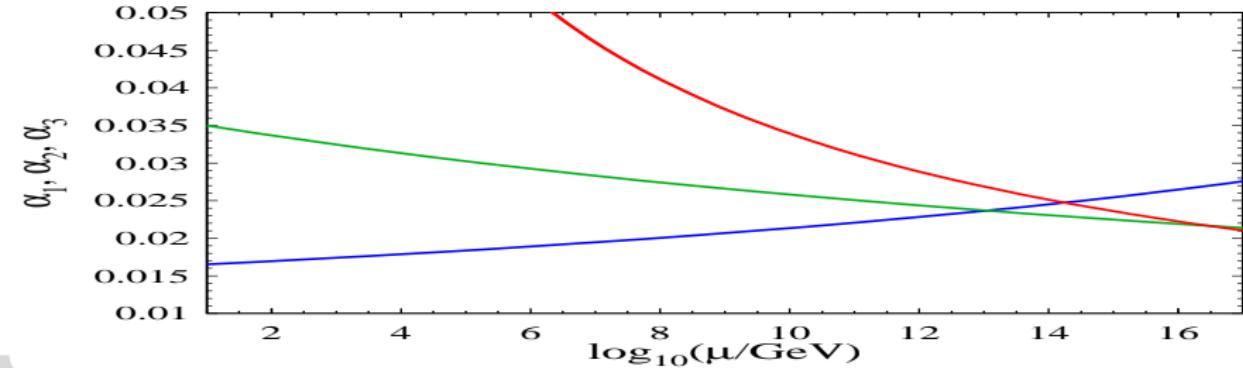


# 3-loop gauge coupling $\beta$ functions in the SM

Frontiers in Perturbative Quantum Field Theory, Bielefeld, September 10-12, 2012

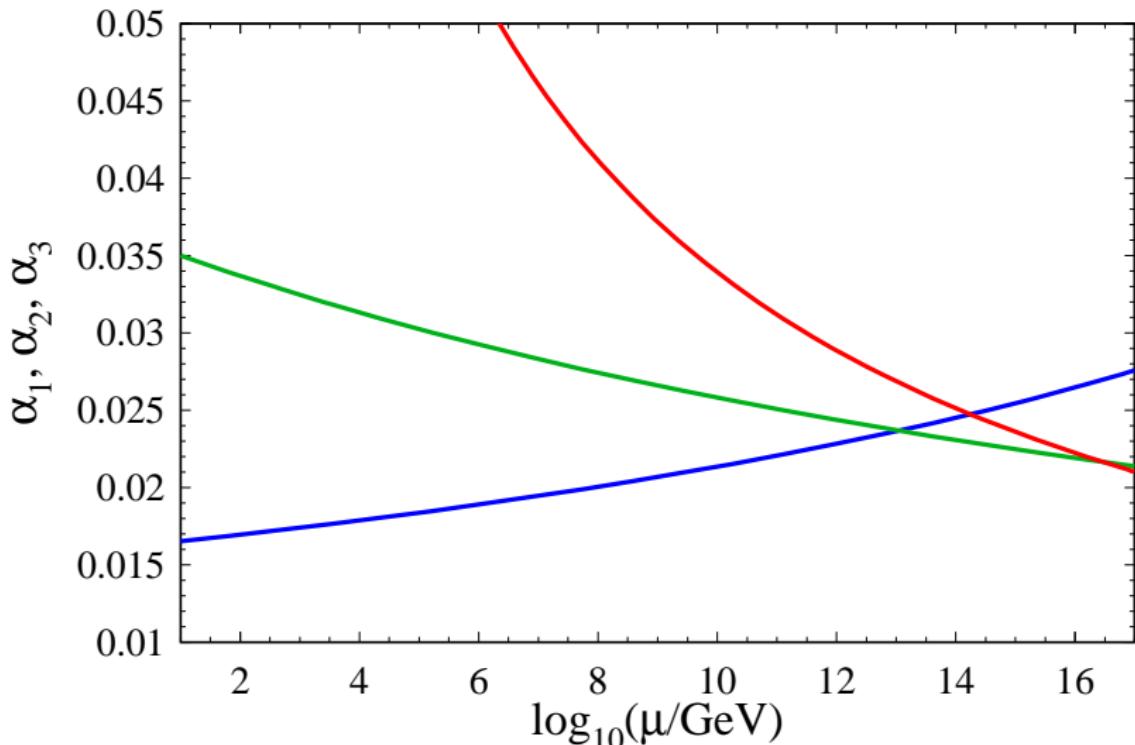
Matthias Steinhauser — TTP Karlsruhe | in collaboration with Luminita Mihaila and Jens Salomon



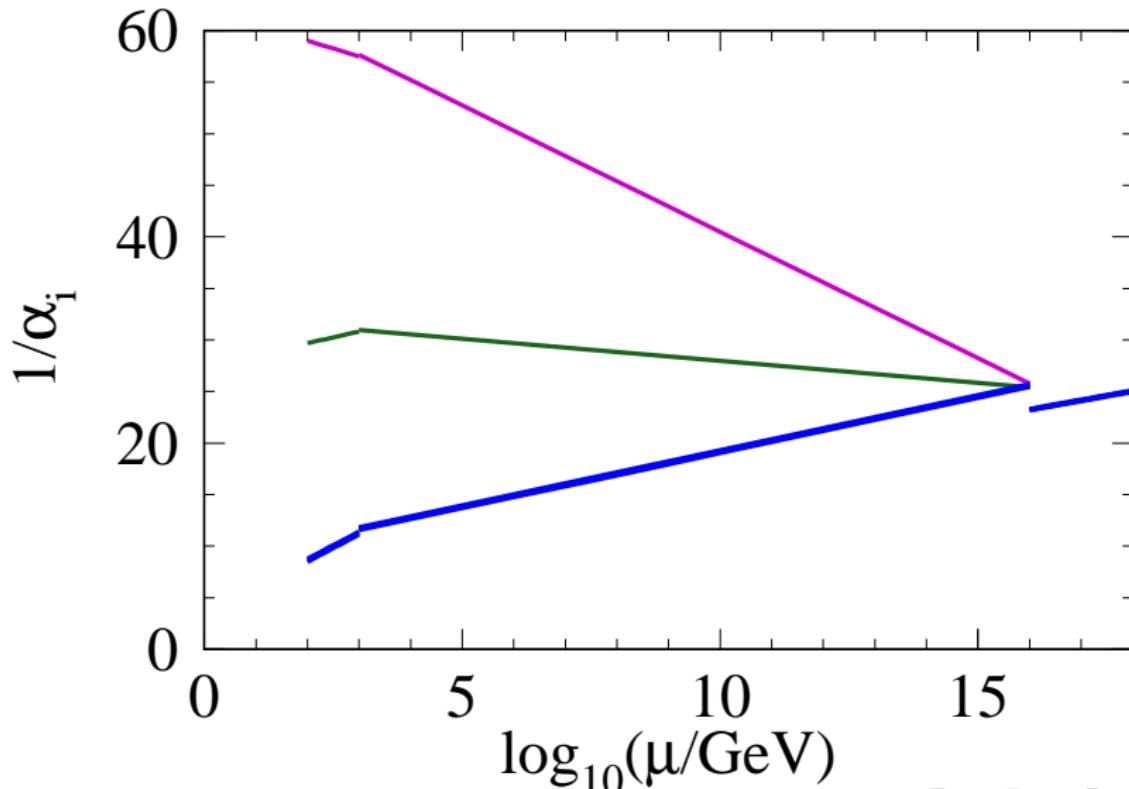
# Outline

- Motivation
- Calculation of  $10^6$  Feynman diagrams
- Results

# Gauge coupling (non) unification

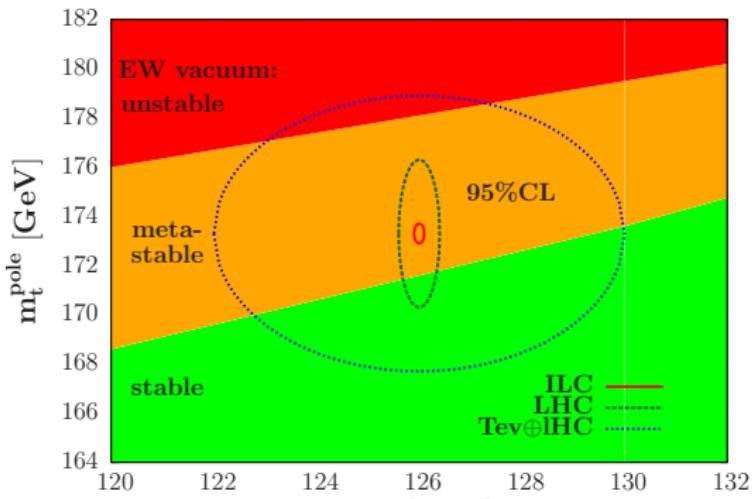


# Gauge coupling (non) unification



# Vacuum stability

## Quartic Higgs coupling at the Planck scale



Ingredients:

- 2-loop effective potential,
- **3-loop beta functions for couplings**,
- 2-loop matching relation for initial values of couplings

[..., Bezrukov,Kalmykov,Kniehl,Shaposhnikov'12; Degrassi,Di Vita,Elias-Miro,Espinosa, Giudice,Isidori,Strumia'12;

Alekhin,Djouadi,Moch'12]

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} \frac{\alpha_s(\mu)}{\pi} &= \beta(\alpha_s) \\ &= -\left(\frac{\alpha_s}{\pi}\right)^2 \left[ \beta_0 + \frac{\alpha_s}{\pi} \beta_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \beta_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \beta_3 + \dots \right] \end{aligned}$$

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} T n_f$$

[Gross,Wilczek'73; Politzer'73]

$$\begin{aligned} \mu^2 \frac{d}{d\mu^2} \frac{\alpha_s(\mu)}{\pi} &= \beta(\alpha_s) \\ &= -\left(\frac{\alpha_s}{\pi}\right)^2 \left[ \beta_0 + \frac{\alpha_s}{\pi} \beta_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \beta_2 + \left(\frac{\alpha_s}{\pi}\right)^3 \beta_3 + \dots \right] \end{aligned}$$

$$\beta_0 = \frac{11}{3} C_A - \frac{2}{3} T n_f$$

[Gross,Wilczek'73; Politzer'73]

$\beta_1$  [Jones'74; Caswell'74; Egorian,Tarasov'78]

$\beta_2$  [Tarasov,Vladimirov,Zharkov'80; Larin,Vermaseren'93]

$\beta_3$  [van Ritbergen,Vermaseren,Larin'97; Czakon'04]

$\beta_2$  computed in many different ways.

Check of  $\beta_3$  with independent method still missing.

# Couplings in the SM

Gauge:

$$\begin{aligned}\alpha_1 &= \frac{5}{3} \frac{\alpha}{\cos^2 \theta_W} && [\text{SU(5)-like normalization}] \\ \alpha_2 &= \frac{\alpha}{\sin^2 \theta_W} \\ \alpha_3 &= \alpha_s\end{aligned}$$

Yukawa:

$$\alpha_x = \frac{\alpha m_x^2}{2 \sin^2 \theta_W M_W^2} \quad x \in \{u, d, c, s, t, b, e, \mu, \tau\}$$

Higgs:

$$\alpha_7 = \frac{\lambda}{4\pi} \quad [\mathcal{L} = -\lambda(\Phi^\dagger \Phi)^2 + \dots]$$

# SM: $\beta$ functions

$$\mu^2 \frac{d}{d\mu^2} \frac{\alpha_i}{\pi} = \beta_i(\alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_b, \alpha_\tau, \dots, \lambda)$$

$$\alpha_i^{\text{bare}} = \mu^{2\epsilon} Z_{\alpha_i} \alpha_i$$

$Z_{\alpha_i}$  charge renormalization constant  
 $\overline{\text{MS}}$  scheme

$$\beta_i = - \left[ \epsilon \frac{\alpha_i}{\pi} + \frac{\alpha_i}{Z_{\alpha_i}} \sum_{j \neq i} \frac{\partial Z_{\alpha_i}}{\partial \alpha_j} \beta_j \right] \left( 1 + \frac{\alpha_i}{Z_{\alpha_i}} \frac{\partial Z_{\alpha_i}}{\partial \alpha_i} \right)^{-1}$$

Calculation of  $\beta_1, \beta_2, \beta_3$  requires:

- $Z_{\alpha_1}, Z_{\alpha_2}, Z_{\alpha_3}$  to 3 loops
- $\beta_t, \beta_b, \beta_\tau$  to 1 loop      dependence of  $Z_{\alpha_1}, Z_{\alpha_2}, Z_{\alpha_3}$  starts at 2 loops
- $\beta_\lambda$  to tree level       $\lambda$  dependence of  $Z_{\alpha_1}, Z_{\alpha_2}, Z_{\alpha_3}$  starts at 3 loops

# SM: known results

## Gauge

**1 loop:** [Gross,Wilczek'73; Politzer'73]

**2 loops:** [Jones'74; Caswell'74; Tarasov,Vladimirov'77; Egorian,Tarasov'79; Jones'81; Fischler,Hill'82; Machacek,Vaughn'83;  
Jack,Osborn'84]

**partial 3 loops:** [Curtright'80; Jones'80; Steinhauser'98; Pickering,Gracey,Jones'01]

**3 loops:** [Mihaila,Salomon,Steinhauser'12]

## Yukawa

**2 loops:** [Fischler,Oliensis'82; Machacek,Vaughn'83; Jack,Osborn'84]

**partial 3 loops:** [Chetyrkin,Zoller'12]

## Higgs

**2 loops:** [Machacek,Vaughn'84; Jack,Osborn'84; Ford,JackJones'92; Luo,Xiao'02]

**partial 3 loops:** [Chetyrkin,Zoller'12]

# 2 approaches

## I. Lorenz gauge,

no spontaneous symmetry breaking,

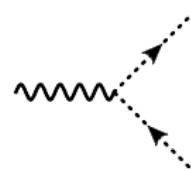
$B$  and  $W$  bosons

## II. Background field gauge,

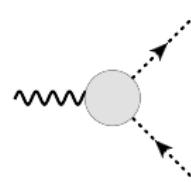
↳ only 2-point functions of background gauge boson  
broken phase of SM,  
 $\gamma$ ,  $W$  and  $Z$  bosons

# Computation of $Z_{\alpha_i}$

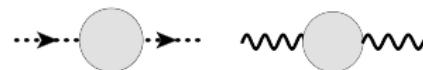
(i) choose vertex involving  $\alpha_i$



(ii) compute  $Z_{\text{vrtx}}$



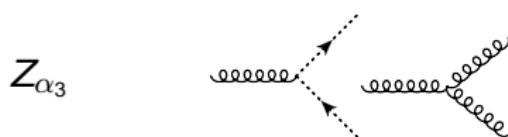
(iii) compute wave function ren.  $Z_{\text{wf},k}$



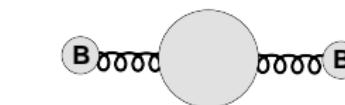
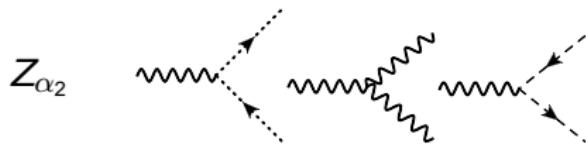
(iv)  $Z_{\alpha_i} = \left( \frac{Z_{\text{vrtx}}}{\prod Z_{\text{wf},k}} \right)^2$

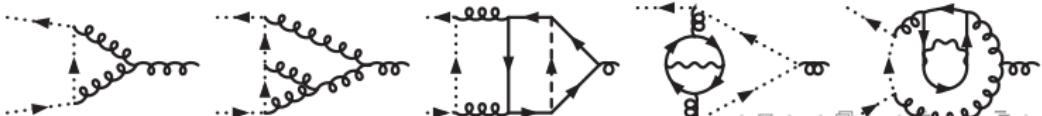
# Vertices

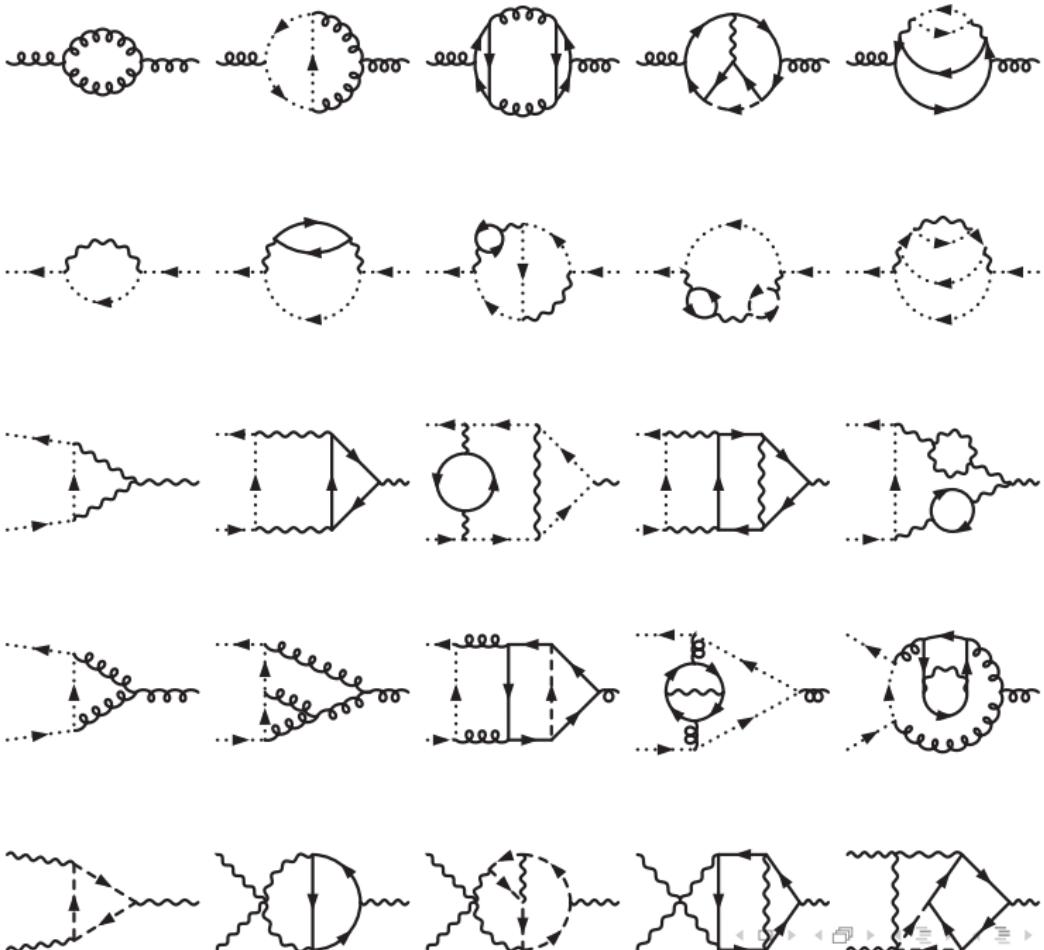
Lorenz gauge

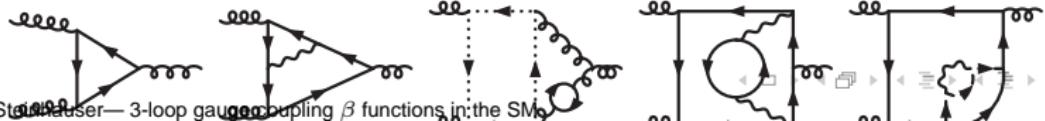
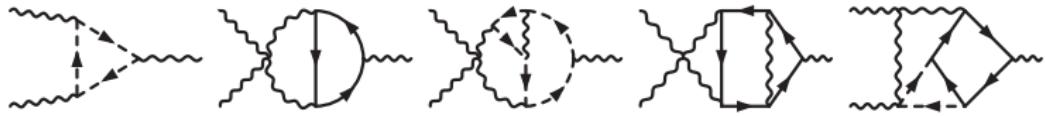
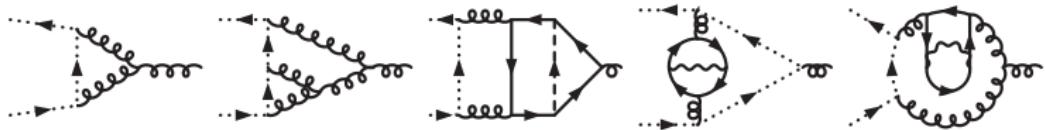


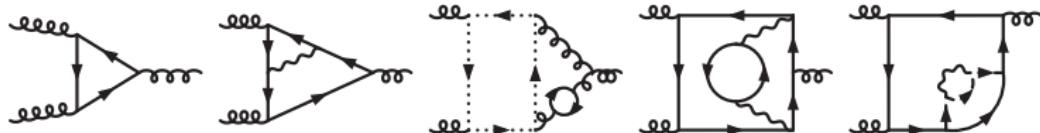
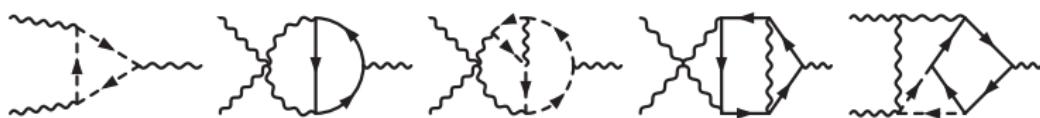
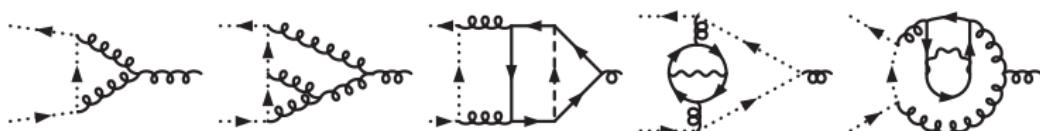
Background field gauge



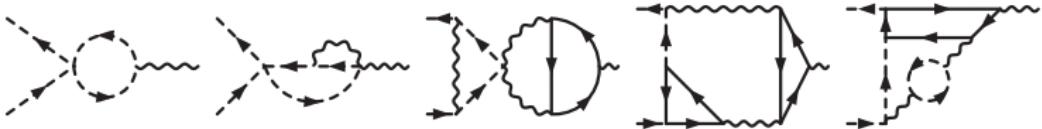
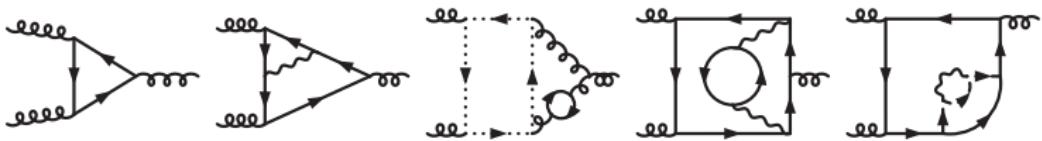
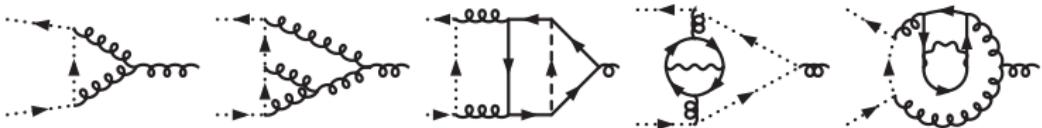








Matthias Steinhauser—3-loop gauge coupling  $\beta$  functions in the SM



# Number of diagrams

Lorenz gauge

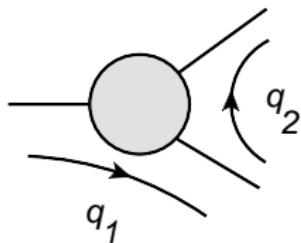
# loops	1	2	3	4
$BB$	14	410	45 926	7 111 021
$W_3 W_3$	17	502	55 063	8 438 172
$gg$	9	188	17 611	2 455 714
$c_g \bar{c}_g$	1	12	521	46 390
$c_{W_3} \bar{c}_{W_3}$	2	42	2 480	251 200
$\phi^+ \phi^-$	10	429	46 418	6 918 256
$BBB$	34	2 172	358 716	73 709 886
$W_1 W_2 W_3$	34	2 216	382 767	79 674 008
$ggg$	21	946	118 086	20 216 024
$c_g \bar{c}_g g$	2	66	4 240	460 389
$c_{W_1} \bar{c}_{W_2} W_3$	2	107	10 577	1 517 631
$\phi^+ \phi^- W_3$	24	2 353	387 338	77 292 771

# Number of diagrams

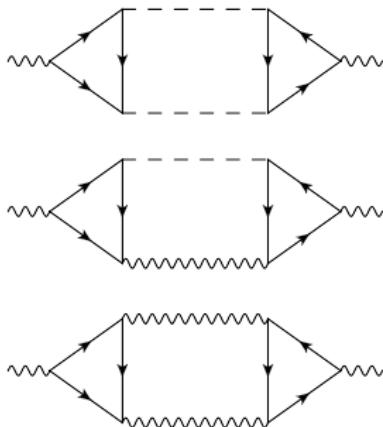
BFG

# loops	1	2	3	4
$\gamma^B \gamma^B$	13	416	61 968	13 683 693
$\gamma^B Z^B$	13	604	100 952	23 640 897
$Z^B Z^B$	20	1064	183 465	44 049 196
$W^{+B} W^{-B}$	18	1438	252 162	42 423 978
$g^B g^B$	10	186	17 494	2 775 946

# Vertex diagrams/loop integrals

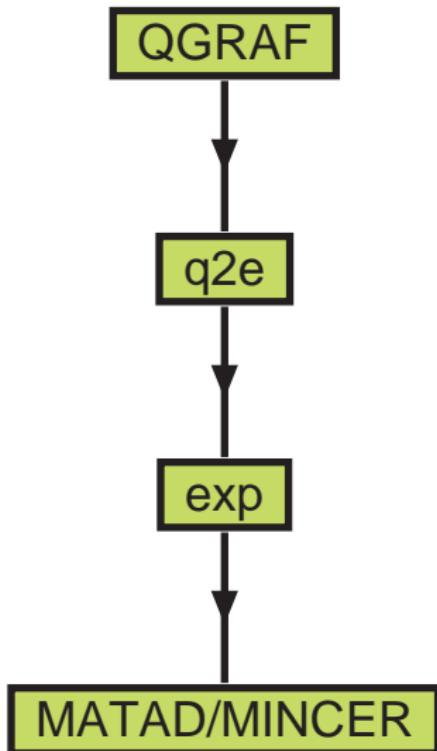


- set all masses to zero
  - set  $q_1 = 0$  or  $q_2 = 0$
  - UV structure not changed
  - make sure that no IR poles are introduced
- ⇒ “MINCER integrals” [Larin, Tkachov, Vermaseren'91]



- anomaly cancellation in fermion triangle
- different for Green's functions with external fermions

# Automation



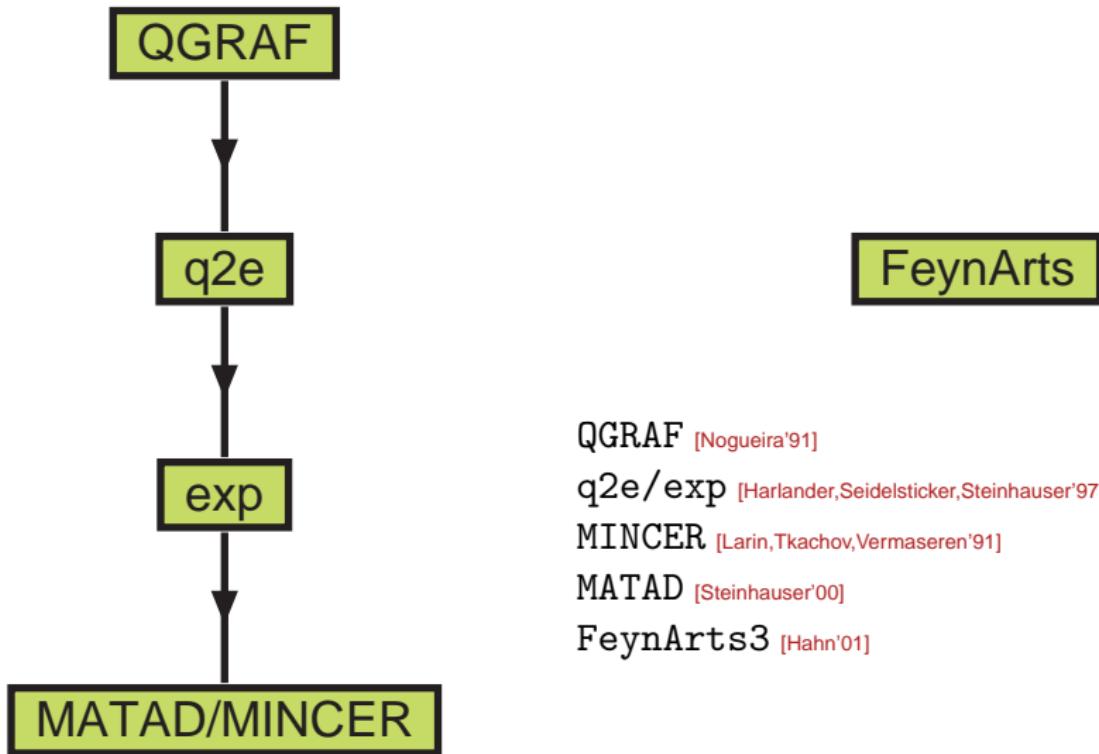
QGRAF [Nogueira'91]

q2e/exp [Harlander,Seidelsticker,Steinhauser'97;Seidelsticker'97]

MINCER [Larin,Tkachov,Vermaseren'91]

MATAD [Steinhauser'00]

# Automation



QGRAF [Nogueira'91]

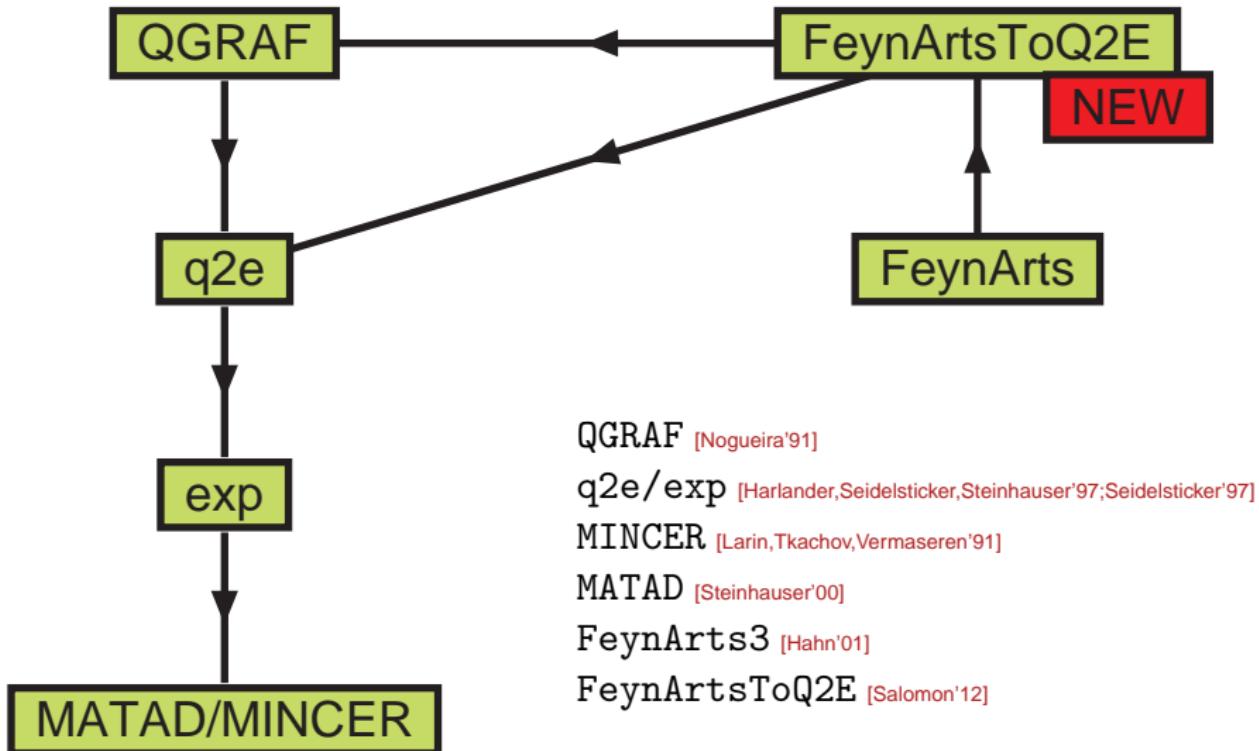
q2e/exp [Harlander,Seidelsticker,Steinhauser'97;Seidelsticker'97]

MINCER [Larin,Tkachov,Vermaseren'91]

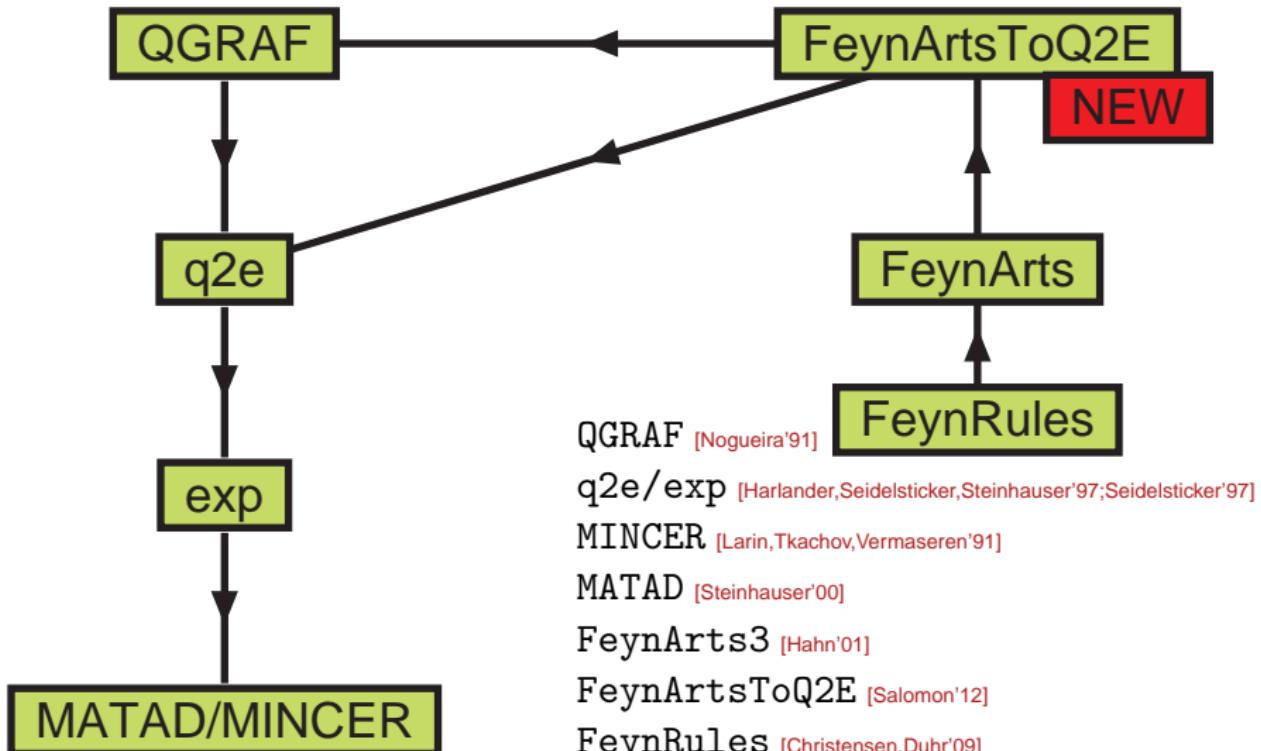
MATAD [Steinhauser'00]

FeynArts3 [Hahn'01]

# Automation



# Automation



# Checks

- 2-loop results for  $\beta_{\text{Yukawa}}$  [Fischler,Oliensis'82; Machacek,Vaughn'83; Jack,Osborn'84]
- 3-loop QCD  $\beta$  function [Tarasov,Vladimirov,Zharkov'80; Larin,Vermaseren'93]
- 3-loop  $\mathcal{O}(\alpha_3^3 \alpha_t)$  to  $\beta_3$  [Steinhauser'98]
- 3-loop results for  $SU(3)$ ,  $SU(2)$ ,  $U(1)$  (only one gauge coupling)
- 3-loop  $\lambda$  terms [Curtright'80; Jones'80] [Pickering,Gracey,Jones'01]

- $Z_{\alpha_i} = \frac{Z_{1,\alpha_i c\bar{c}}}{Z_{2c}\sqrt{Z_{3,\alpha_i}}} = \frac{Z_{1,\alpha_i \alpha_j \alpha_j}}{\left(\sqrt{Z_{3,\alpha_i}}\right)^3}$
  - calculation for arbitrary  $\xi_{\text{QCD}}, \xi_W, \xi_B$ 
    - ⇒  $\beta$  functions are  $\xi$  independent
  - $BBB$  vertex ⇒ zero to 3 loops (= sum of 300 000 diagrams)
  - IR safe: introduce mass  $m \neq 0$ ; asymptotic expansion for  $q^2 \gg m^2$ 
    - ⇒ NO  $\ln(m^2/\mu^2)$  terms!
- (Note: up to 35 sub-diagrams/diagram!)

# Results: $\beta_2$ as an example

$$\begin{aligned}
 \beta_2 = & \frac{\alpha_2^2}{(4\pi)^2} \left\{ -\frac{86}{3} + \frac{16n_G}{3} \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^3} \left\{ \frac{6\alpha_1}{5} - \frac{518\alpha_2}{3} - 6\text{tr}\hat{T} - 6\text{tr}\hat{B} - 2\text{tr}\hat{L} + n_G \left[ \frac{4\alpha_1}{5} + \frac{196\alpha_2}{3} + 16\alpha_3 \right] \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^4} \left\{ \frac{163\alpha_1^2}{400} + \frac{561\alpha_1\alpha_2}{40} - \frac{667111\alpha_2^2}{432} + \frac{6\alpha_1\hat{\lambda}}{5} + 6\alpha_2\hat{\lambda} - 12\hat{\lambda}^2 - \frac{593\alpha_1\text{tr}\hat{T}}{40} \right. \\
 & - \frac{729\alpha_2\text{tr}\hat{T}}{8} - 28\alpha_3\text{tr}\hat{T} - \frac{533\alpha_1\text{tr}\hat{B}}{40} - \frac{729\alpha_2\text{tr}\hat{B}}{8} - 28\alpha_3\text{tr}\hat{B} - \frac{51\alpha_1\text{tr}\hat{L}}{8} \\
 & - \frac{243\alpha_2\text{tr}\hat{L}}{8} + \frac{57\text{tr}\hat{B}^2}{4} + \frac{45(\text{tr}\hat{B})^2}{2} + 15\text{tr}\hat{B}\text{tr}\hat{L} + \frac{19\text{tr}\hat{L}^2}{4} + \frac{5(\text{tr}\hat{L})^2}{2} + \frac{27\text{tr}\hat{T}\hat{B}}{2} \\
 & + \frac{57\text{tr}\hat{T}^2}{4} + 45\text{tr}\hat{T}\text{tr}\hat{B} + 15\text{tr}\hat{T}\text{tr}\hat{L} + \frac{45(\text{tr}\hat{T})^2}{2} \\
 & + n_G \left[ -\frac{28\alpha_1^2}{15} + \frac{13\alpha_1\alpha_2}{5} + \frac{25648\alpha_2^2}{27} - \frac{4\alpha_1\alpha_3}{15} + 52\alpha_2\alpha_3 + \frac{500\alpha_3^2}{3} \right] \\
 & \left. + n_G^2 \left[ \begin{array}{ccc} 44\alpha_1^2 & 1660\alpha_2^2 & 176\alpha_3^2 \end{array} \right] \right\}
 \end{aligned}$$

$n_G$ : # of generations

# Results: $\beta_2$ as an example

$$\begin{aligned}
 \beta_2 = & \frac{\alpha_2^2}{(4\pi)^2} \left\{ -\frac{86}{3} + \frac{16n_G}{3} \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^3} \left\{ \frac{6\alpha_1}{5} - \frac{518\alpha_2}{3} - 6\text{tr}\hat{T} - 6\text{tr}\hat{B} - 2\text{tr}\hat{L} + n_G \left[ \frac{4\alpha_1}{5} + \frac{196\alpha_2}{3} + 16\alpha_3 \right] \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^4} \left\{ \frac{163\alpha_1^2}{400} + \frac{561\alpha_1\alpha_2}{40} - \frac{667111\alpha_2^2}{432} + \frac{6\alpha_1\hat{\lambda}}{5} + 6\alpha_2\hat{\lambda} - 12\hat{\lambda}^2 - \frac{593\alpha_1\text{tr}\hat{T}}{40} \right. \\
 & \left. - \frac{729\alpha_2\text{tr}\hat{T}}{8} - 28\alpha_3\text{tr}\hat{T} - \frac{533\alpha_1\text{tr}\hat{B}}{40} - \frac{729\alpha_2\text{tr}\hat{B}}{8} - 28\alpha_3\text{tr}\hat{B} - \frac{51\alpha_1\text{tr}\hat{L}}{8} \right. \\
 & \left. - \frac{243\alpha_2\text{tr}\hat{L}}{8} + \frac{57\text{tr}\hat{B}^2}{4} + \frac{45(\text{tr}\hat{B})^2}{2} + 15\text{tr}\hat{B}\text{tr}\hat{L} + \frac{19\text{tr}\hat{L}^2}{4} + \frac{5(\text{tr}\hat{L})^2}{2} + \frac{27\text{tr}\hat{T}\hat{B}}{2} \right. \\
 & \left. + \frac{57\text{tr}\hat{T}^2}{4} + 45\text{tr}\hat{T}\text{tr}\hat{B} + 15\text{tr}\hat{T}\text{tr}\hat{L} + \frac{45(\text{tr}\hat{T})^2}{2} \right. \\
 & \left. + n_G \left[ -\frac{28\alpha_1^2}{15} + \frac{13\alpha_1\alpha_2}{5} + \frac{25648\alpha_2^2}{27} - \frac{4\alpha_1\alpha_3}{15} + 52\alpha_2\alpha_3 + \frac{500\alpha_3^2}{3} \right] \right\} \\
 & + \boxed{1 \text{ loop: } \frac{\alpha_2^2}{(4\pi)^2} \left\{ -\frac{86}{3} + \frac{16n_G}{3} \right\}}
 \end{aligned}$$

# Results: $\beta_2$ as an example

$$\begin{aligned}
 \beta_2 = & \frac{\alpha_2^2}{(4\pi)^2} \left\{ -\frac{86}{3} + \frac{16n_G}{3} \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^3} \left\{ \frac{6\alpha_1}{5} - \frac{518\alpha_2}{3} - 6\text{tr}\hat{T} - 6\text{tr}\hat{B} - 2\text{tr}\hat{L} + n_G \left[ \frac{4\alpha_1}{5} + \frac{196\alpha_2}{3} + 16\alpha_3 \right] \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^4} \left\{ \frac{163\alpha_1^2}{400} + \frac{561\alpha_1\alpha_2}{40} - \frac{667111\alpha_2^2}{432} + \frac{6\alpha_1\hat{\lambda}}{5} + 6\alpha_2\hat{\lambda} - 12\hat{\lambda}^2 - \frac{593\alpha_1\text{tr}\hat{T}}{40} \right. \\
 & \left. - \frac{729\alpha_2\text{tr}\hat{T}}{8} - 28\alpha_3\text{tr}\hat{T} - \frac{533\alpha_1\text{tr}\hat{B}}{40} - \frac{729\alpha_2\text{tr}\hat{B}}{8} - 28\alpha_3\text{tr}\hat{B} - \frac{51\alpha_1\text{tr}\hat{L}}{8} \right. \\
 & \left. - \frac{243\alpha_2\text{tr}\hat{L}}{8} + \frac{57\text{tr}\hat{B}^2}{4} + \frac{45(\text{tr}\hat{B})^2}{2} + 15\text{tr}\hat{B}\text{tr}\hat{L} + \frac{19\text{tr}\hat{L}^2}{4} + \frac{5(\text{tr}\hat{L})^2}{2} + \frac{27\text{tr}\hat{T}\hat{B}}{2} \right. \\
 & \left. + \text{tr}\hat{T}, \text{tr}\hat{B}, \text{tr}\hat{L}: \text{Yukawa couplings} \right. \\
 & \left. + \text{example: keep only } \alpha_t, \alpha_b, \alpha_\tau \right. \\
 & \left. \Rightarrow \text{tr}\hat{T} \rightarrow \alpha_t, \text{tr}\hat{B} \rightarrow \alpha_b, \text{tr}\hat{L} \rightarrow \alpha_\tau \right. \\
 & + n_G^2 \left[ -\frac{44\alpha_1^2}{45} - \frac{1660\alpha_2^2}{27} - \frac{176\alpha_3^2}{9} \right]
 \end{aligned}$$

# Results: $\beta_2$ as an example

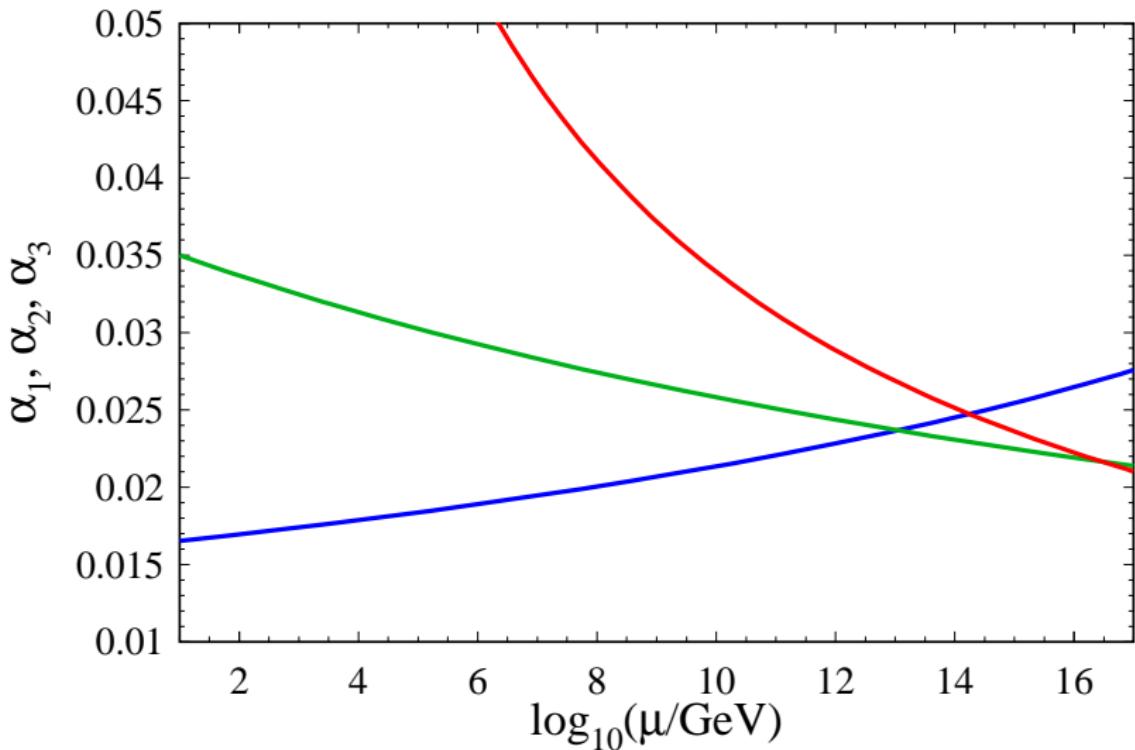
$$\begin{aligned}
 \beta_2 = & \frac{\alpha_2^2}{(4\pi)^2} \left\{ -\frac{86}{3} + \frac{16n_G}{3} \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^3} \left\{ \frac{6\alpha_1}{5} - \frac{518\alpha_2}{3} - 6\text{tr}\hat{T} - 6\text{tr}\hat{B} - 2\text{tr}\hat{L} + n_G \left[ \frac{4\alpha_1}{5} + \frac{196\alpha_2}{3} + 16\alpha_3 \right] \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^4} \left\{ \frac{163\alpha_1^2}{400} + \frac{561\alpha_1\alpha_2}{40} - \frac{667111\alpha_2^2}{432} + \frac{6\alpha_1\hat{\lambda}}{5} + 6\alpha_2\hat{\lambda} - 12\hat{\lambda}^2 - \frac{593\alpha_1\text{tr}\hat{T}}{40} \right. \\
 & \left. - \frac{729\alpha_2\text{tr}\hat{T}}{8} - 28\alpha_3\text{tr}\hat{T} - \frac{533\alpha_1\text{tr}\hat{B}}{40} - \frac{729\alpha_2\text{tr}\hat{B}}{8} - 28\alpha_3\text{tr}\hat{B} - \frac{51\alpha_1\text{tr}\hat{L}}{8} \right. \\
 & \left. - \frac{243\alpha_2\text{tr}\hat{L}}{8} + \frac{57\text{tr}\hat{B}^2}{4} + \frac{45(\text{tr}\hat{B})^2}{2} + 15\text{tr}\hat{B}\text{tr}\hat{L} + \frac{19\text{tr}\hat{L}^2}{4} + \frac{5(\text{tr}\hat{L})^2}{2} + \frac{27\text{tr}\hat{T}\hat{B}}{2} \right. \\
 & \left. + \frac{57\text{tr}\hat{T}^2}{4} + 45\text{tr}\hat{T}\text{tr}\hat{B} + 15\text{tr}\hat{T}\text{tr}\hat{L} + \frac{45(\text{tr}\hat{T})^2}{2} \right. \\
 & \left. + n_G \left[ -\frac{28\alpha_1^2}{15} + \frac{13\alpha_1\alpha_2}{5} + \frac{25648\alpha_2^2}{27} - \frac{4\alpha_1\alpha_3}{15} + 52\alpha_2\alpha_3 + \frac{500\alpha_3^2}{3} \right] \right\} \\
 & + r^2 \left[ \begin{array}{ccc} 44\alpha_1^2 & 1660\alpha_2^2 & 176\alpha_3^2 \end{array} \right]
 \end{aligned}$$

2 loops:  $\propto \alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_b, \alpha_\tau$

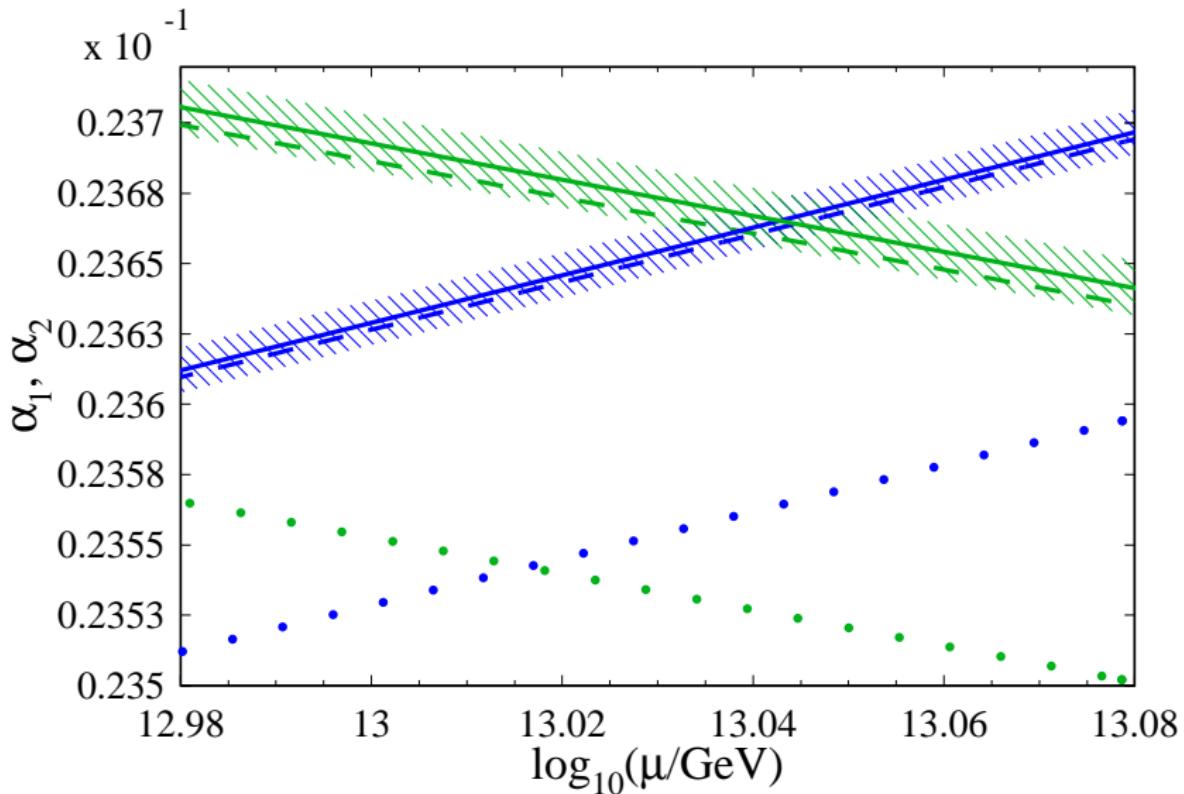
# Results: $\beta_2$ as an example

$$\begin{aligned}
 \beta_2 = & \frac{\alpha_2^2}{(4\pi)^2} \left\{ -\frac{86}{3} + \frac{16n_G}{3} \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^3} \left\{ \frac{6\alpha_1}{5} - \frac{518\alpha_2}{3} - 6\text{tr}\hat{T} - 6\text{tr}\hat{B} - 2\text{tr}\hat{L} + n_G \left[ \frac{4\alpha_1}{5} + \frac{196\alpha_2}{3} + 16\alpha_3 \right] \right\} \\
 & + \frac{\alpha_2^2}{(4\pi)^4} \left\{ \frac{163\alpha_1^2}{400} + \frac{561\alpha_1\alpha_2}{40} - \frac{667111\alpha_2^2}{432} + \frac{6\alpha_1\hat{\lambda}}{5} + 6\alpha_2\hat{\lambda} - 12\hat{\lambda}^2 - \frac{593\alpha_1\text{tr}\hat{T}}{40} \right. \\
 & \left. - \frac{729\alpha_2\text{tr}\hat{T}}{8} - 28\alpha_3\text{tr}\hat{T} - \frac{533\alpha_1\text{tr}\hat{B}}{40} - \frac{729\alpha_2\text{tr}\hat{B}}{8} - 28\alpha_3\text{tr}\hat{B} - \frac{51\alpha_1\text{tr}\hat{L}}{8} \right. \\
 & \left. - \frac{243\alpha_2\text{tr}\hat{L}}{8} + \frac{57\text{tr}\hat{B}^2}{4} + \frac{45(\text{tr}\hat{B})^2}{2} + 15\text{tr}\hat{B}\text{tr}\hat{L} + \frac{19\text{tr}\hat{L}^2}{4} + \frac{5(\text{tr}\hat{L})^2}{2} + \frac{27\text{tr}\hat{T}\hat{B}}{2} \right. \\
 & \left. + \frac{57\text{tr}\hat{T}^2}{4} + 45\text{tr}\hat{T}\text{tr}\hat{B} + 15\text{tr}\hat{T}\text{tr}\hat{L} + \frac{45(\text{tr}\hat{T})^2}{2} \right\} \\
 & + \boxed{3 \text{ loops: } \alpha_i\alpha_j(\dots) + \lambda\alpha_i(\dots) + \lambda^2(\dots) \left[ \frac{100\alpha_3^2}{3} \right]} \\
 & + n_G^2 \left[ -\frac{44\alpha_1^2}{45} - \frac{1660\alpha_2^2}{27} - \frac{176\alpha_3^2}{9} \right]
 \end{aligned}$$

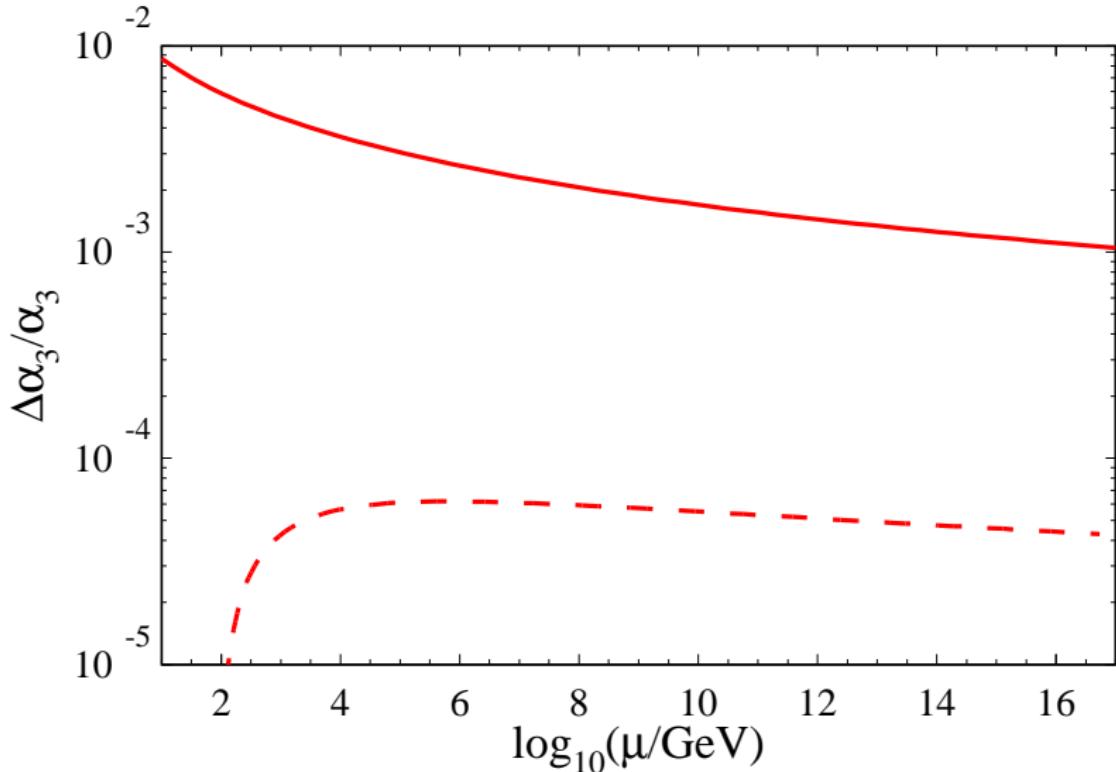
# Numerics



# Numerics



# Numerics



# Dominant terms

Relative contribution to the difference

“3-loop running” – “2-loop running” ( $\mu = M_Z \rightarrow \mu = 10^{16} \text{ GeV}$ )

$\alpha_1$	$\alpha_1^2 \alpha_3^2$	> 90%
$\alpha_2$	$\alpha_2^2 \alpha_3^2$	+56%
	$\alpha_2^4$	+37%
	$\alpha_2^3 \alpha_3$	+13%
$\alpha_3$	$\alpha_3^4$	+137%
	$\alpha_3^3 \alpha_t$	-112%
	$\alpha_3^3 \alpha_2$	+45%
	$\alpha_3^2 \alpha_t^2$	+28%
	$\alpha_3^2 \alpha_2^2$	+17%
	$\alpha_3^2 \alpha_2 \alpha_t$	-16%
$\alpha_3$	$\alpha_3^5$	-58%

# Conclusions

- $\beta_{\alpha_1}, \beta_{\alpha_2}, \beta_{\alpha_3}$  in SM to 3 loops
- fundamental quantity of SM
- automated setup
- “3 loops – 2 loops”  $\sim$  experimental uncertainty