Solitons in AdS spacetime

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Models of Gravity

References

Work done in collaboration with:

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Université de Mons, Belgium

Betti Hartmann
Jacobs University Bremen, Germany

Papers

1. Yves Brihaye, Betti Hartmann, **Sardor Tojiev**
Formation of scalar hair on Gauss-Bonnet solitons and black holes.
Phys. Rev. D87 : 024040, 2013

2. Yves Brihaye, Betti Hartmann, **Sardor Tojiev**
Stability of charged solitons and formation of boson stars in 5-dimensional Anti-de Sitter space-time.
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3. Yves Brihaye, Betti Hartmann, **Sardor Tojiev**
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Outline

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Introduction

Motivation

formation of scalar "hair" -superconductivity.

effective mass $m^2_{eff.}$ of the scalar field drops below the Breitenlohner - Freedman (BF) bound under certain circumstances and the black holes or solitons - become unstable to the formation of scalar hair.

$$m^2_{BF} \geq -\frac{d-1}{L^2}$$

the stability of classical field theory solutions - e.g. black hole uniqueness.

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The model

Electrically charged solitons in 5d AdS spacetime. The action reads

$$S = \frac{1}{\gamma}\int d^5x \sqrt{-g}\,(R - 2\Lambda + \gamma\mathcal{L}_{matter})$$

The matter Lagrangian

$$\mathcal{L}_{matter} = -\frac{1}{4}F_{MN}F^{MN} - (D_M\psi)^* D^M\psi - m^2\psi^*\psi$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$, and $D_M\psi = \partial_M\psi - ieA_M\psi$

We choose the following spherically symmetric Ansatz for the metric :

$$ds^2 = -a^2 f dt^2 + \frac{1}{f}dr^2 + r^2 d\Omega_3^2 \, ,$$

For the electromagnetic field and the scalar field, we have :

$$A_M dx^M = \phi(r)\,dt \quad , \quad \psi = \psi(r) \, .$$

The ansatz and equations

The coupled gravity and matter field equations are :

$$
\begin{aligned}
f' &= \frac{2}{r}\left(1 - f + 2\frac{r^2}{L^2}\right) - \gamma\frac{r}{2fa^2}\left(2e^2\phi^2\psi^2 + f(2m^2a^2\psi^2 + \phi'^2) + 2f^2a^2\psi'^2\right) \\
a' &= \gamma\frac{r(e^2\phi^2\psi^2 + a^2f^2\psi'^2)}{af^2} \\
\phi'' &= -\left(\frac{3}{r} - \frac{a'}{a}\right)\phi' + 2\frac{e^2\psi^2}{f}\phi \\
\psi'' &= -\left(\frac{3}{r} + \frac{f'}{f} + \frac{a'}{a}\right)\psi' - \left(\frac{e^2\phi^2}{f^2a^2} - \frac{m^2}{f}\right)\psi
\end{aligned}
$$

The system possesses two scaling symmetries:

$$
\begin{aligned}
&r \to \lambda r \quad , \quad t \to \lambda t \quad , \quad L \to \lambda L \quad , \quad e \to e/\lambda \\
&\phi \to \lambda\phi \quad , \quad \psi \to \lambda\psi \quad , \quad e \to e/\lambda \quad , \quad \gamma \to \gamma/\lambda^2
\end{aligned}
$$

The metric functions have the following asymptotic behaviour for $r \gg 1$

$$f(r) = \frac{r^2}{L^2} + 1 + \frac{f_2}{r^2} + ..., \qquad a(r) = 1 + \frac{c}{r^4}, \qquad M = -\frac{f_2}{2}$$

Asymptotically, the matter fields behave as follows

$$\phi(r \gg 1) = \mu - \frac{Q}{r^2} + \dots \quad , \quad \psi(r \gg 1) = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}$$

with

$$\lambda_- = 2 - \sqrt{4 + m^2 L^2}, \quad \lambda_+ = 2 + \sqrt{4 + m^2 L^2}$$

Boundary conditions

To find soliton solution of the equations of motion

$$f(0) = 1, \quad \psi(0) = \psi_0, \quad \phi'(0) = 0, \quad \psi'(0) = 0, \quad a(r \gg 1) = 1, \quad \psi_- = 0$$

The solitons can then be characterized by the values of the matter & metric functions at the origin $\phi(0), \psi(0), a(0)$ which depend on the choice of e^2, Q and m^2.

Numerical results

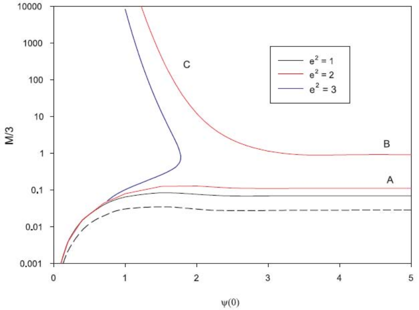
Numerical results

- $m > 0$
- $m < 0$
- $m = 0$

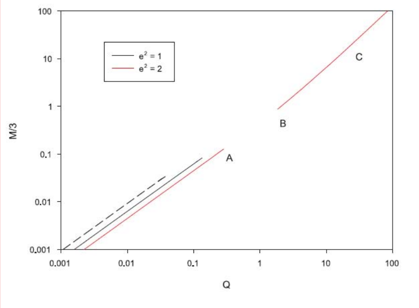
$m^2 = 1$

$m^2 = 1$. The solid lines represent the fundamental solutions, while the dashed lines correspond to the first excited solutions (here given for $e^2 = 1$)

Mass M as function of $\psi(0)$



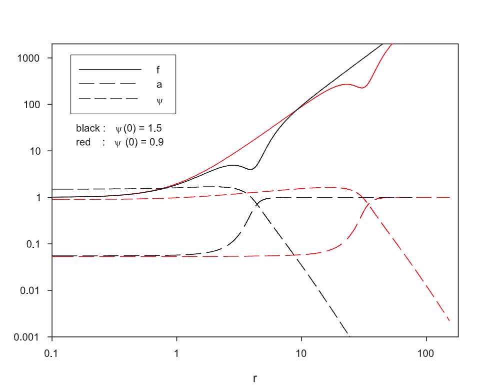
Mass M as function of Q



$m^2 = 1$

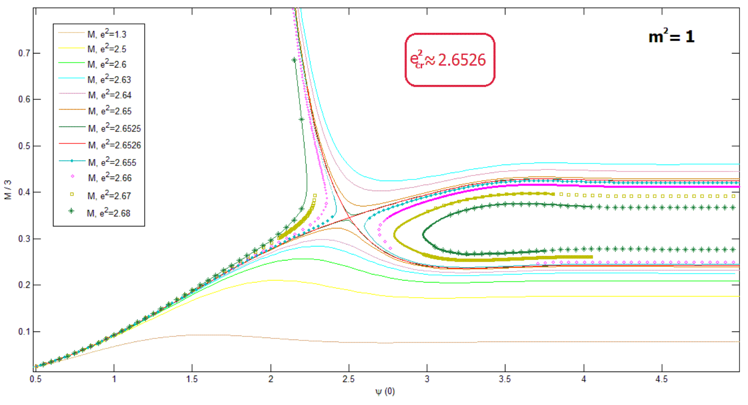
$m^2 = 1$

The profiles of the metric functions: $e^2 = 2$ and for $\psi(0) = 1.5$(black) and $\psi(0) = 0.9$(red)



$m^2 = 1$

The mass M is shown as functions of ψ(0) for $m^2 = 1$ and several values of e^2 close to the critical value e_{cr}



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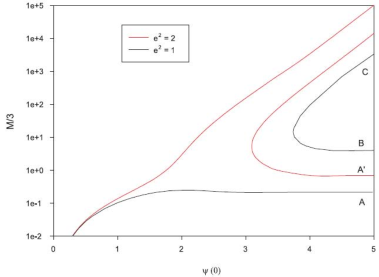
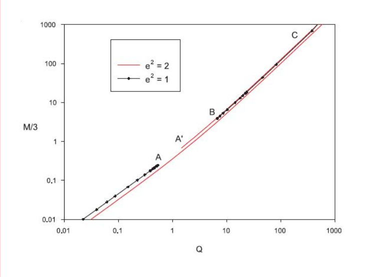
$m^2 = -3$

$m^2 = -3$. The labels A, B and C represent the branches of fundamental solutions, while A′ corresponds to a branch of first excited solutions(here given for $e^2 = 2$): $e^2_{cr} \approx 1.3575$

Mass M as function of $\psi(0)$



Mass M as function of Q



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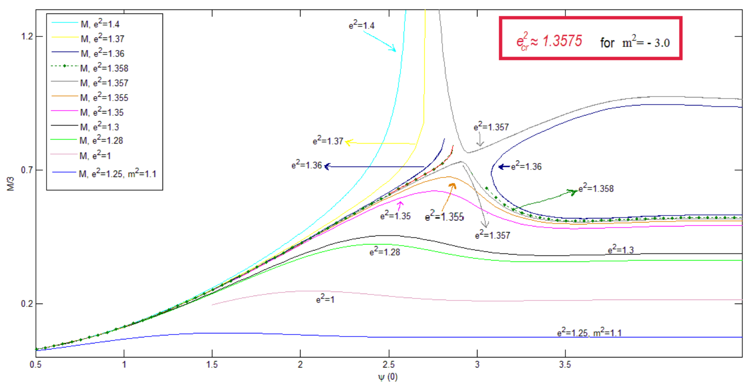
$m^2 = -3$

The mass M is shown as functions of ψ(0) for $m^2 = -3$ and several values of e^2 close to the critical value e_{cr}



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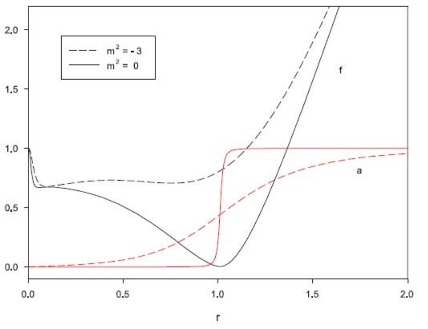
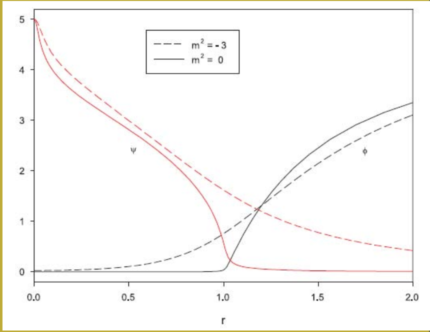
Dependence on m^2

Here $\psi(0) = 5.0$, $e^2 = 2$ and $m^2 = -3$(dashed lines) and $m^2 = 0$(solid lines).

$f(r)$(black) and $a(r)$(red) as function of r

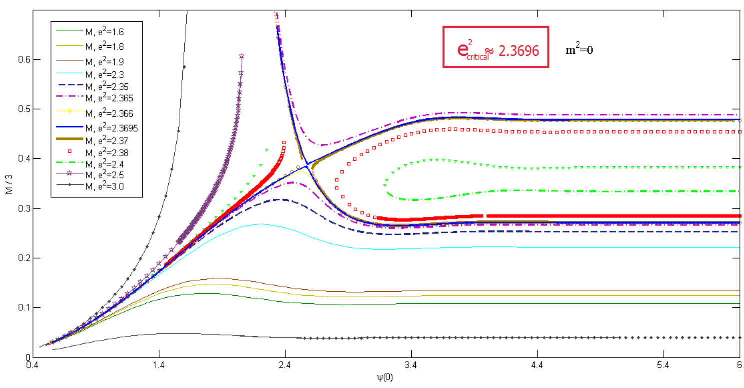


$\psi(r)$(red) and $\phi(r)$(black) as function of r



$m^2 = 0$

The mass M is shown as functions of ψ(0) for $m^2 = 0$ and several values of e^2 close to the critical value e_{cr}



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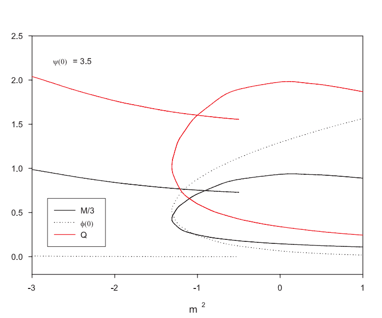
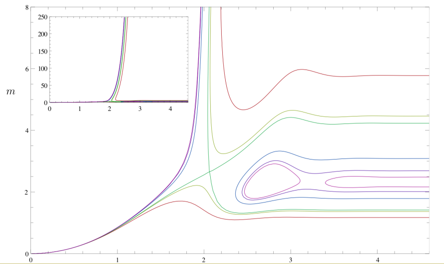
All phenomena are summarized

The mass M(black solid), the value $\phi(0)$(black dotted) and the charge Q(red solid) of the hairy soliton in dependence on m^2 for $e^2 = 2$, $\psi(0) = 3.5$





Gentle (4d),et al. ’12,
Dias (5d),et al. ’12

Summary

1. We have studied the formation of scalar hair on charged soliton in global AdS and the dependence of the solutions on the choice of the charge e^2 and the mass m^2 of the scalar field.

2. We find that the pattern of solutions depends crucially on the choice of e^2 and m^2 with a critical value $e^2_{cr} \approx 2.4 + m^2/3$ dividing this pattern into distinct types.

3. Interestingly, we observe that boson stars in AdS can have arbitrary large M and Q; however, also that a 'forbidden band' of the M and Q at intermidate values of the mass and charge exists.

4. It would be interesting:
   - the effects of Gauss-Bonnet term.
   - holographic interpretation.
   - . . .

Thank You
for your attention !

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