{4r(-2+3r^2)}\Delta E' = 0}$$

$$[i\,j]\quad \Psi_f + \Phi_f + \frac{a^2\left(1+3r^2\right)\beta_1}{-4r+6r^3}\Delta E = 0$$

$$[i\;i]\quad \Phi + \frac{1}{2}\Psi + \frac{1}{3}k^2\Delta E + \left(-1 + \frac{2k^2 r\left(-2+3r^2\right)}{3a^2(1+3r^2)\beta_1}\right)\Phi_f + \left(1 - \frac{3}{1+3r^2} + \frac{2k^2 r\left(-2+3r^2\right)}{3a^2(1+3r^2)\beta_1}\right)\Psi_f = 0$$

Perturbation in QS limit

- 6 equations - 1 degeneracy
- 5 unknown fields ($\Phi, \Psi, \Phi_f, \Psi_f$ and ΔE)
- analytic solution (only $\delta\rho$ dependency)

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$$\Psi = -\frac{\mathcal{H}^2\Omega_m\delta\left(2k^2r^3\left(11+6r^2\right)+3\beta_1a^2\left(1+7r^2-6r^4\right)\right)}{2k^2\left(\beta_1a^2\left(1+r^2\right)^2\left(1+3r^2\right)+k^2r^3\left(7+3r^2\right)\right)}$$

Perturbation in QS limit

modified gravity parameters:

$$\eta \equiv -\frac{\Phi}{\Psi} = H_2 \frac{1 + H_4 (k/\mathcal{H})^2}{1 + H_3 (k/\mathcal{H})^2}$$

$$Y \equiv -\frac{2k^2 \Psi}{3\mathcal{H}^2 \Omega_m \delta_m} = H_1 \frac{1 + H_3 (k/\mathcal{H})^2}{1 + H_5 (k/\mathcal{H})^2}$$

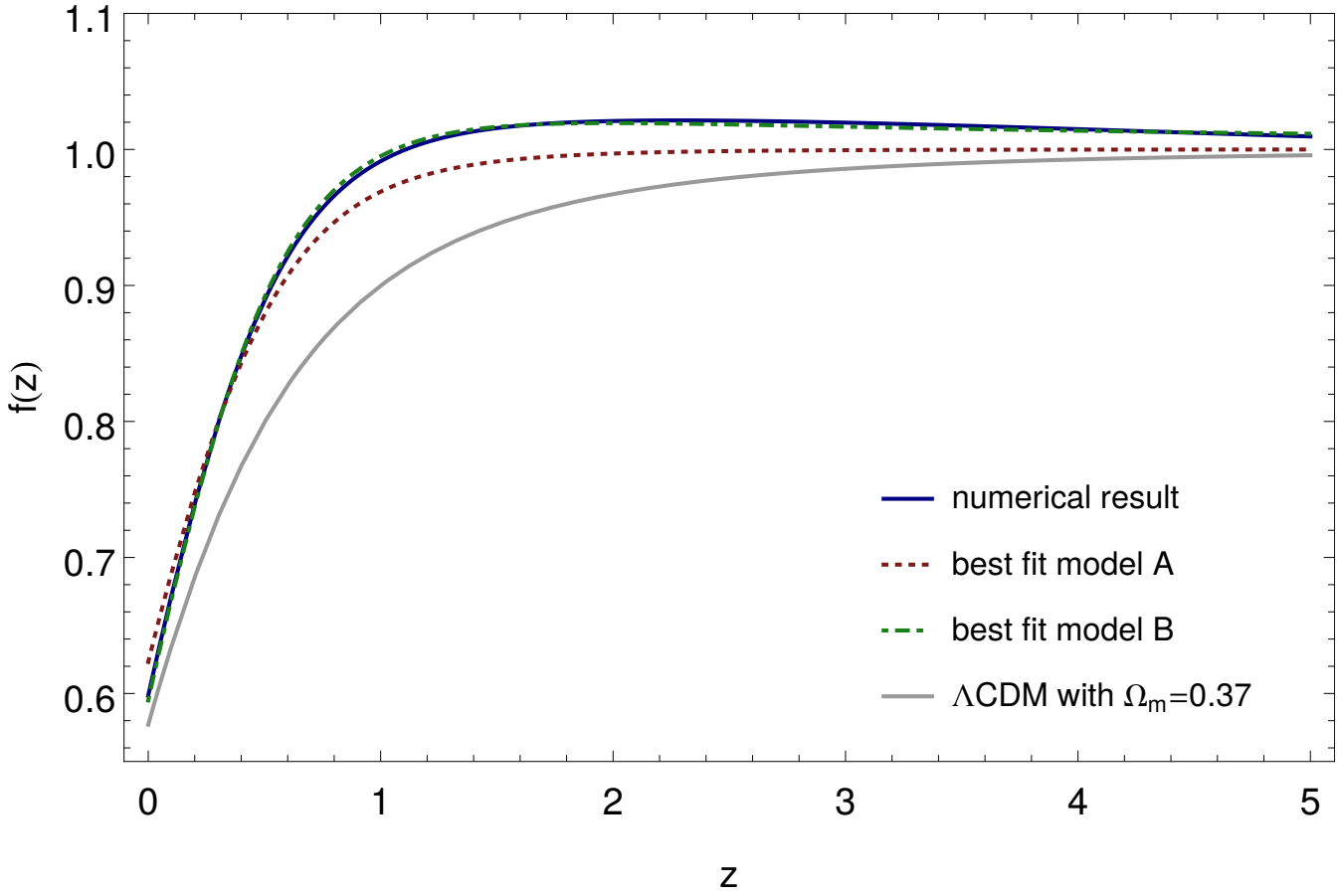
$$H_1 \equiv \frac{1 + 7r^2 - 6r^4}{(1 + r^2)^2 (1 + 3r^2)}$$

$$H_2 \equiv \frac{1 + 4r^2 + 3r^4}{1 + 7r^2 - 6r^4}$$

$$H_3 \equiv \frac{2\mathcal{H}^2 r^3 \left(11 + 6r^2\right)}{3\beta_1 a^2 (1 + 7r^2 - 6r^4)}$$

$$H_4 \equiv \frac{2\mathcal{H}^2 r^3 \left(10 + 3r^2\right)}{3\beta_1 a^2 (1 + 4r^2 + 3r^4)}$$

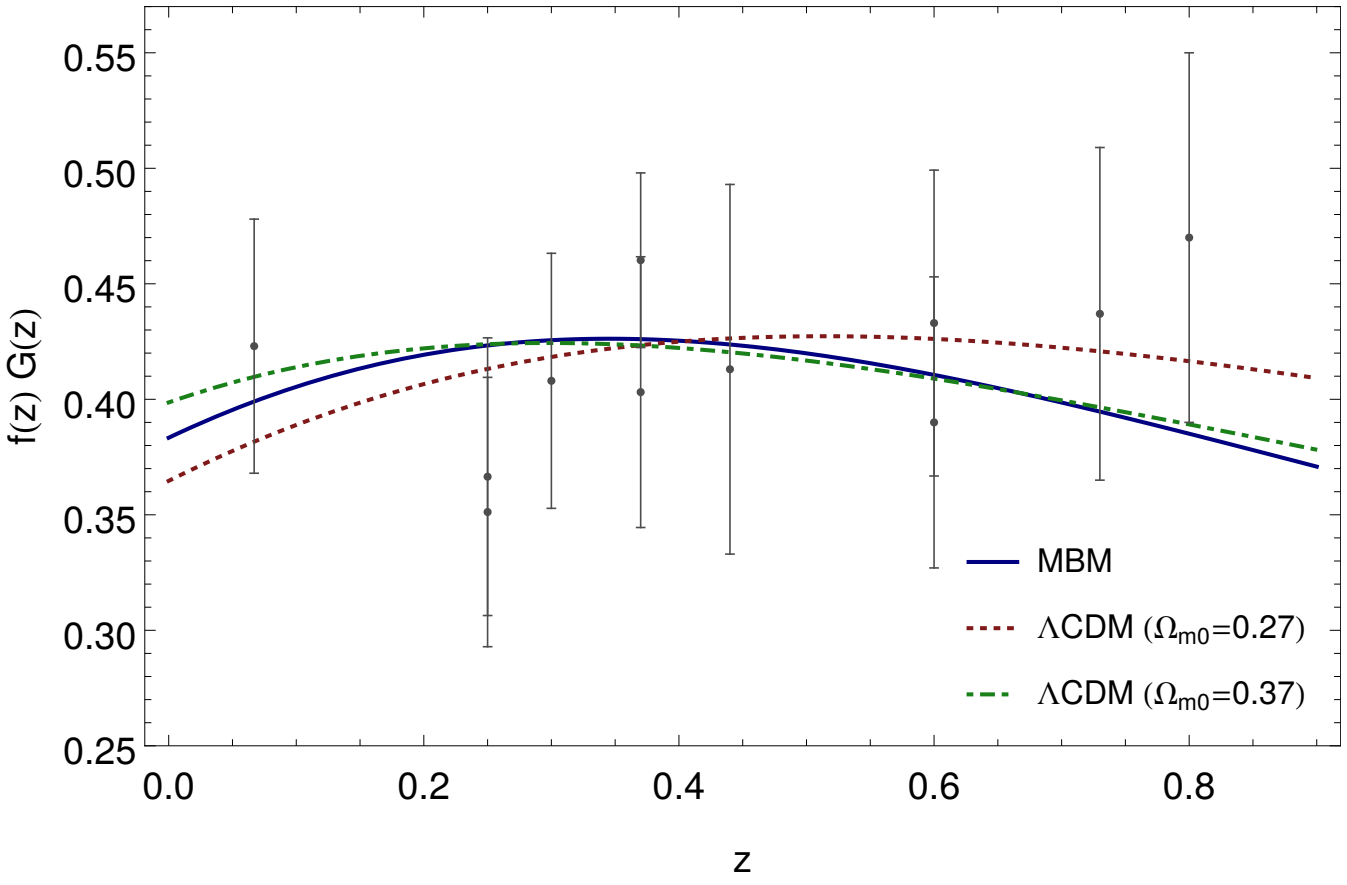
$$H_5 \equiv \frac{\mathcal{H}^2 r^3 \left(7 + 3r^2\right)}{\beta_1 a^2 (1 + r^2)^2 (1 + 3r^2)}$$

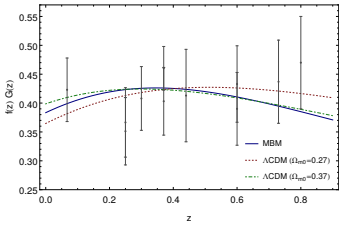


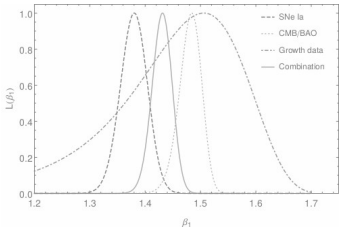
Fit model $f = \Omega_m^{\gamma_0}\left(1 + \frac{\gamma_1}{z+1}\right)$ $\quad (\gamma_0 \simeq 0.58,\ \gamma_1 \simeq 0.07)$

Comparison with measured growth data

growth histories from 6dFGS, LRG_{200}, LRG_{60}, BOSS, WiggleZ and VIPERS (compiled by Macaulay et al. (arXiv 1303.6583))







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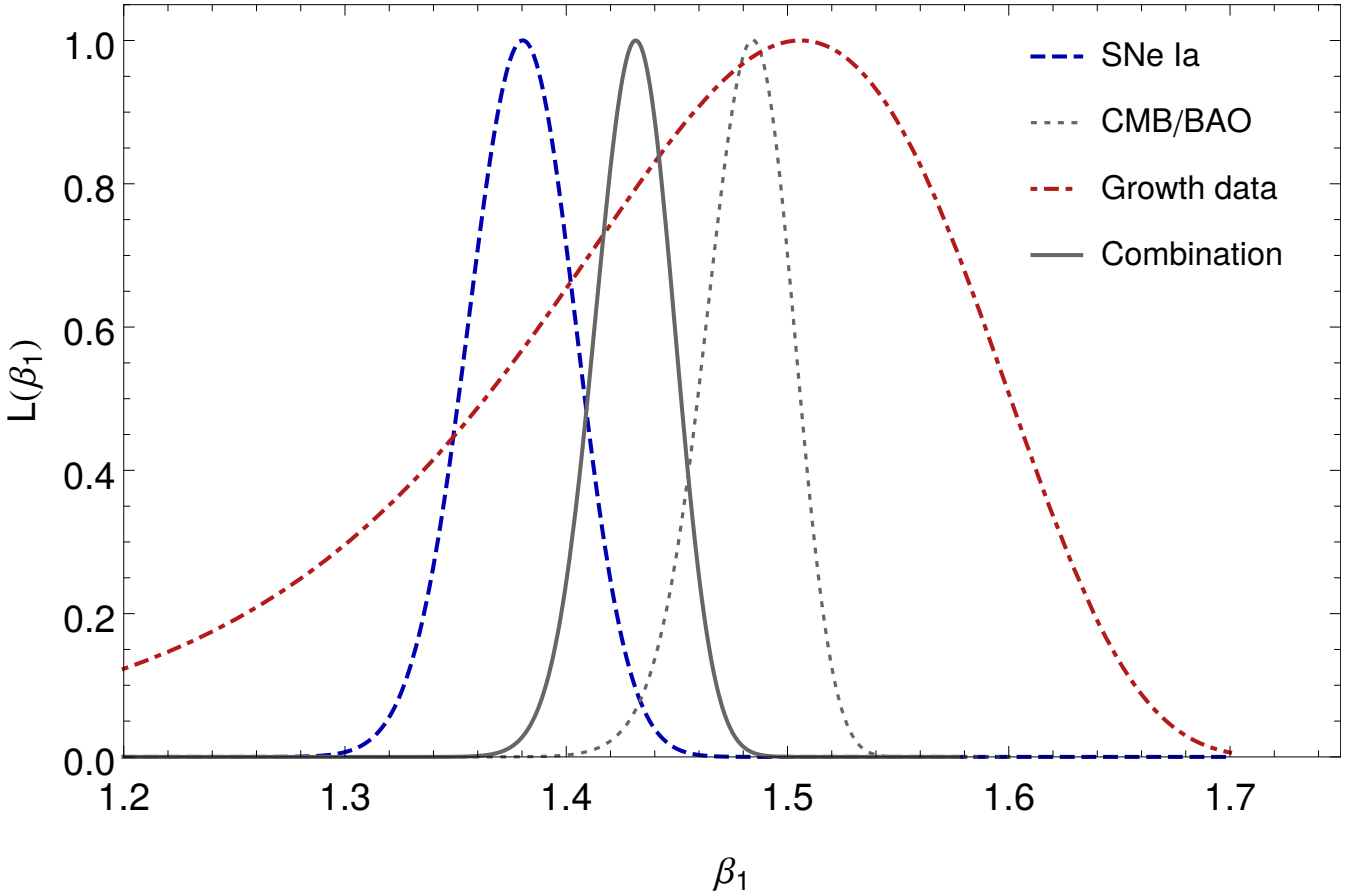

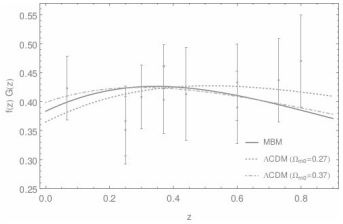

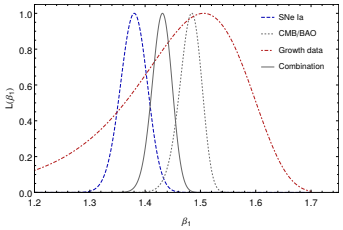

Thank you for your attention!