Gauss–Bonnet Boson Stars in AdS

Jürgen Riedel

in collaboration with

Yves Brihaye[(1)], **Betti Hartmann**[(2)(3)] **, and Raluca Suciu**[(2)]

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Faculty of Physics, University Oldenburg, Germany
Models of Gravity

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(1) Physique-Mathématique, Université de Mons, 7000 Mons, Belgium
(2) School of Engineering and Science, Jacobs University Bremen, Germany
(3) Universidade Federal do Espirito Santo (UFES), Departamento de Fisica, Vitoria (ES), Brazil

Agenda

- Describe the **model** to construct **non-rotating Gauss-Bonnet boson stars in AdS**
- Describe **effect of Gauss-Bonnet term** to boson star solutions
- **Stability analysis of rotating Gauss-Bonnet boson star solutions** (ignoring the Gauss-Bonnet coupling for now)
- Outlook

Solitons in non-linear field theories

General properties of soliton solutions

- **Localized, finite energy, stable, regular solutions** of non-linear field equations
- Can be viewed as **models of elementary particles**

Examples

- **Topological solitons**: Skyrme model of hadrons in high energy physics one of first models and magnetic monopoles, domain walls etc.
- **Non-topological solitons**: Q-balls (named after Noether charge Q) (flat space-time) and boson stars (generalisation in curved space-time)

Non-topolocial solitons

Properties of non-topological solitons

- Solutions possess the same **boundary conditions at infinity as the physical vacuum state**
- Degenerate vacuum states **do not necessarily exist**
- Require an **additive conservation law**, e.g. **gauge invariance** under an **arbitrary global phase transformation**

S. R. Coleman, Nucl. Phys. B **262** (1985), 263, R. Friedberg, T. D. Lee and A. Sirlin, Phys. Rev. D **13** (1976) 2739), D. J. Kaup, Phys. Rev. **172** (1968), 1331, R. Friedberg, T. D. Lee and Y. Pang, Phys. Rev. D **35** (1987), 3658, P. Jetzer, Phys. Rept. **220** (1992), 163, F. E. Schunck and E. Mielke, Class. Quant. Grav. **20** (2003) R31, F. E. Schunck and E. Mielke, Phys. Lett. A **249** (1998), 389.

Why study boson stars?

Boson stars

- Simple **toy models for a wide range of objects** such as particles, compact stars, e.g. neutron stars and even centres of galaxies
- We are interested in the **effect of Gauss-Bonnet gravity** and will study these objects in **the minimal number of dimensions** in which the term does not become a **total derivative.**
- Toy models for **studying properties of AdS space-time**
- Toy models for **AdS/CFT correspondence.** Planar **boson stars** in AdS have been interpreted as Bose-Einstein condensates of glueballs

Model for Gauss–Bonnet Boson Stars

- **Action**

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R - 2\Lambda + \alpha\mathcal{L}_{GB} + 16\pi G_5\mathcal{L}_{\rm matter}\right)$$
$$\mathcal{L}_{GB} = \left(R^{MNKL}R_{MNKL} - 4R^{MN}R_{MN} + R^2\right) \quad (1)$$

- **Matter Lagrangian** $\mathcal{L}_{matter} = -\left(\partial_\mu\psi\right)^*\partial^\mu\psi - U(\psi)$
- **Gauge mediated potential**

$$U_{\rm SUSY}(|\psi|) = m^2\eta_{\rm susy}^2\left(1 - \exp\left(-\frac{|\psi|^2}{\eta_{\rm susy}^2}\right)\right) \quad (2)$$

$$U_{\rm SUSY}(|\psi|) = m^2|\psi|^2 - \frac{m^2|\psi|^4}{2\eta_{\rm susy}^2} + \frac{m^2|\psi|^6}{6\eta_{\rm susy}^4} + O\left(|\psi|^8\right) \quad (3)$$

A. Kusenko, Phys. Lett. B **404** (1997), 285; Phys. Lett. B **405** (1997), 108, L. Campanelli and M. Ruggieri, Phys. Rev. D **77** (2008), 043504

Model for Gauss–Bonnet Boson Stars

- **Einstein Equations are derived from the variation of the action with respect to the metric fields**

$$G_{MN} + \Lambda g_{MN} + \frac{\alpha}{2} H_{MN} = 8\pi G_5 T_{MN} \quad (4)$$

where H_{MN} is given by

$$\begin{aligned} H_{MN} &= 2\left(R_{MABC}R_N^{ABC} - 2R_{MANB}R^{AB} - 2R_{MA}R_N^A + RR_{MN}\right) \\ &- \frac{1}{2}g_{MN}\left(R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD}\right) \quad (5) \end{aligned}$$

- **Energy-momentum tensor**

$$\begin{aligned} T_{MN} &= -g_{MN}\left[\frac{1}{2}g^{KL}\left(\partial_K\psi^*\partial_L\psi + \partial_L\psi^*\partial_K\psi\right) + U(\psi)\right] \\ &+ \partial_M\psi^*\partial_N\psi + \partial_N\psi^*\partial_M\psi \quad (6) \end{aligned}$$

Model continued

- The **Klein-Gordon equation** is given by:

$$\left(\Box - \frac{\partial U}{\partial |\psi|^2}\right)\psi = 0 \qquad (7)$$

- $\mathcal{L}_{matter}$ is invariant under the **global U(1) transformation**

$$\psi \to \psi e^{i\chi} \quad . \qquad (8)$$

- Locally conserved **Noether current** j^M

$$j^M = -\frac{i}{2}\left(\psi^* \partial^M \psi - \psi \partial^M \psi^*\right) \; ; \; j^M_{;M} = 0 \qquad (9)$$

- The globally conserved **Noether charge** Q reads

$$Q = -\int d^4x \sqrt{-g} j^0 \; . \qquad (10)$$

Ansatz non-rotating

- **Metric Ansatz**

$$ds^2 = -N(r)A^2(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2 + \sin^2\theta\sin^2\varphi d\chi^2\right) \quad (11)$$

where

$$N(r) = 1 - \frac{2n(r)}{r^2} \quad (12)$$

- **Stationary Ansatz for complex scalar field**

$$\psi(r,t) = \phi(r)e^{i\omega t} \quad (13)$$

Ansatz rotating

- **Metric Ansatz**

$$
\begin{aligned}
ds^2 &= -A(r)dt^2 + \frac{1}{N(r)}dr^2 + G(r)d\theta^2 \\
&+ H(r)\sin^2\theta\,(d\varphi_1 - W(r)dt)^2 \\
&+ H(r)\cos^2\theta\,(d\varphi_2 - W(r)dt)^2 \\
&+ (G(r) - H(r))\sin^2\theta\cos^2\theta(d\varphi_1 - d\varphi_2)^2, \quad (14)
\end{aligned}
$$

with $\theta = [0,\pi/2]$ and $\varphi_1,\varphi_2 = [0, 2\pi]$.

- **Cohomogeneity-1 Ansatz for Complex Scalar Field**

$$\Phi = \phi(r)e^{i\omega t}\hat{\Phi}, \quad (15)$$

with

$$\hat{\Phi} = (\sin\theta e^{i\varphi_1}, \cos\theta e^{i\varphi_2})^t \quad (16)$$

Boundary Conditions for asymptotic AdS space-time

- If $\Lambda < 0$ the scalar field function **falls of** with

$$\phi(r >> 1) = \frac{\phi_\Delta}{r^\Delta} \ , \quad \Delta = 2 + \sqrt{4 + L_{eff}^2} \ . \qquad (17)$$

- Where L_{eff} **is the effective AdS-radius**:

$$L_{eff}^2 = \frac{2\alpha}{1 - \sqrt{1 - \frac{4\alpha}{L^2}}} ; L^2 = \frac{-6}{\Lambda} \qquad (18)$$

- **Chern-Simons limit**:

$$\alpha = \frac{L^2}{4} \qquad (19)$$

- **Mass for** $\kappa > 0$ we define the **gravitational mass** at AdS boundary

$$M_G \sim n(r \to \infty)/\kappa \qquad (20)$$

Finding solutions: fixing ω

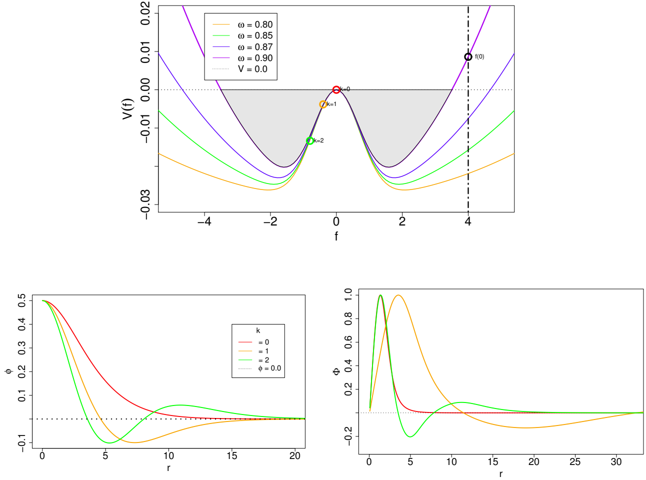


Figure : Effective potential $V(f) = \omega^2\phi^2 - U(f)$.

Gauss–Bonnet Boson Stars with $\Lambda < 0$

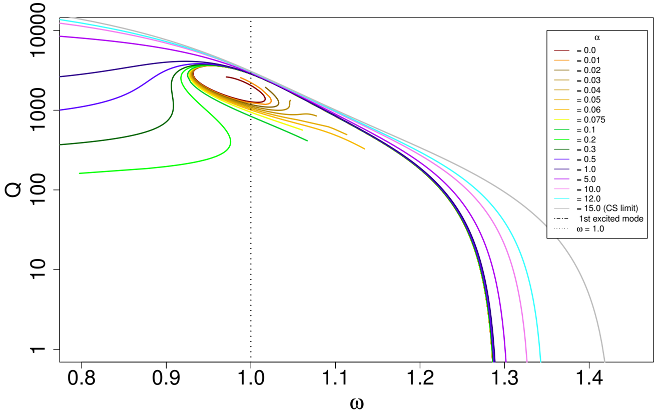


Figure : Charge Q in dependence on the frequency ω for $\Lambda = -0.01$, $\kappa = 0.02$ and different values of α. ω_{max} **shift**: $\omega_{max} = \frac{\Delta}{L_{eff}}$

Gauss–Bonnet Boson Stars with $\Lambda < 0$ Animation

(Loading Video...)

Excited Gauss–Bonnet Boson Stars with $\Lambda < 0$

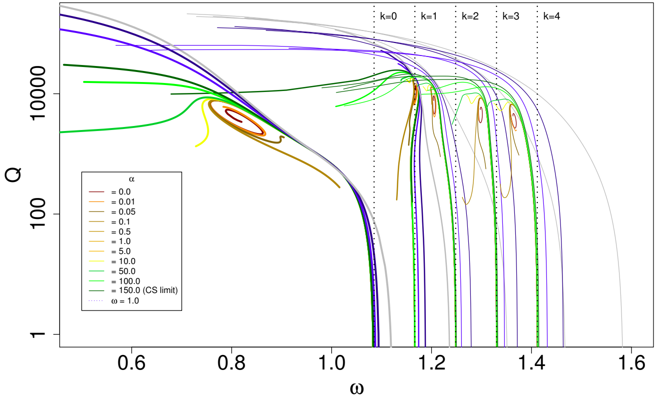


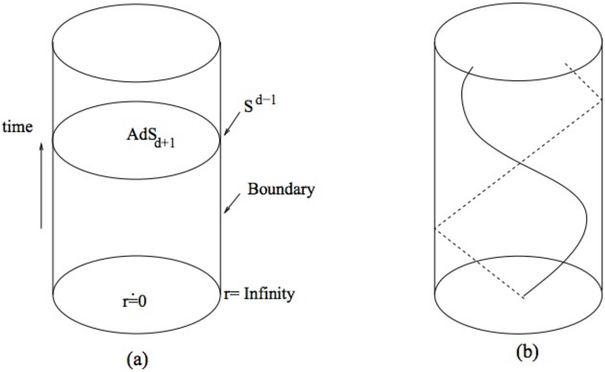
Figure : Charge Q in dependence on the frequency ω for $\Lambda = -0.01$, $\kappa = 0.02$, and different values of α. ω_{max} **shift**: $\omega_{max} = \frac{\Delta+2k}{L_{eff}}$

Summary Effect of Gauss-Bonnet Correction

Boson stars

- **Small coupling to GB term, i.e. small** α, we find **similar spiral like characteristic** as for boson stars in **pure Einstein gravity**.
- When the **Gauss-Bonnet parameter** α **is large** enough the **spiral 'unwinds'**.
- When α **and the coupling to gravity (**κ**)** are of the **same magnitude**, only **one branch of solutions** survives.
- The **single branch** extends to the **small values of the frequency** ω.

AdS Space-time



(4)

Figure : (a) Penrose diagram of AdS space-time, (b) massive (solid) and massless (dotted) geodesic.

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(4) J. Maldacena, The gauge/gravity duality, arXiv:1106.6073v1

Stability of AdS

- It has been shown that **AdS is linearly stable** (Ishibashi and Wald 2004).
- The metric **conformally approaches** the static cylindrical boundary, waves **bounce off at infinity** will **return in finite time.**
- **general result on the non-linear stability** does not yet exist for AdS.
- It is speculated that **AdS is nonlinearly unstable** since in AdS one would expect **small perturbations to bounce of the boundary**, to interact with themselves and **lead to instabilities**.
- **Conjecture**: small finite Q**-balls of AdS in** $(3+1)$**-dimensional AdS** eventually **lead to the formation of black holes** (Bizon and Rostworowski 2011).
- Related to the fact that **energy is transferred** to **smaller and smaller scales**, e.g. pure gravity in AdS (Dias et al).

Erogoregions and Instability

- If the **boson star mass** M **is smaller than** $m_B Q$ one would expect the **boson star to be classically stable** with respect to decay into Q individual scalar bosons.
- **Solutions to Einsteins field equations** with **sufficiently large angular momentum** can suffer from superradiant instability (Hawking and Reall 2000).
- Instablity occurs at **ergoregion** (Friedman 1978) where the **relativistic frame-dragging** is so strong that **stationary orbits are no longer possible.**
- At ergoregion infalling **bosonic waves** are **amplified when reflected.**
- The **boundary of the ergoregion** is defined as where the **covariant tt-component becomes zero**, i.e. $g_{tt} = 0$
- **Analysis for boson stars in** $(3+1)$ **dimensions** has been done (Cardoso et al 2008, Kleihaus et al 2008)

Stability of Boson Stars with AdS Radius very large

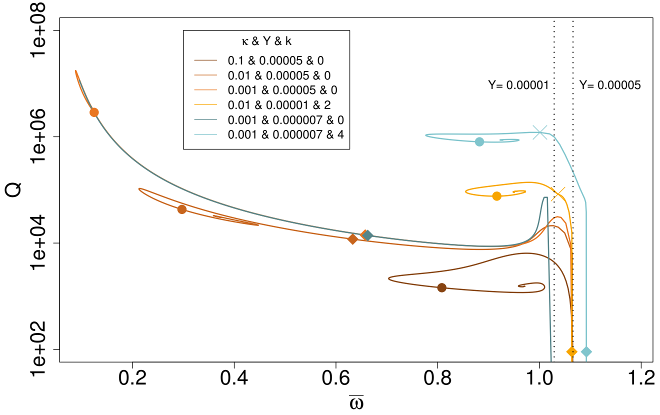

Figure : Charge Q in dependence on the frequency ω for different values of κ and $Y = -6\Lambda$.

Ergoregions Animation

(Loading Video...)

Stability for Boson Stars in AdS

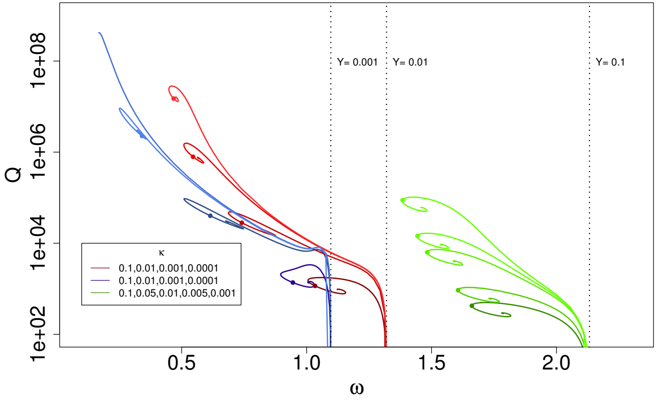

Figure : Charge Q in dependence on the frequency ω for different values of κ and $Y = -6\Lambda$.

Stability Analysis for Radial Excited Boson Stars in AdS

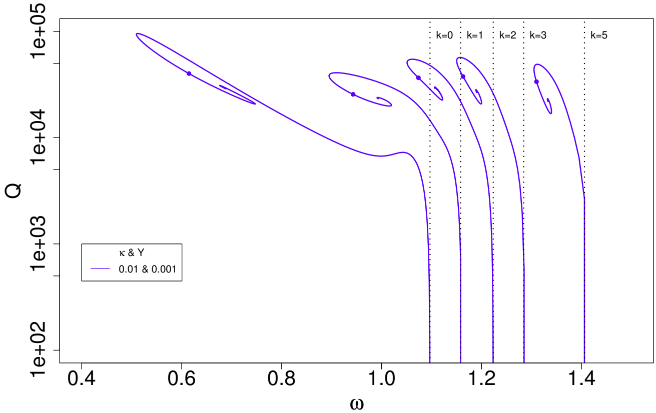

Figure : Charge Q in dependence on the frequency ω for different excited modes k.

Summary Stability Analysis

- **Strong coupling to gravity:** self-interacting **rotating boson stars are destabilized.**
- **Sufficiently small AdS radius**: self-interacting **rotating boson stars are destabilized.**
- **Sufficiently strong rotation** stabilizes self-interacting rotating boson stars.
- **Onset of ergoregions** can occur **on the main branch of boson star solutions**, supposed to be **classically stable.**
- **Radially excited self-interacting rotating boson stars** can be **classically stable** in aAdS for **sufficiently large AdS radius** and **sufficiently small backreaction.**

Outlook

- Analysis the **effect of the Gauss-Bonnet term** on **stability of boson stars.**
- To do a **stabiliy analysis** similar to the one done for non-rotating minimal boson stars by **Bucher et al 2013 ([arXiv:1304.4166 [gr-qc]])**
- See whether **our arguments** related to the **classical stability** of our solutions **agrees with a full perturbation analysis**
- Gauss-Bonnet Boson Stars and **AdS/CFT correspondance**
- Boson Stars in **general Lovelock theory**

Thank You!