Finite Temperature Lattice QCD

Introduction

The QCD transition

The transition temperature

The Equation of State

Screening masses

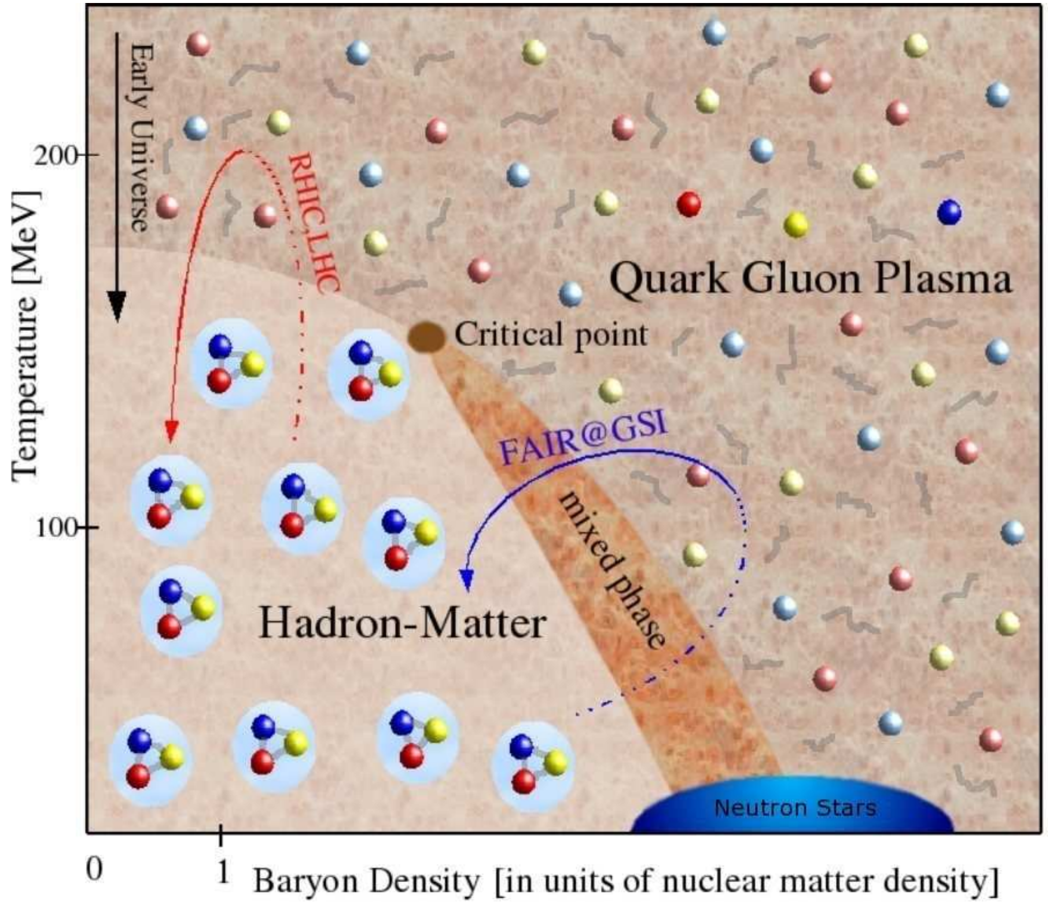
finite density $\longrightarrow$ Ch. Schmidt

topological aspects

$\longrightarrow$ M. Müller-Preußker

Introduction

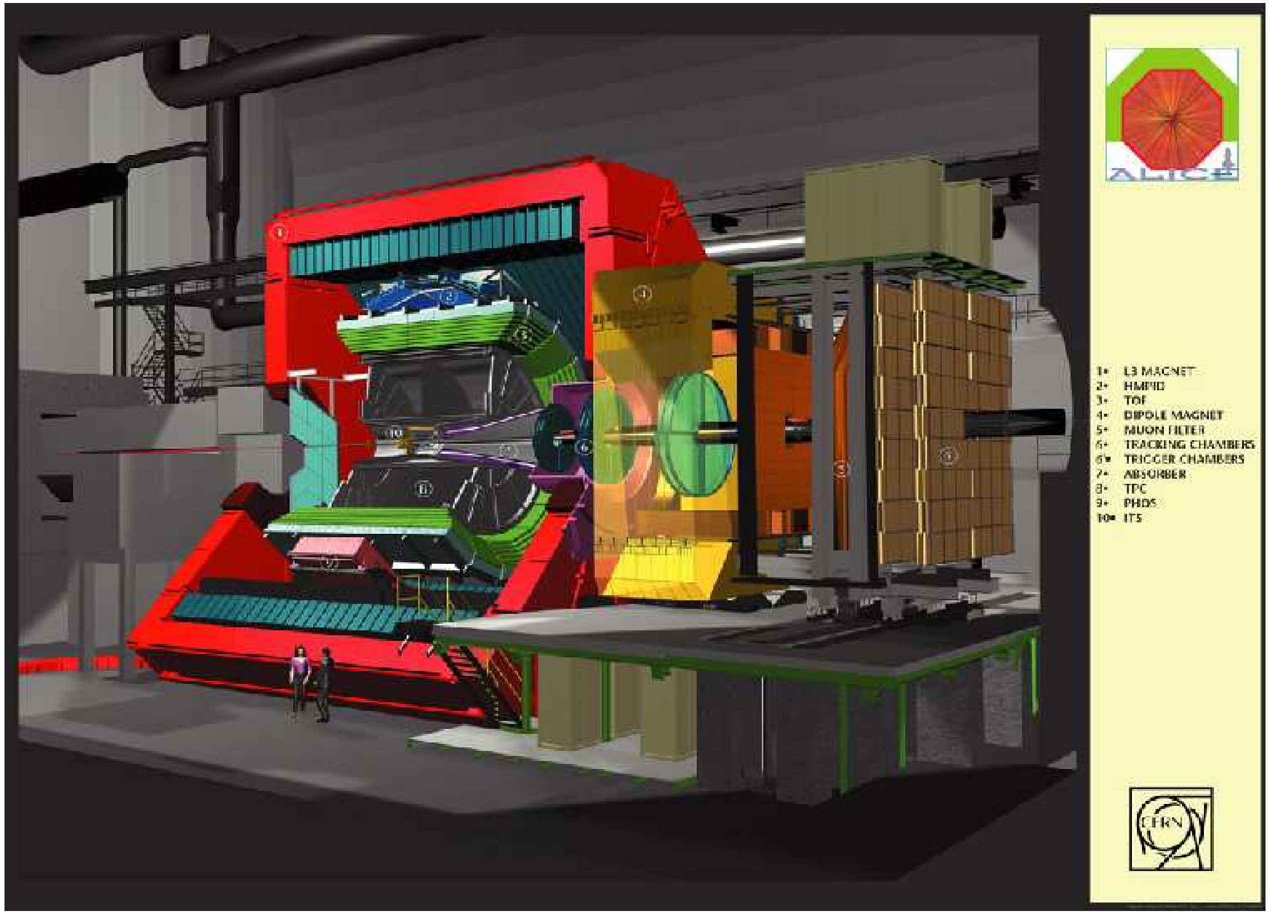
asymptotic freedom $\alpha_s(Q \to \infty) = 0$, at finite T: $Q \sim T$

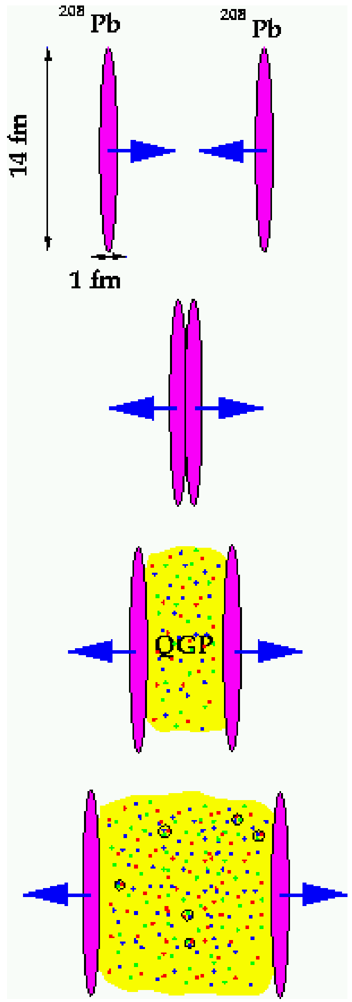


- universe $n_B/n_\gamma \simeq 10^{-9}$ $\rightsquigarrow$ cosmology
- at high density
  - probably rich phase structure (color superconductivity, color-flavor locking ...)
  - neutron stars $\rightsquigarrow$ astrophysics
  - difficult for the lattice (sign problem) $\to$ lectures by G. Aarts, Ch.Schmidt
- heavy ion collisions (LHC, RHIC, FAIR)

  FAIR: $N_B/S \simeq 0.03$, $\mu_B/T \simeq 2.5$

  RHIC: $N_B/S \simeq 0.003$, $\mu_B/T \simeq 0.25$

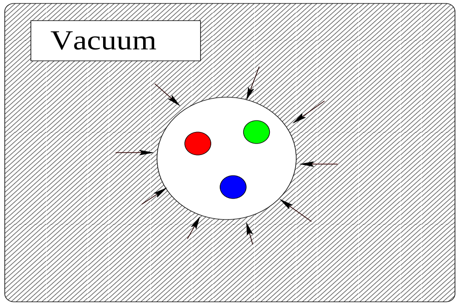




– at mid rapidity, baryon density is small

– short equilibration time $\tau_{\rm equi} \lesssim 1$ fm (strong interaction !)

– expansion is described by hydrodynamics $\longrightarrow$ Equation of State estimates from a simple model: bag model



hadron view

$$p_H = d_H \frac{\pi^2}{90} T^4$$

$$d_H = \#\text{d.o.f.} = 3 \quad (\pi^\pm, \pi^0)$$

quark-gluon view

$$p_P = d_P \frac{\pi^2}{90} T^4 - B$$

$$d_P = 2 \cdot 8 + 7/8 \cdot 2 \cdot 2 \cdot 3 \cdot N_F \quad (G, q)$$

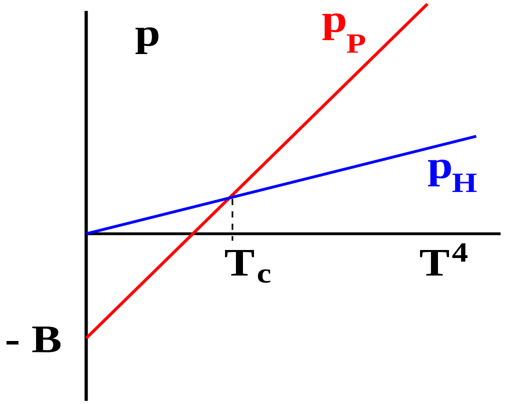
at $T \neq 0$, instead of energy E, free energy

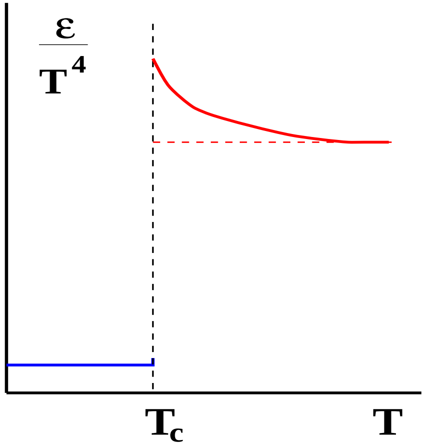
$F = E - TS = -pV$ minimal

$\Rightarrow$ here **1st order phase transition**

since $\epsilon = E/V \sim T^2 \partial(p/T)/\partial T$

has a discontinuity





$$p_H(T_c) = p_P(T_c) \;\Leftrightarrow\; T_c = \left(\frac{90}{\pi^2(d_P - d_H)}\right)^{1/4} B^{1/4} \simeq 0.7 B^{1/4} \simeq 140 \text{ MeV}$$

$$M_H = \frac{16\pi}{3} R^3 B$$

NOTE: we have a scale here, the temperature T

Quantum statistics in equilibrium:

the central quantity is the **partition function** $Z = \mathrm{tr}\left\{\exp(-\hat{H}/T)\right\} = \Sigma_n \langle n|\exp(-\hat{H}/T)|n\rangle$

(at $\mu = 0$)

compare with time evolution operator $\exp(-it\hat{H})$

- (inverse) temperature direction $\equiv$ imaginary time $\tau = it$
- temporal extent limited by $0 \leq \tau \leq 1/T$
- trace requests periodic b.c. in time for bosons $\phi(\tau = 1/T) = \phi(0)$ (commutators)
  anti-periodic b.c. in time for fermions $\psi(\tau = 1/T) = -\psi(0)$ (anti-commutators)

→ Feynman path integral

$$Z(T,V) = \int_{\text{periodic}} \mathcal{D}\phi(\vec{x},\tau)\exp\left\{-\int_0^{1/T} d\tau \int_V d^3\vec{x}\,\mathcal{L}_E[\phi(\vec{x},\tau)]\right\}$$

where $\mathcal{L}_E$ is the euclidean Lagrange density

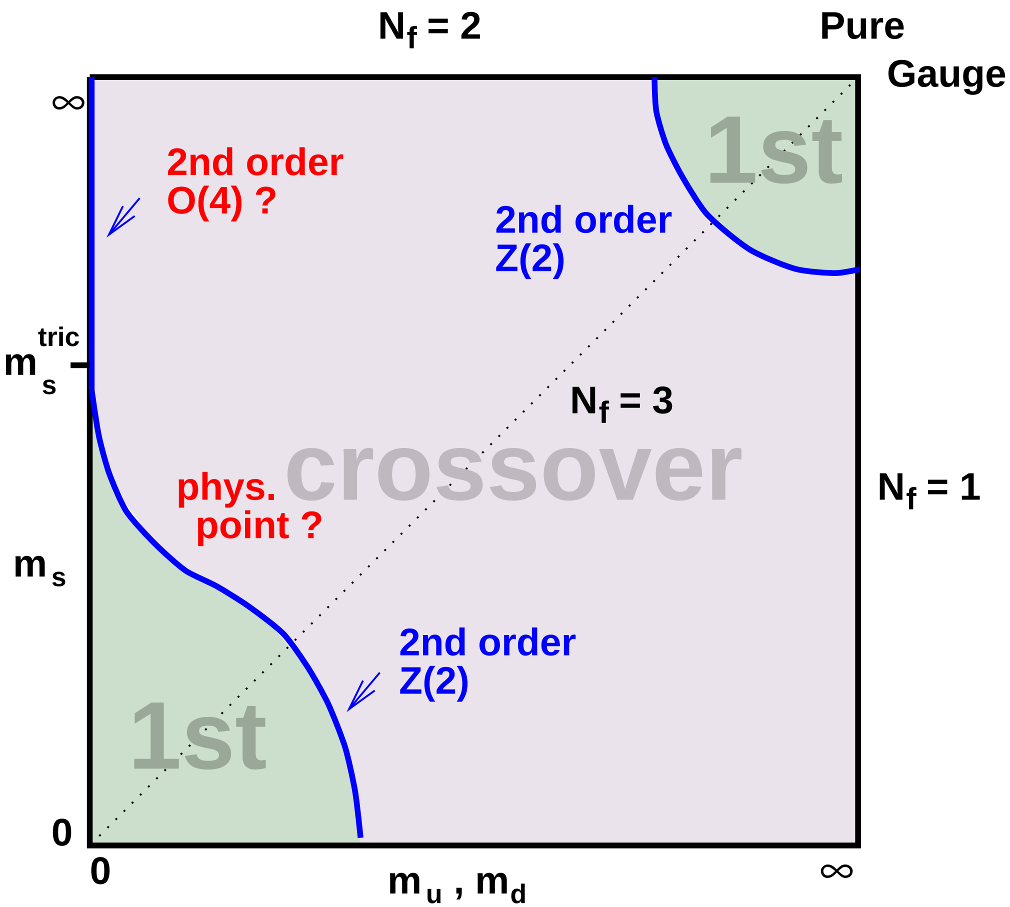
apply standard thermodynamic relations, e.g. energy density $\epsilon = \frac{T^2}{V}\frac{\partial \ln Z}{\partial T}\Big|_V$

specific heat $c_V = \frac{1}{VT^2}\frac{\partial^2 \ln Z}{\partial(1/T)^2}\Big|_V$

on the **lattice** as at zero temperature, but $a(\beta)T = \frac{1}{N_\tau}$ and $LT = \frac{N_\sigma}{N_\tau}$

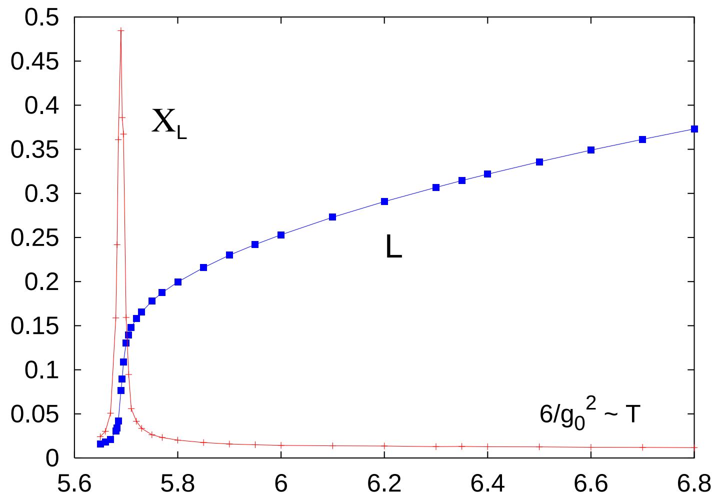
- charm at $m_c/T_c = \mathcal{O}(10)$ presumably too heavy for the dynamics at T_c
- leaves light $m_u \simeq m_d \equiv m_l$ and strange m_s quark

- the nature of the transition is expected to depend strongly on m_l and m_s :

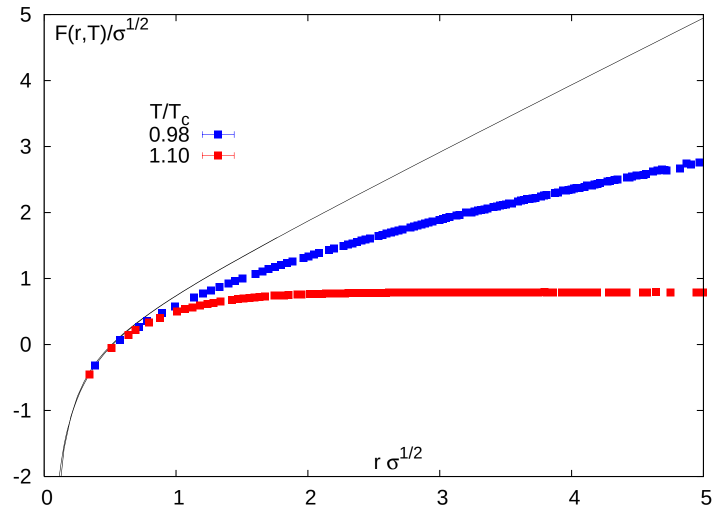


- the action is invariant under the global transformation $U_\tau(\vec{x}, \tau = N_\tau) \to zU_\tau(\vec{x}, \tau = N_\tau)$
  with $z \,\epsilon\, Z(3) = \{\exp(in2\pi/3), n = 0, 1, 2\}$, the center group of $SU(3)$
- the Polyakov loop $L(\vec{x}) = tr\, \Pi_\tau\, U_\tau(\vec{x}, \tau)$ is not

$\Rightarrow$ $\langle L(\vec{x})\rangle$ is an order parameter for the spontaneous breakdown of the $Z(3)$ symmetry



corrolary $\langle L(r)L(0)\rangle \sim \exp(-F_{Q\bar{Q}}(r)/T)$



– the rise of L signals a phase transition

– the susceptibility $\chi_L \sim \langle L^2\rangle - \langle L\rangle^2$
measures thermal fluctuations
which grow large at T_c

measures the free energy of static quarks at separation r

$F_{Q\bar{Q}}(r \to \infty) \to \infty$ for $T < T_c$

$F_{Q\bar{Q}}(r \to \infty) \to$ finite for $T > T_c$

i.e. in pure Yang-Mills, $\langle L\rangle$ detects the confinement - deconfinement transition

with fermions, $\langle L\rangle$ is not an order parameter, $Z(3)$ broken by fermion action

- at and in the vicinity of a phase transition thermal fluctuations grow large or diverge
- i.e. correlation lengths ξ become large or diverge
- in this limit microscopic interaction details don't matter $\rightsquigarrow$ series of "spin blockings" and RG flow
- what counts are global symmetries and the dimension

a universality class consists of all those models which flow into the same critical fixed point under repeated renormalization group transformations

$\longrightarrow$ hence the same critical behavior in terms of scaling with universal critical exponents and certain other universal quantities/functions

- equilibrium QCD (Yang-Mills) at high T is expected to be 3 dimensional generically: $\phi(\tau) = \sum_n \phi(\omega_n) \exp(i\omega_n \tau)$ with $\omega_n = 2\pi T n \rightsquigarrow$ only static modes ($n = 0$) survive

$\Rightarrow$ compare with (classical) 3d systems with the same global symmetries,

- pure $SU(3)$ Yang-Mills: the same universality class as the 3d $Z(3)$ (3-state) Potts spin model (weakly) 1st order

---

* the theory behind this is Wilson's renormalization group
there is a huge condensend matter literature
for introductory lectures see e.g. J. Engels at
http://www.physik.uni-bielefeld.de/igs/schools/Fall2006/phase.html

the free energy density $f = -\frac{T}{V} \ln Z$ consists of a regular and a singular part, $f = f_{\rm reg} + f_{sing}$, where f_{sing} develops the singularities which drive the transition

scaling: in the vicinity of a critical point $(T_c, H = 0)$, under an arbitrary scale change b

$$f_{sing}(t, h) = b^{-d} f_{sing}(b^{y_t} t, b^{y_h} h)$$

where y_t, y_h are universal critical exponents

$t = \frac{1}{t_0} \frac{T - T_c}{T_c}$ (normalized) reduced temperature

$h = \frac{1}{h_0} H$ (normalized) external symmetry breaking, e.g. magnetic field, m_q in QCD

t_0, h_0 non-universal metric factors i.e. need to be adjusted for each model

$$f_{sing}(t,h) = b^{-d} f_{sing}(b^{y_t} t, b^{y_h} h)$$

- choose $b = |t|^{-1/y_t}$: $f_{sing}(t,h) = |t|^{d/y_t} f_{sing}(\pm 1, |t|^{-y_h/y_t} h)$

  $\Rightarrow$ for magnetization $M = -\partial f/\partial H$ find at $t < 0, H = 0$: $M \sim (-t)^{\beta}$ with $\beta = (d - y_h)/y_t$

  $\Rightarrow$ for susceptibility $\chi = \partial M/\partial H \sim \langle(\delta M)^2\rangle$ find at $H = 0$: $\chi \sim |t|^{-\gamma}$ with $\gamma = (2y_h - d)/y_t$

- choose $b = |h|^{-1/y_h}$: $f_{sing}(t,h) = |h|^{d/y_h} f_{sing}(t|h|^{-y_t/y_h}, \pm 1)$

  $\Rightarrow$ for magnetization find at $t = 0$: $M \sim |h|^{1/\delta}$ with $1/\delta = d/y_h - 1$

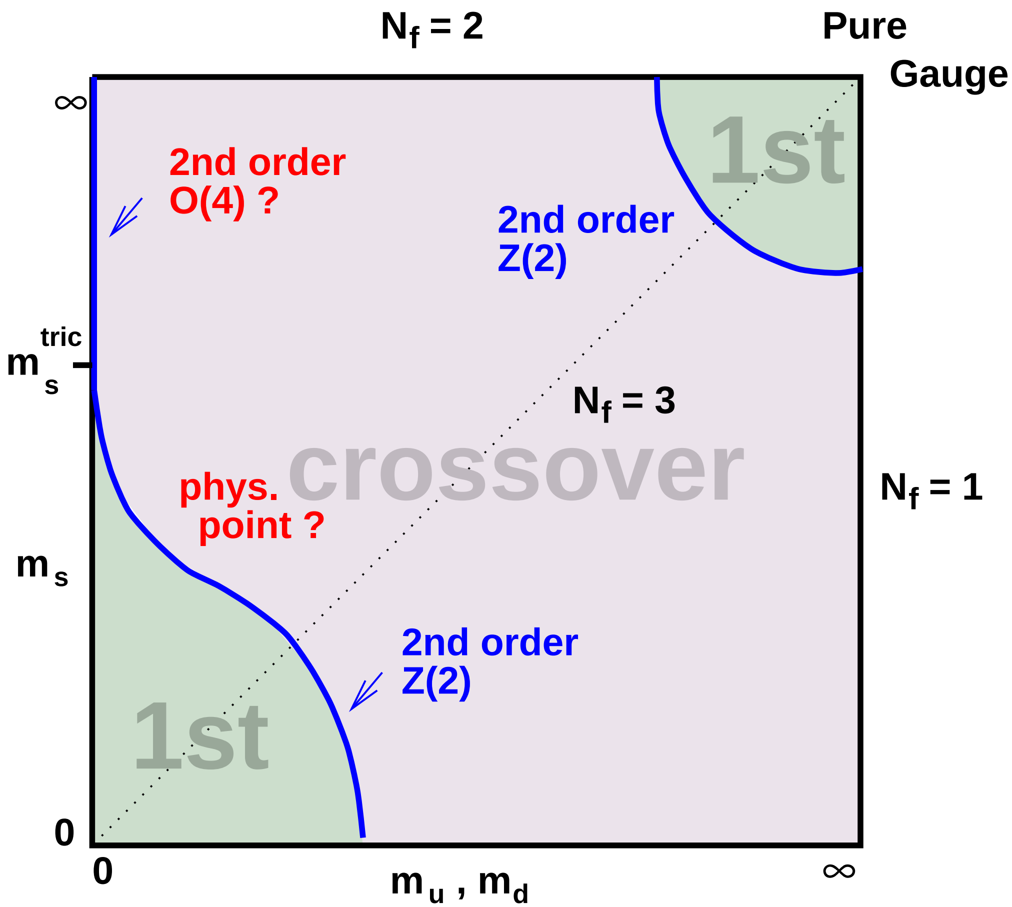
  $\Rightarrow$ note that $\beta + \gamma = y_h/y_t = \beta\delta$ "hyperscaling relation"

  $\Rightarrow$ at $t \simeq 0$: $M(t,h) = |h|^{1/\delta} f_G(z)$ with $f_G(z) = f_{sing}(z) + z f'_{sing}(z)$ universal and $z = t|h|^{-1/\beta\delta}$

- similarly, e.g. the so-called Binder cumulant $B_4 = \dfrac{\langle(\delta M)^4\rangle}{\langle(\delta M)^2\rangle^2}$ with $\delta M = M - \langle M\rangle$ is a universal number

- keep in mind though that corrections to scaling might be important, and there are also the regular terms

conjectured landscape of transitions



arising from the chiral symmetry of QCD at $m_q = 0$

$$SU_L(N_f) \times SU_R(N_f) \times U_A(1)$$

– its spontaneous breakdown at low T

– its restoration at high T

– and its modelling by σ models (universality)

– note: $U_A(1)$ broken by triangle anomaly but could be effectively restored at high T (no topologically non-trivial configurations)

$N_F = 2$ — Wilczek, Pisarski

– if $U_A(1)$ effectively restored then 1st order

– if 2nd order then in $SU(2) \times SU(2) \simeq O(4)$ class

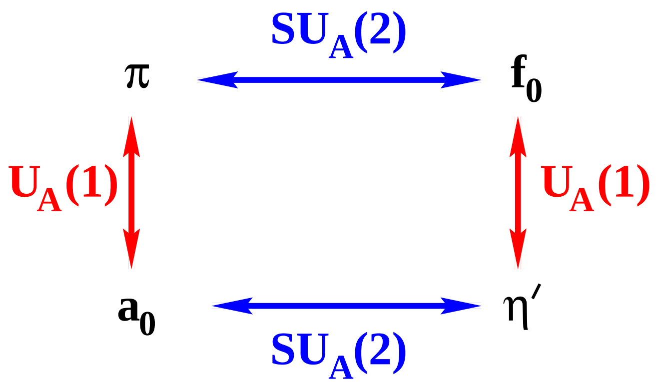
$N_F = 3$

– 1st order — Wilczek, Pisarski

– at critical end point: $Z(2)$ class — Gavin et al.

$N_F = 2$

- conflicting results for critical behavior of $M = \langle \bar{q}q \rangle$, all from coarse lattices with un-improved actions
  - Wilson M scales $\sim O(4)$ in $m \sim H$ at $T > T_c$ [Iwasaki et al.]
  - staggered χ_m, χ_t do not scale $\sim O(4)$ in m [Karsch,EL; JLQCD; MILC]
  - staggered M scales $\sim O(4)$ in L [Engels et al.]
  - staggered c_V scales as 1st order [Di Giacomo et al.]
  - staggered M is as in $O(2)$ at finite L [Kogut, Sinclair]
- $U_A(1)$:
  - if effectively restored, then degeneracy in 2-point function (mass spectrum) [Shuryak; Cohen et al;, ...]



there are indications that $U_A(1)$ is not restored

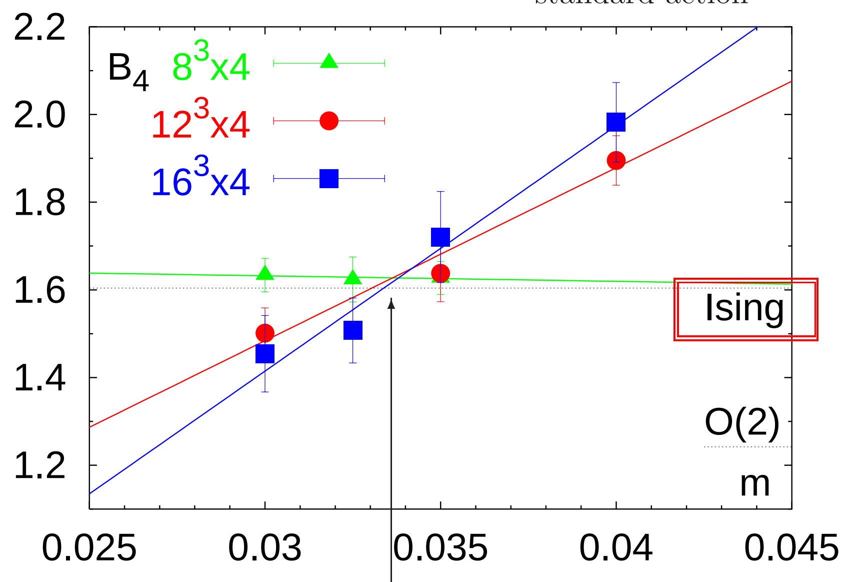
- current projects with Wilson-type quarks [QCDSF-DIK,tmfT,WHOT] have not yet addressed this question

$N_F = 3$

Binder cumulant B_4

- intersection for various V yields critical value of m
- value of B_4 is universal
- corrections from V finite and 'order parameter not matched correctly'

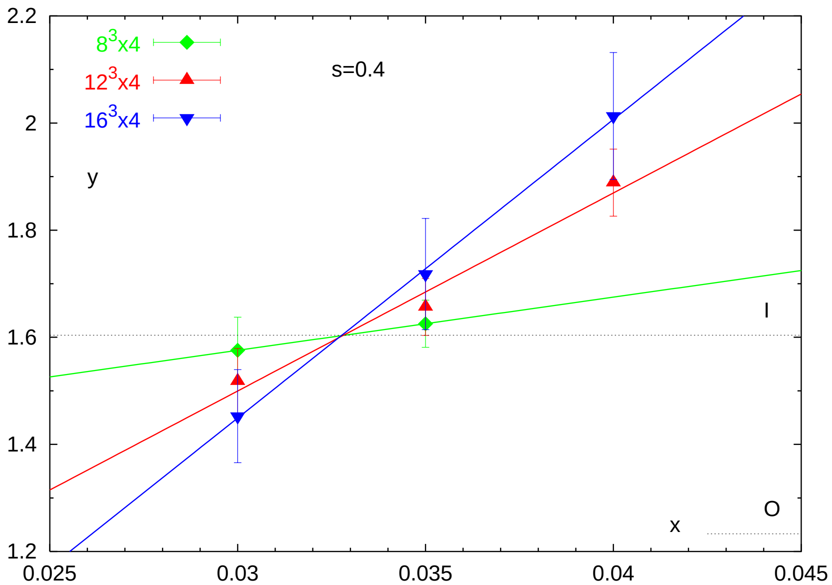
standard action



$m_c \simeq 4\, m_u^{phys} \Leftrightarrow m_{PS} \simeq 290$ MeV

[Bielefeld; deForcrand,Philipsen]

but: – m_c not universal
– improved action: $m_{PS} \simeq 70$ MeV



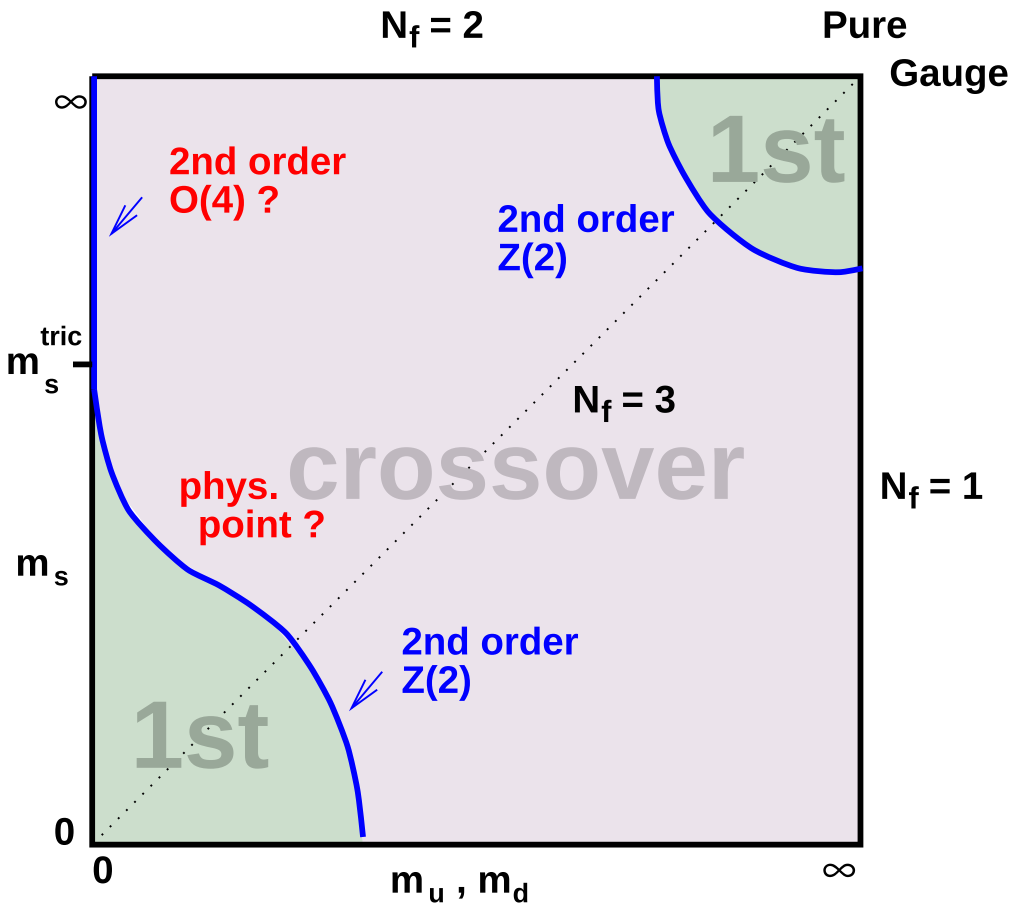
magnetization-like order parameter $\mathcal{M}$

not identical with chiral condensate $\langle \overline{q} q \rangle$

(chiral symmetry broken by $m_q \neq 0$ anyway)

$$\mathcal{M} = \langle \overline{q} q \rangle + s\, S$$

conjectured landscape of transitions



$N_F = 2$

– unclear

$N_F = 3$

– $Z(2)$ critical endpoint at $m_\pi < m_\pi^{phys}$ (?)

– 1st order in chiral limit

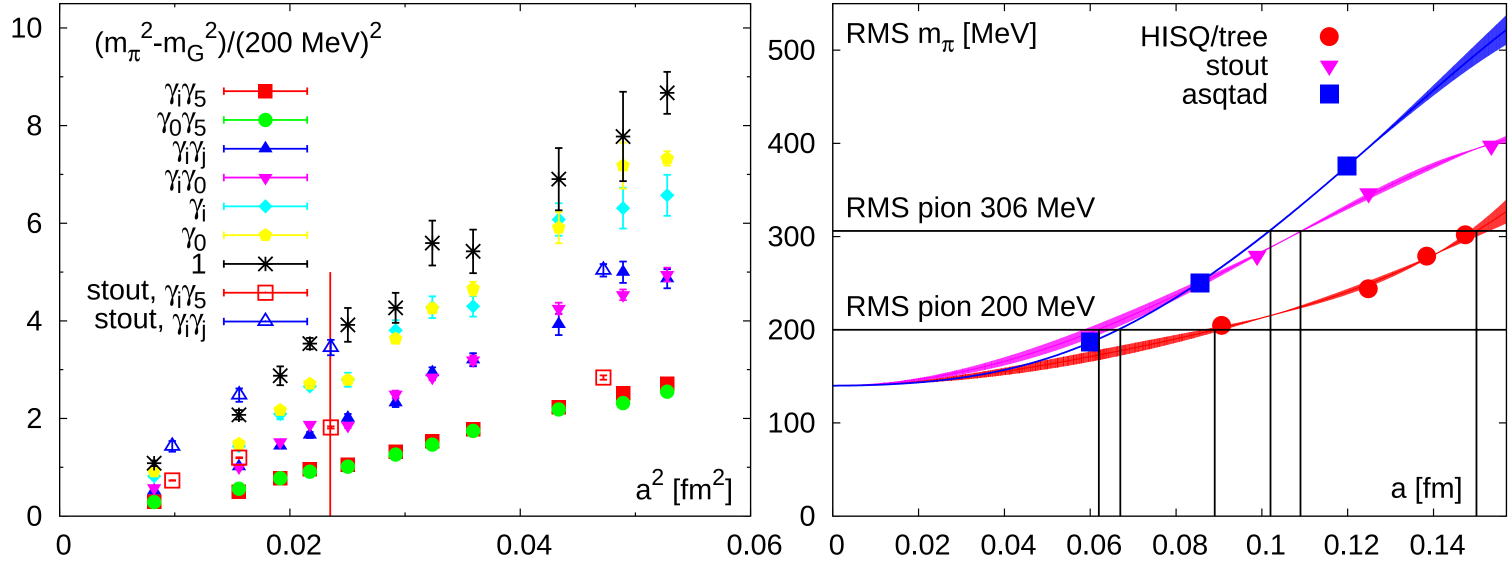
$N_F = 2+1$

recent projects all use improved staggered quarks (p4fat3, asqtad, stout, HISQ)*, $m_s = m_s^{phys}$

at finite lattice spacing: taste violations $\mathcal{O}(a^2\alpha_s)$

$\Rightarrow$ the pion 16-multiplet $\longrightarrow$ 8 different representations, *staggχPT*: 5 different reps.

only 1 pion is true Goldstone boson also at $a \neq 0$: $m_{\gamma_5}^2 = m_G^2 \sim m_q$ in chiral limit



$m_G \simeq 140$ MeV

$$m_\pi^{RMS} = \sqrt{\sum_{i=1}^{16} m_{\pi,i}^2}$$

$\Rightarrow$ the expected $O(4)$ for 2 light flavors in the limit $m_l \longrightarrow 0$ goes over into $O(2)$ at $a \neq 0$

* first results from DWF fermions, not yet on the nature of the transition [Christ, Karsch et al.]
first results from improved Wilson quarks, mainly equation of state [WHOT]

in $O(N)$ spin models

"magnetic equation of state" $M(t,h) = h^{1/\delta} f_G(z)$ with $z = t/h^{1/\beta\delta}$ and f_G universal

$\Rightarrow$ $M \sim (-t)^\beta$ at $h = 0, t < 0$

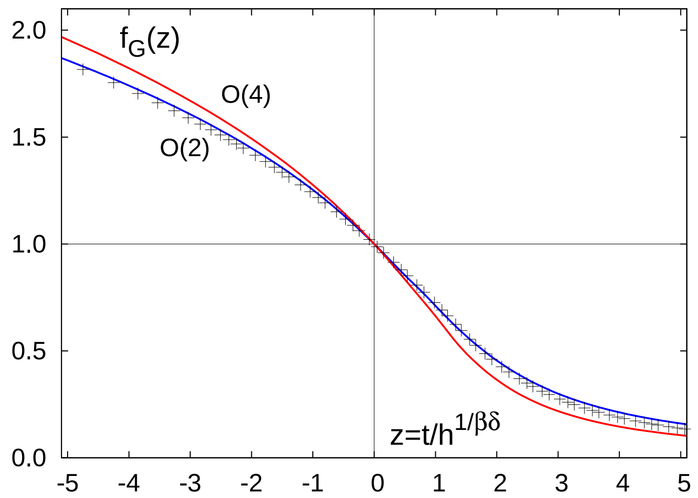
$M \sim h^{1/\delta}$ at $t = 0$

$M \simeq h^{1/\delta}(-z)^\beta[1 + \tilde{c}_2\beta\,(-z)^{-\beta\delta/2} + \ldots] = M(t,0) + c(t)\sqrt{h} + \ldots$ at $z \to -\infty$ Zia, Wallace

$\sqrt{h}$ behavior is known as Goldstone effect in d=3 ($\sim h \ln h$ in d=4) Gasser, Leutwyler, Hasenfratz

critical exponents β, δ and $f_G(z)$ known from spin model simulations [Engels et al.]

N	β	δ	$\tilde{c}_2$	z_p
2	0.349	4.7798	0.592(10)	1.56(10)
4	0.380	4.824(9)	0.666(6)	1.33(5)



in QCD, the chiral condensate $\langle\bar{q}q\rangle \sim \langle \mathrm{Tr} D_q^{-1}\rangle$ is the order parameter in the limit $m_l \longrightarrow 0$

i.e. takes the role of the magnetization $M \sim \langle\bar{q}q\rangle$ and $H \sim m_q$

complications: – needs multiplicative renormalization $\langle\bar{q}q\rangle^R = Z_m^{-1}\langle\bar{q}q\rangle$

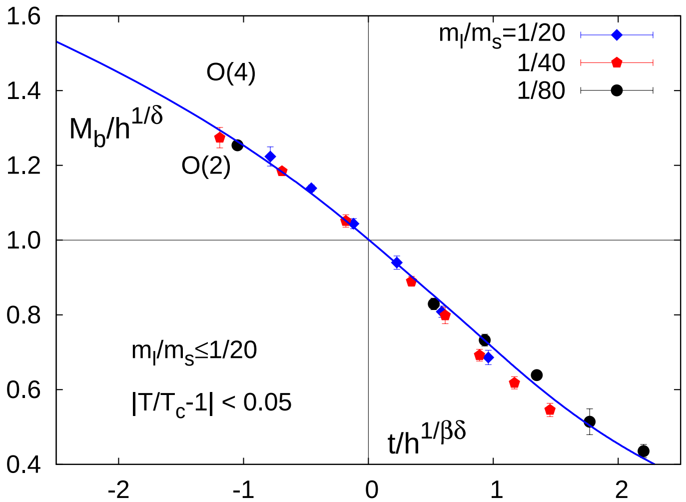
where Z_m is mass renormalization $m_q^R = Z_m m_q$

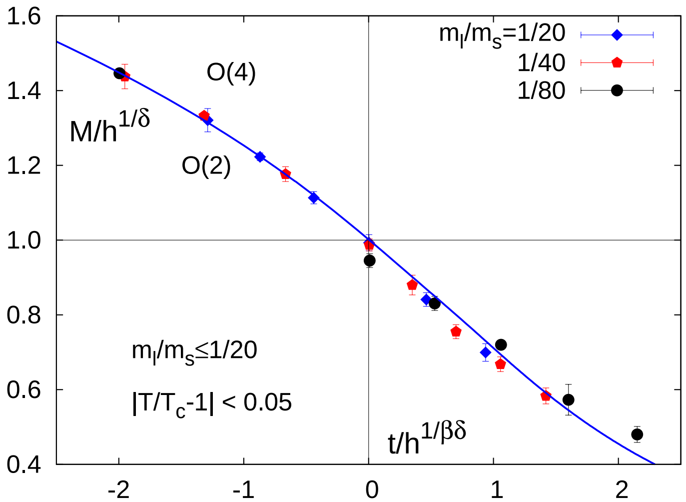
– away from chiral limit receives additive power divergence $\langle\bar{q}q\rangle(m_q) = \langle\bar{q}q\rangle(0) + m_q/a^2 + \cdots$

$\rightsquigarrow$ subtracted condensate $M = m_s\left[\langle\bar{l}l\rangle - \frac{m_l}{m_s}\langle\bar{s}s\rangle\right]$, difference to $M_b = m_s\langle\bar{l}l\rangle$ small and vanishing with $m_l \longrightarrow 0$

fit data to the scaling function f_G with 3 unknowns: t_0, h_0, the metric constants

T_c, the critical temperature in the chiral limit





data is well described by $O(2)$ scaling

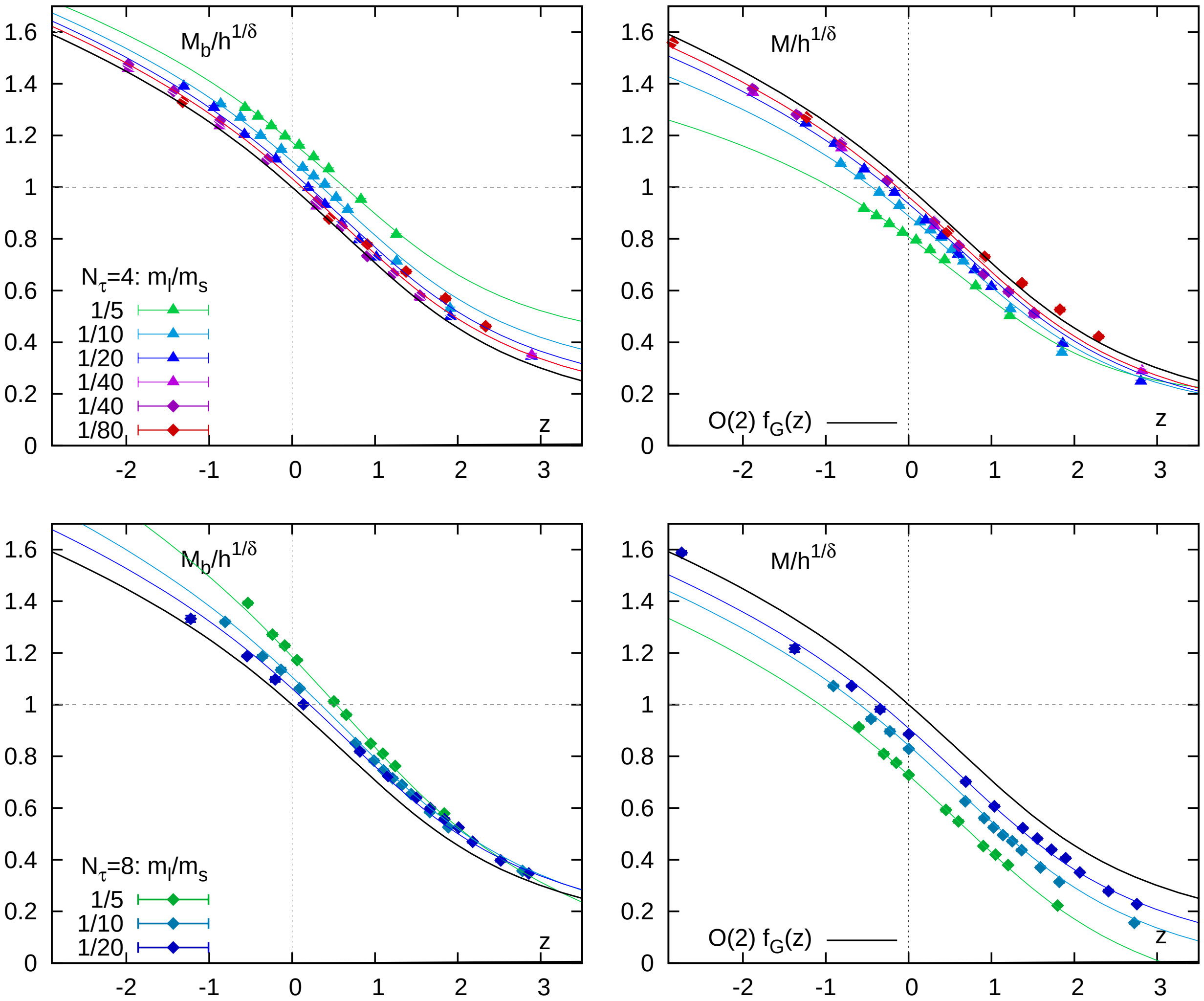
note: $(m_l/m_s)^{phys} \simeq 1/27$

at $m_l/m_s \geq 1/10$ corrections to scaling need to be taken into account

$$M(t,h) = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + a_t t h + b_1 h + b_3 h^3 + b_5 h^5$$

$N_\tau = 4$

$N_\tau = 8$



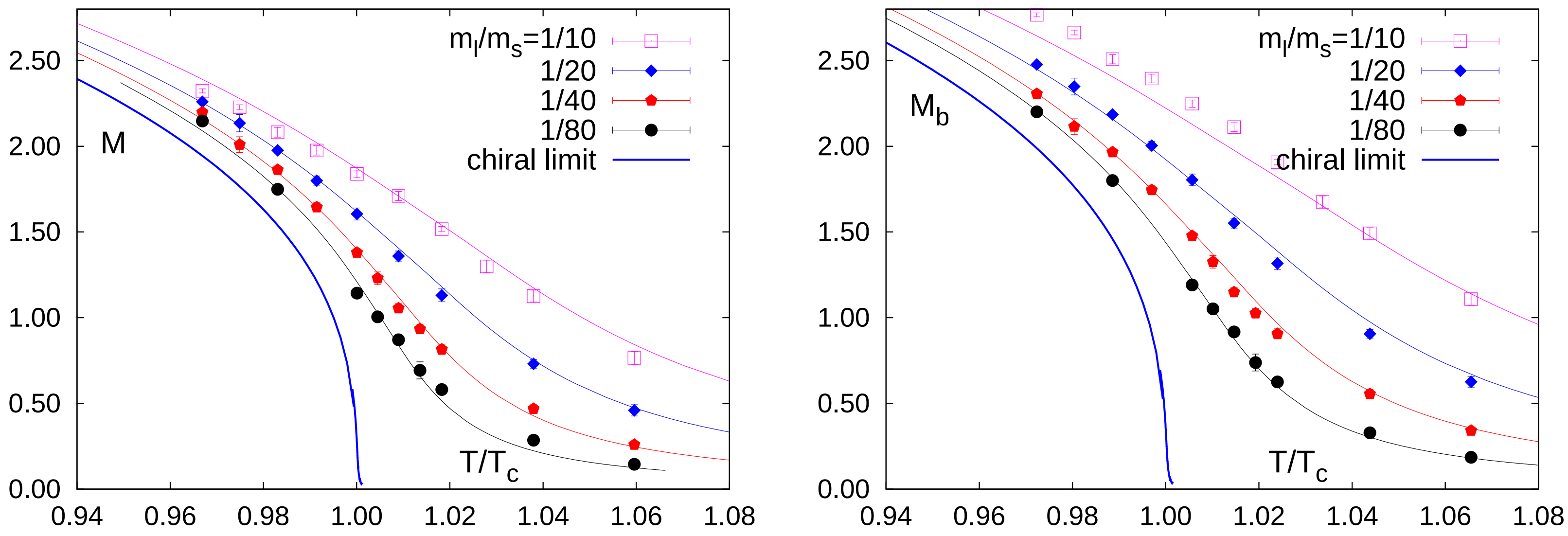
$b_1 \simeq 0$ for (subtracted) M

p4fat3 action

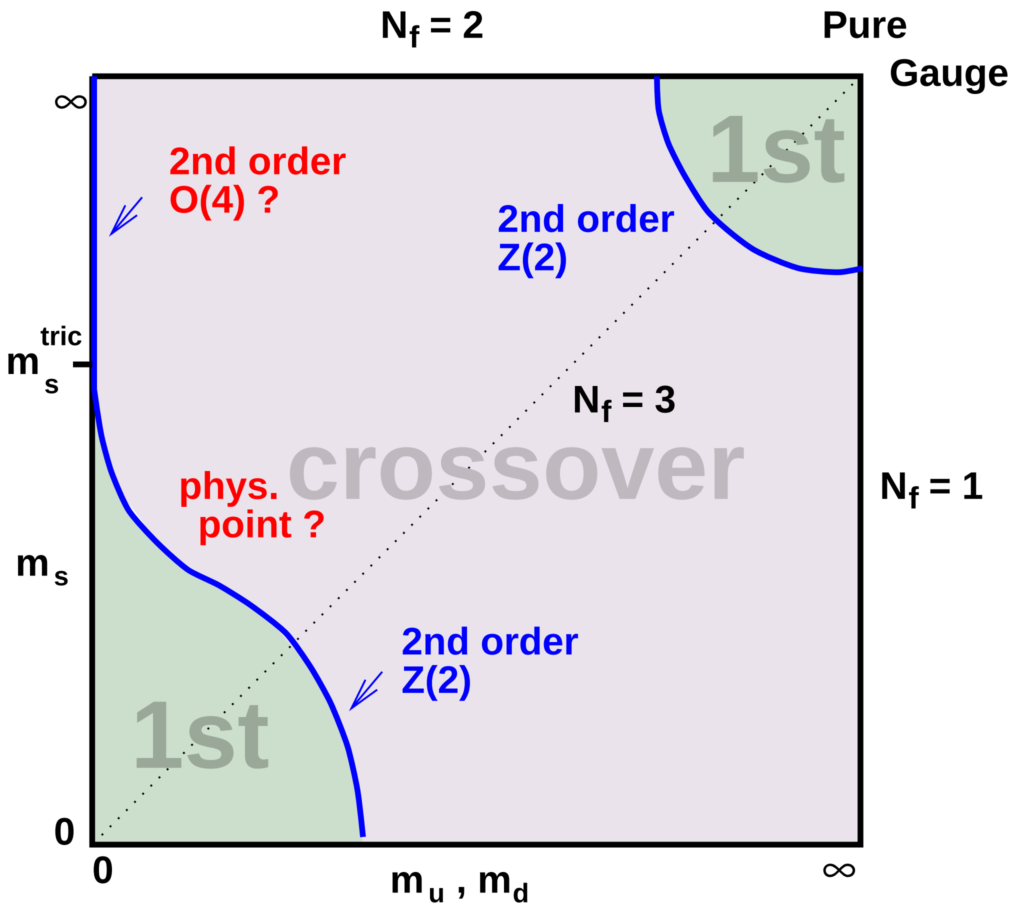
similar for asqtad, HISQ

soon to be published

once one has scaling (possibly including corrections) the chiral limit is under control



conjectured landscape of transitions



$N_F = 2+1$ at $m_s = m_s^{phys}$

– m_l^{phys} compatible with being in the scaling region of a chiral $O(N)$ phase transition at $m_l = 0$

– 1st order at $m_l = 0$ not completely ruled out yet though

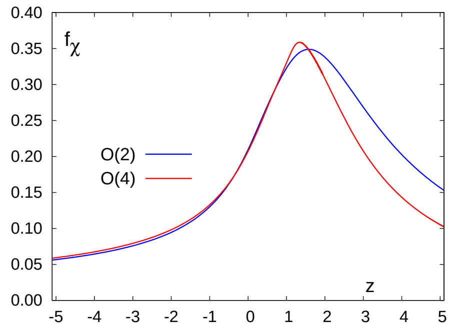
$N_F = 2$

– unclear

$N_F = 3$

– $Z(2)$ critical endpoint at $m_\pi < m_\pi^{phys}$ (?)

– 1st order in chiral limit

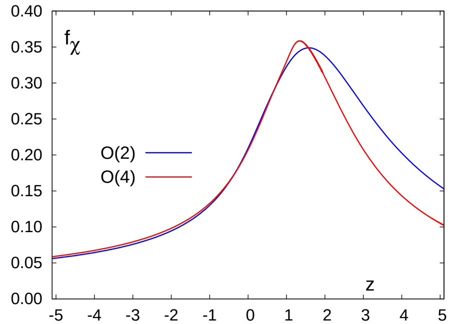


the chiral susceptibility χ_M is now fixed completely

$$\chi_M(t,h) = \frac{\partial M}{\partial H} = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \quad \text{with} \quad f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f'_G(z)\right)$$

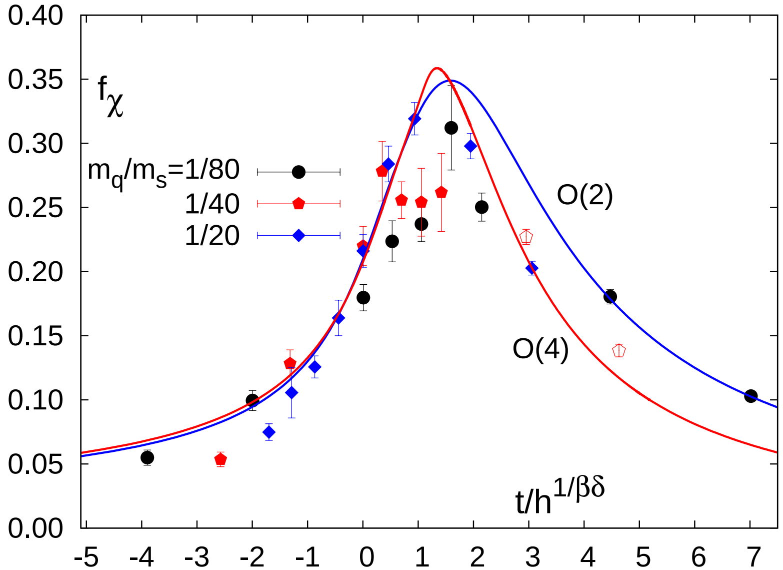
the chiral susceptibility χ_M is now fixed completely

$$\chi_M(t,h) = \frac{\partial M}{\partial H} = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \quad \text{with} \quad f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f'_G(z)\right)$$



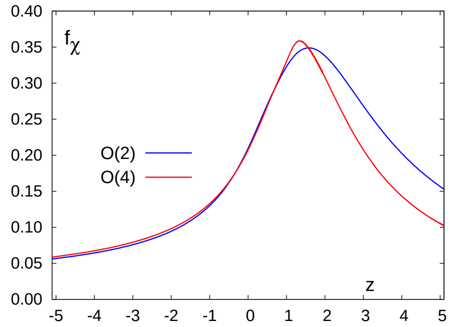
v.v., reconstruct scaling fct f_χ from the data on χ_M: $f_\chi = \chi_M \, h_0 \, h^{1-1/\delta}$

and compare with f_χ from the $O(N)$ models:



p4fat3

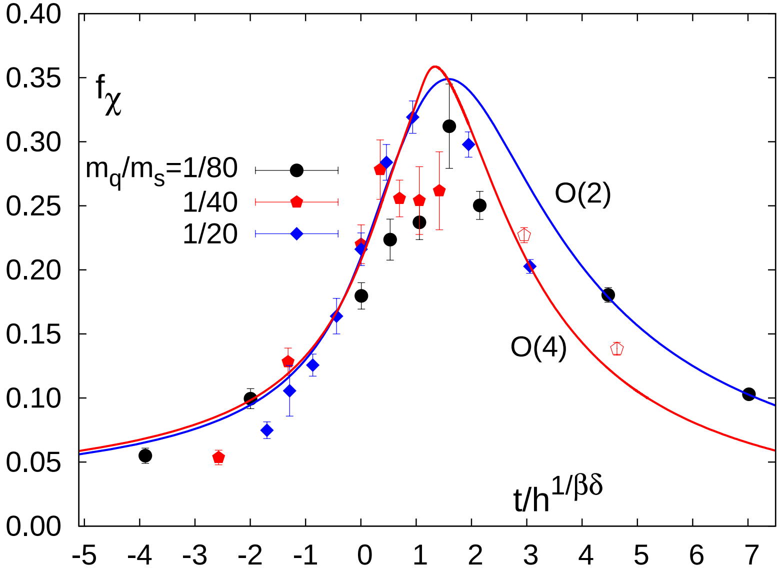
the chiral susceptibility χ_M is now fixed completely



$$\chi_M(t,h) = \frac{\partial M}{\partial H} = \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \quad \text{with} \quad f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f_G'(z) \right)$$

v.v., reconstruct scaling fct f_χ from the data on χ_M: $f_\chi = \chi_M \, h_0 \, h^{1-1/\delta}$

and compare with f_χ from the $O(N)$ models:



p4fat3

peaks in χ_M at pseudocritical temperatures $T_p(m_l \neq 0)$ are caused by the peak in f_χ at $z = z_p$

$$\frac{t_p}{h^{1/\beta\delta}} = \frac{\frac{1}{t_0}\frac{T_p - T_c}{T_c}}{\left(\frac{H}{h_0}\right)^{1/\beta\delta}} = z_p \quad \Rightarrow \quad \frac{T_p(H) - T_c}{T_c} = z_p \, H^{1/\beta\delta} \, \frac{t_0}{h_0^{1/\beta\delta}}$$

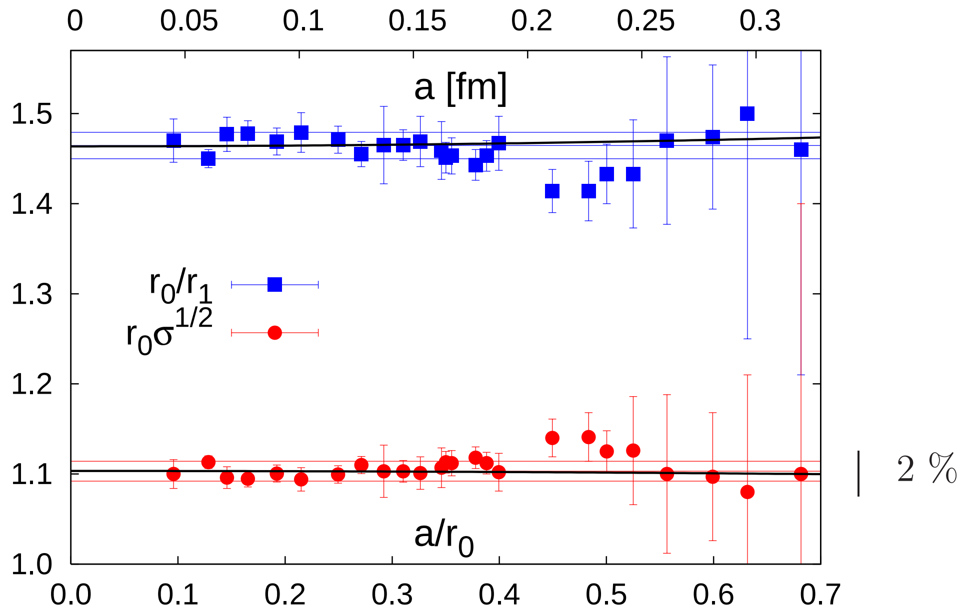
$\longrightarrow$ predict $T_p(m_l)$ from scaling of $\langle \bar{q}q \rangle$

The transition temperature

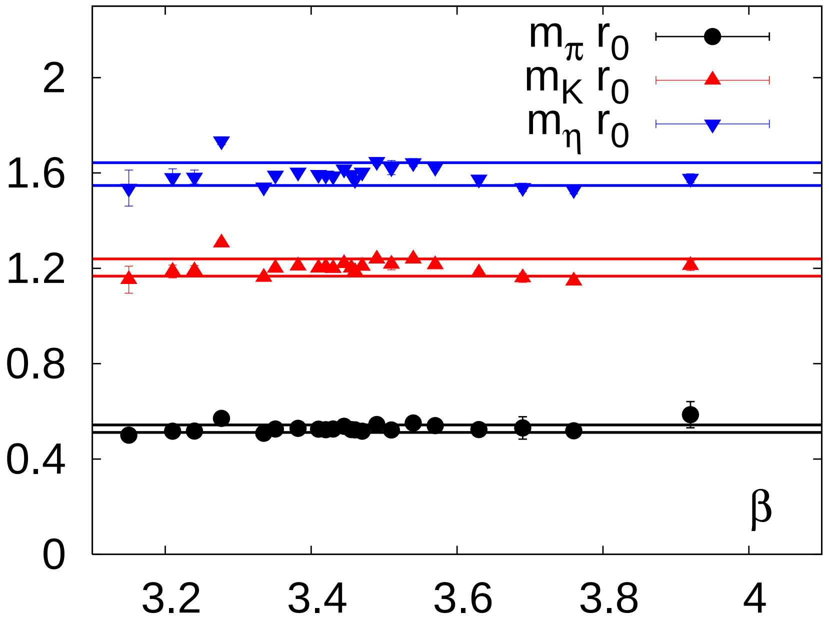
- ⋆ scan through $\beta \longrightarrow a(\beta) \longrightarrow T = 1/N_\tau a(\beta)$
- ⋆ T = 0 scale taken from $\Upsilon\, 2S - 1S$ splitting [A. Gray et al.] via the heavy quark potential $V(r)$

$r_{0,1}/a$ from $r^2 \frac{dV(r)}{dr}\Big|_{r=r_{0,1}} = 1.65(1.0)$

for absolute values (in MeV) we use $r_0 = 0.469(7)$ fm [A. Gray et al.]



- ⋆ fine tune $\hat{m}_{l,s}(\beta)$ such that $m_{\pi,K} = const$



$m_K \simeq m_K^{\rm phys}$

$m_\pi \simeq 220$ MeV

plots for p4fat3 action at $m_l/m_s = 0.1$
similar for other actions, different m_l/m_s
may take other scale setting quantity e.g. f_K

⋆ investigate quantities sensitive to the transition

renormalized chiral condensate

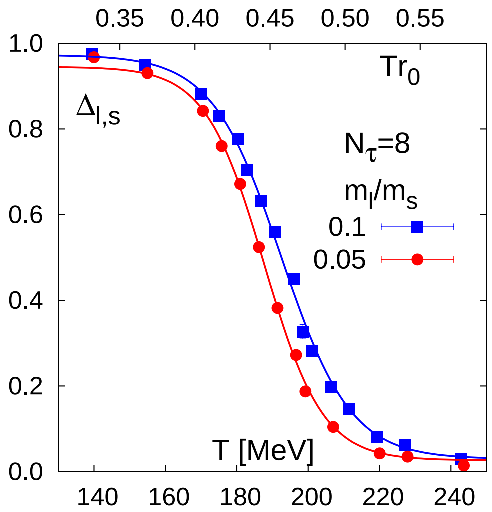
$$\Delta_{l,s}(T) = \frac{\langle\bar{\psi}\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,T}}{\langle\bar{\psi}\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_{s,0}}$$

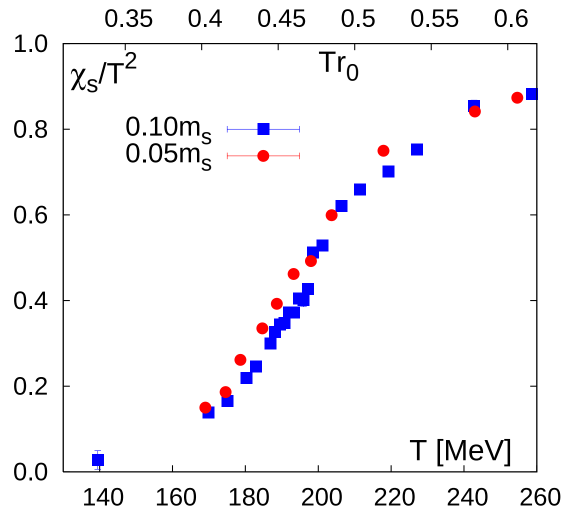
subtracting power-law additive and cancelling multiplicative UV divergencies

strange number susceptibility

$$\chi_s(T)/T^2 \sim \langle n_s^2\rangle - \langle n_s\rangle^2$$

measures strangeness fluctuations
indicative of deconfinement and
in principle sensitive to singular behavior



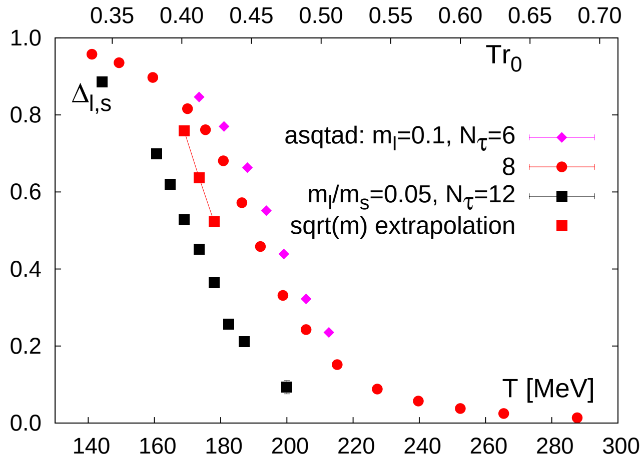


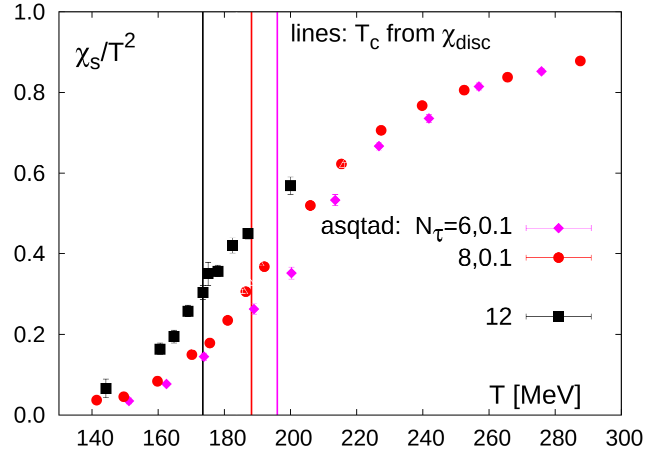
p4fat3 action

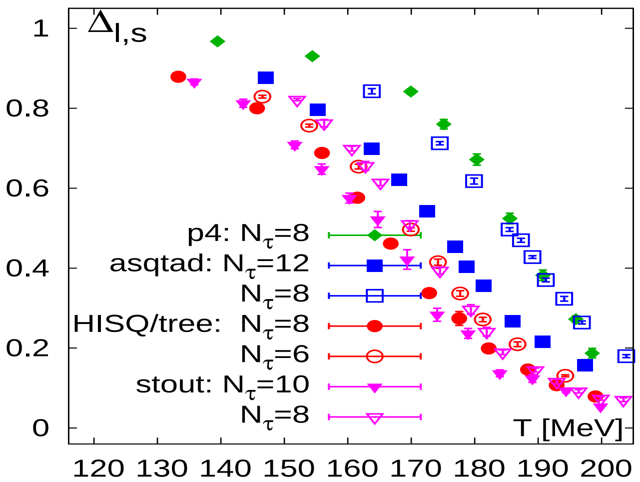
for both variables: region of (rapid) change moves to lower T when m_l is decreased

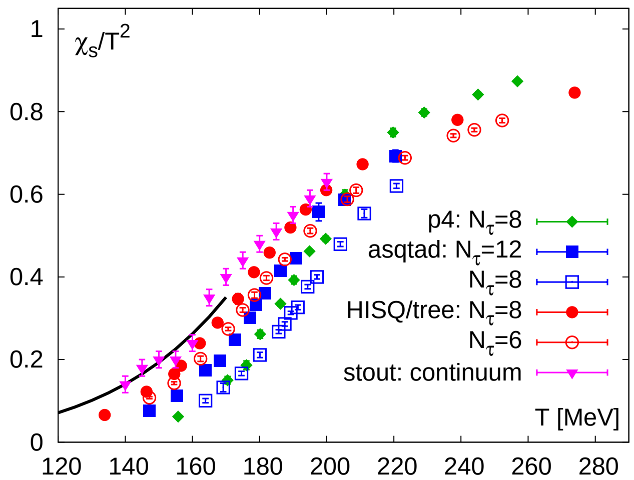
OK, see $T_p(m_l)$

in both variables: region of (rapid) change moves to lower T when a is decreased: discretization effects, not known a priori





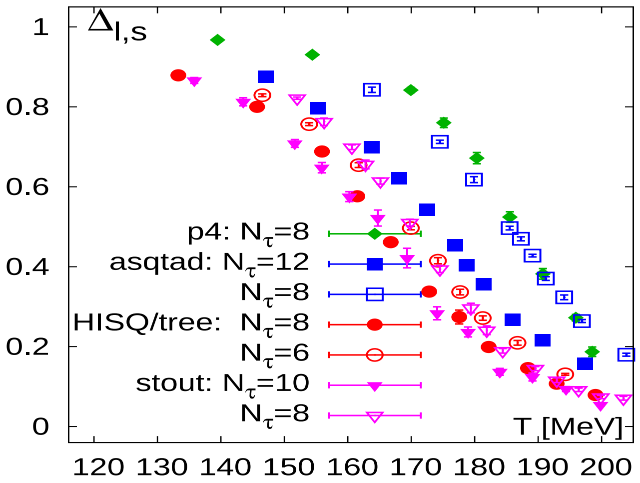




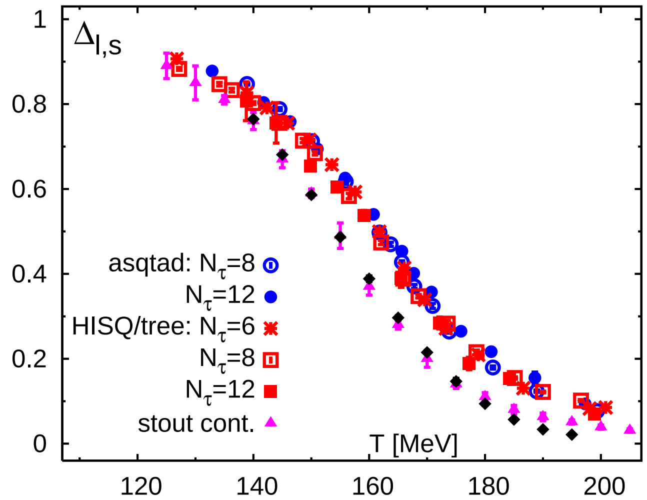
NB: difficult to identify a transition temperature e.g. by locating an inflection point:

curvature = 2nd derivative changes sign $\Rightarrow$ 1st one has maximum situation may look better when a different quantity is used to set the T scale

$r_{0,1}$ scale



f_K scale

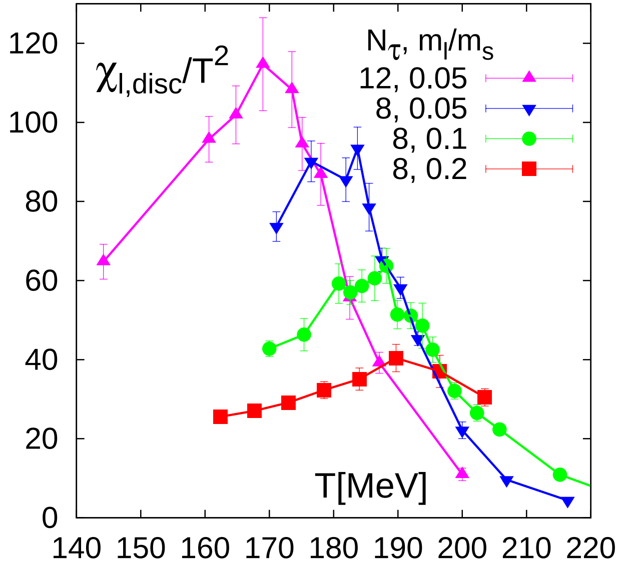


explanation: f_K is affected by taste violations roughly in the same way as $\Delta_{l,s}$

r_i i.e. static quark potential much less sensitive → in principle more reliable

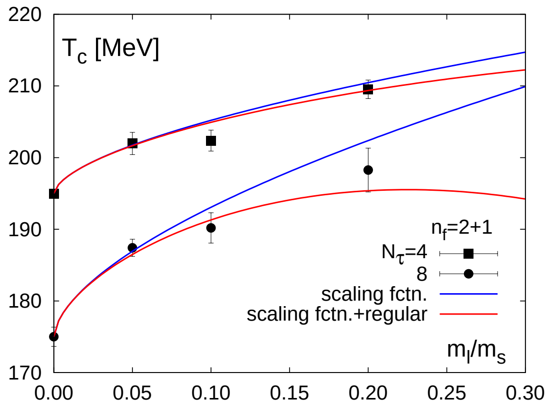
in any case, a continuum extrapolation is needed, and a better way to locate the transition

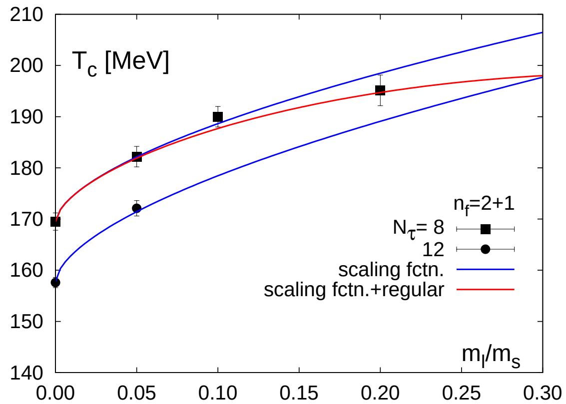
$\rightsquigarrow$ determine $T_p(m_l, N_\tau)$ from the location of peaks of the susceptibility χ_{dis}



asqtad action

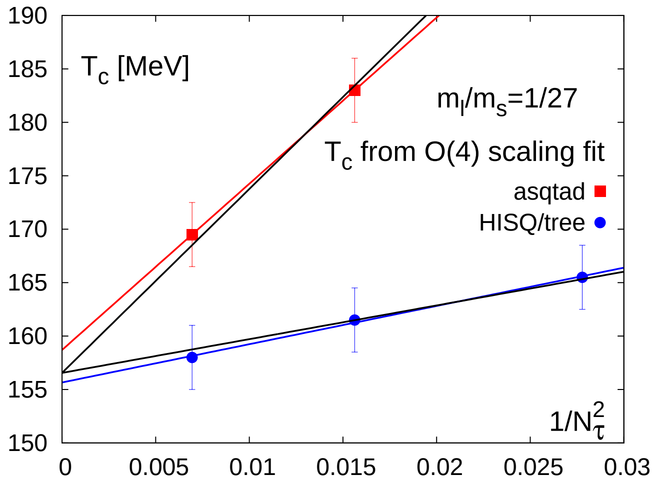
and compare with predictions from $O(N)$ scaling fits to $\langle\bar{q}q\rangle$





i.e. m_l dependence under control

extrapolation to continuum limit at physical mass $m_l/m_s = 1/27$



$T_c = 157$ (4 stat) (3 extrapolation) (1 scale) MeV — asqtad, HISQ

$T_c = 147$ (2 stat) (3 syst) MeV — stout χ_M

$T_c = 155$ (3 stat) (3 syst) MeV — stout $\Delta_{l,s}$

Meson screening masses

aim at **analyzing existence and properties of hadronic excitations at $T > 0$**

$\rightsquigarrow$ Correlator in momentum space defined at (boson) Matsubara frequencies $\omega_n = 2\pi T n$

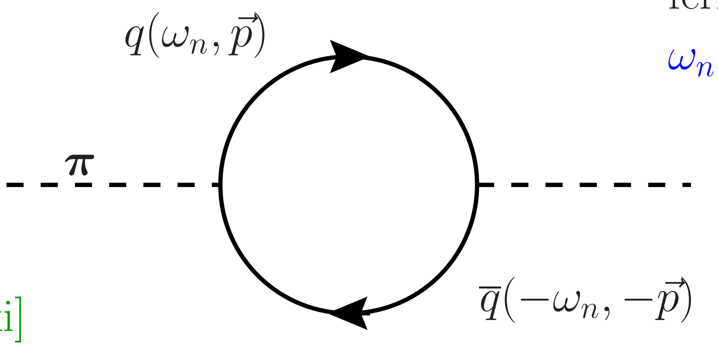
$$\begin{aligned}\Delta(i\omega_n, \vec{p}) &= \oint \frac{dp_0}{2\pi i} \frac{\Delta(p_o, \vec{p})}{p_0 - i\omega_n} \\ &= \int \frac{dp_0}{2\pi} \frac{1}{p_0 - i\omega_n} \underbrace{\frac{1}{i}\left[\Delta(p_0 + i\epsilon, \vec{p}) - \Delta(p_0 - i\epsilon, \vec{p})\right]}_{\textbf{spectral density } \boldsymbol{\sigma(p_0, \vec{p})}} + \ Subtr.\end{aligned}$$

because of limited physical extension $1/T$ in the temporal direction

$\rightsquigarrow$ spatial correlator $\quad G^S(z) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \Delta(0, p) = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} e^{ipz} \int_{-\infty}^{+\infty} dp_0 \frac{\sigma(p_0, p)}{p_0}$

- is determined by the spectral density which encodes the complete spectral information
- is sensitive to symmetries
- can be compared with free quark propagation
- but exponential decay $\sim \exp(-m_{screen} z)$ depends on m_{screen} which may be different from mass

At T large: expect free quarks



fermion Matsubara
$\omega_n = \pi T(2n+1)$

- free Continuum: [Friman, Florkowski]

$$G_\pi^S(z) = \frac{N_c T}{2\pi z^2 \sinh(2\pi T z)} \left[1 + 2\pi T z \coth(2\pi T z)\right]$$

define effective (z-dependent) screening mass

$$m_{\text{screen}}^{\text{eff}}(z) = -\frac{1}{G(z)} \frac{\partial G(z)}{\partial z} \simeq 2\pi T \left\{1 + \frac{1}{2\pi T z} + \ldots\right\}$$

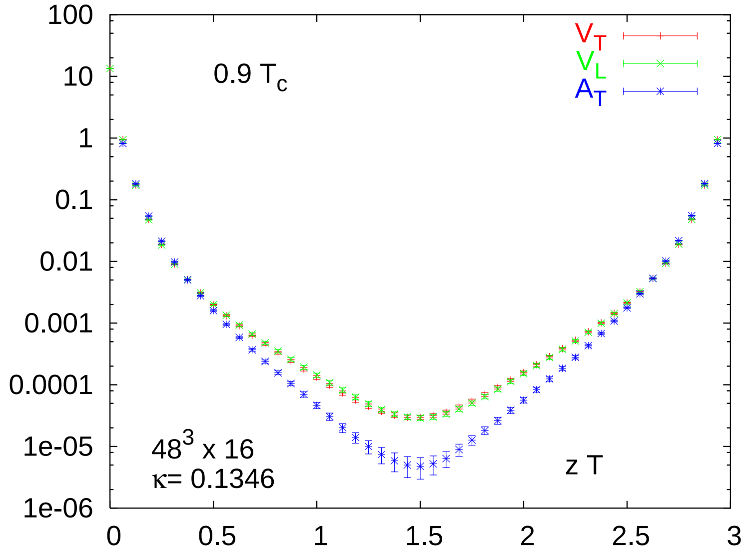
- lowest order correction: [Laine, Vepsäläinen]

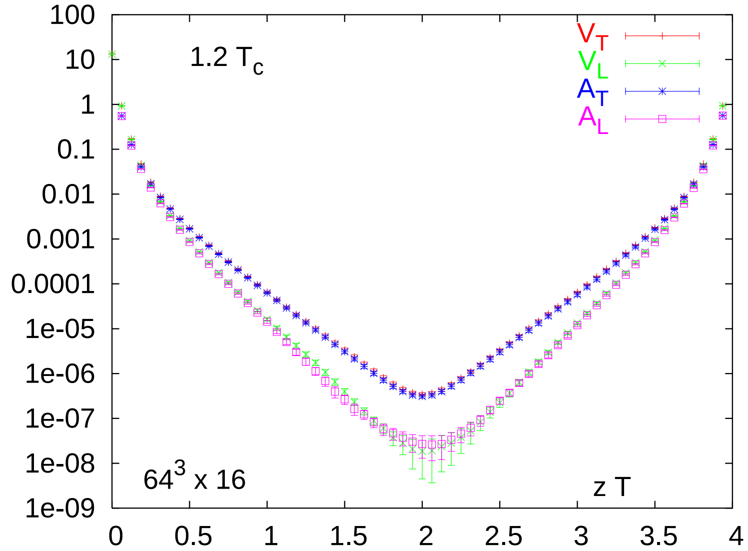
$$m_{\text{screen}} = 2\pi T(1 + g^2 \times \left\{ \begin{array}{l} 0.022(N_F = 0) \\ 0.033(N_F = 3) \end{array} \right\})$$

Symmetry restorations

- at $T \neq 0$, for spatial correlations: rotational $SO(3) \rightarrow SO(2) \times Z(2)$ [S.Gupta]

$$\Rightarrow \quad V_T \neq V_L, \quad A_T \neq A_L \quad \text{possible}$$



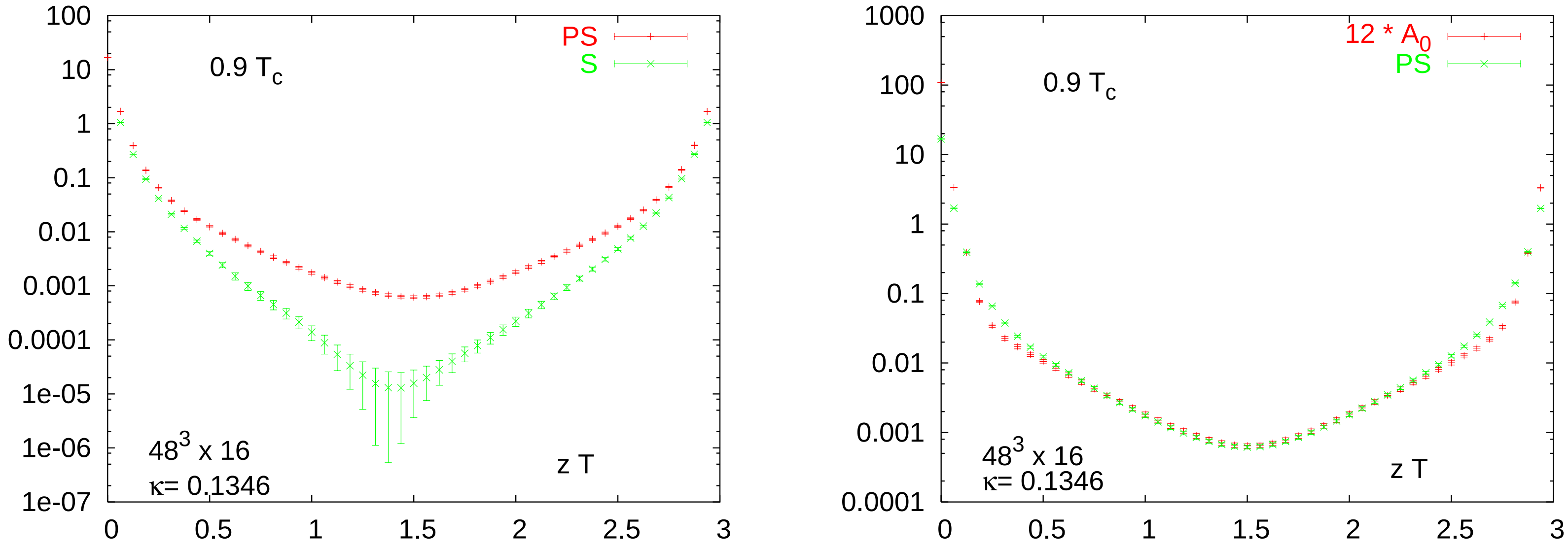


$T < T_c$: $\quad V_T = V_L \neq A_T = A_L$ $\qquad$ $T > T_c$: $\quad V_T = A_T \neq V_L = A_L$

- at $T > T_c$, chiral symmetry restoration $SU_V(N_F) \rightarrow SU_L(N_F) \times SU_R(N_F)$

Pseudoscalar - Scalar (connected: a_0/δ) sector

- below T_c

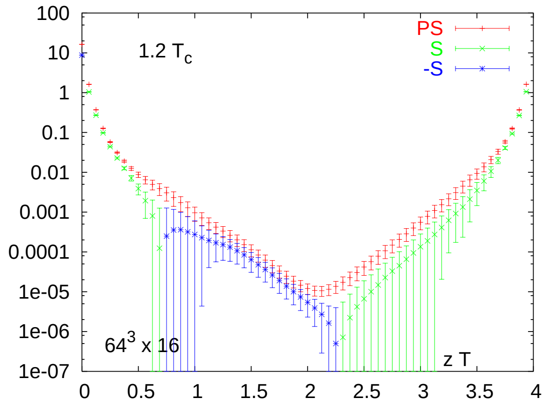


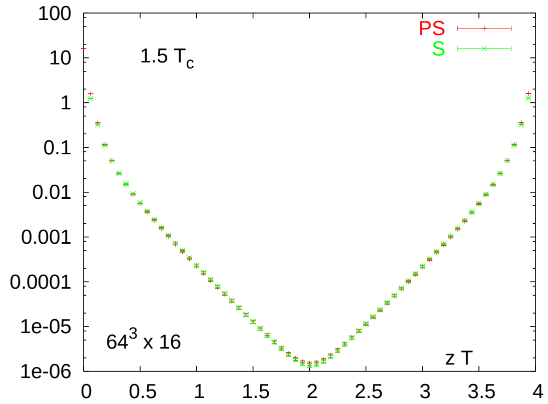
- pseudoscalar π much lighter than scalar a_0/δ
- π couples to axialvector 0-component A_0 with relative strength $1/12 \sim f_\pi$

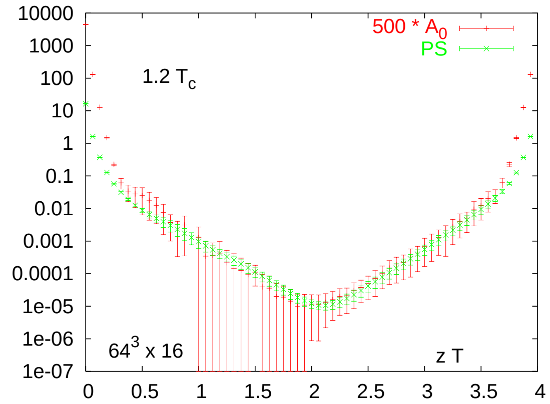
- at/above T_c

- for $N_F = 2$, effective $U_A(1)$ restoration would predict $\pi - a_0/\delta$ degeneracy

- not observed at $T = 1.2T_c$:
  topologically non-trivial configurations survive up to (at least) $1.2T_c$







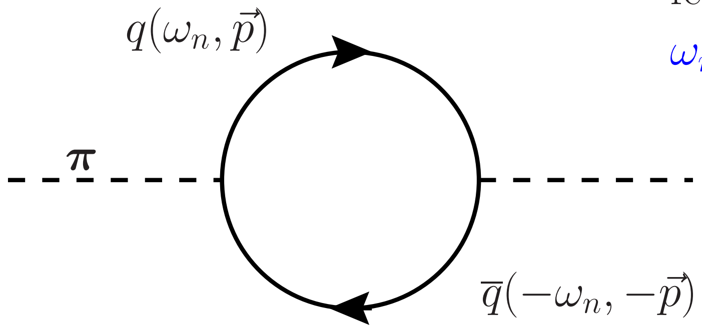
– at $1.5T_c$: $m_{\rm PS} = m_{\rm S}$

– A_0 couples to PS with much less strength $\simeq 1/500$

fermion Matsubara

$\omega_n = \pi T(2n+1)$

free lattice Wilson quarks



$$G_M^S(z) = \frac{1}{N_\sigma^2 N_\tau} \sum_{k_1,k_2,\omega_n} \frac{1}{(1+M)^2} \frac{1}{\sinh^2(EN_\sigma/2)} \left\{ b_M \cosh\left[2E\left(\frac{N_\sigma}{2} - z\right)\right] + d_M \right\}$$

where $M = \Sigma_{1,2,4}(1-\cos(k_i))$

$$\cosh(E) = 1 + \frac{\Sigma_{1,2,4}\sin^2(k_i) + M^2}{2(1+M)}$$

	b_M	d_M
π	1	0
$\frac{1}{2}(\rho_1+\rho_2)$	$1 - \frac{1}{2}\frac{\sin^2(k_1)+\sin^2(k_2)}{\sinh^2 E}$	$-\frac{1}{2}\frac{\sin^2(k_1)+\sin^2(k_2)}{\sinh^2 E}$
ρ_3	0	1
ρ_4	$1 - \frac{1}{2}\frac{\sin^2(k_4)}{\sinh^2 E}$	$-\frac{1}{2}\frac{\sin^2(k_4)}{\sinh^2 E}$

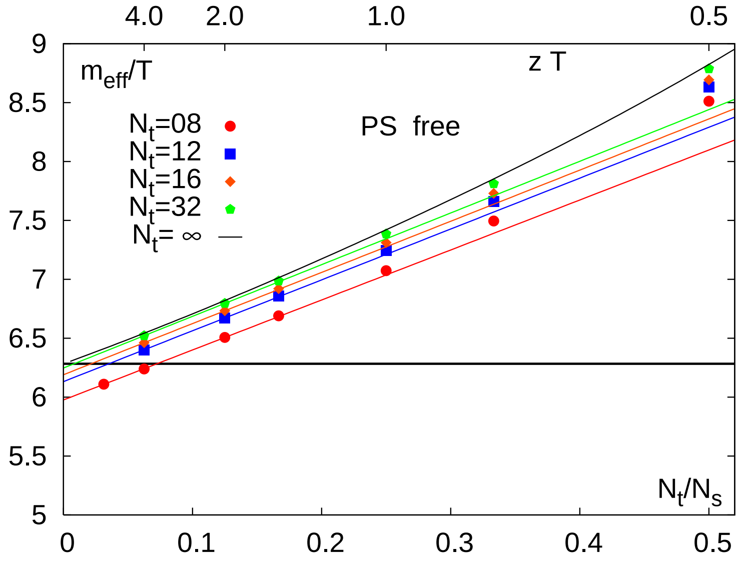
In the following, we used

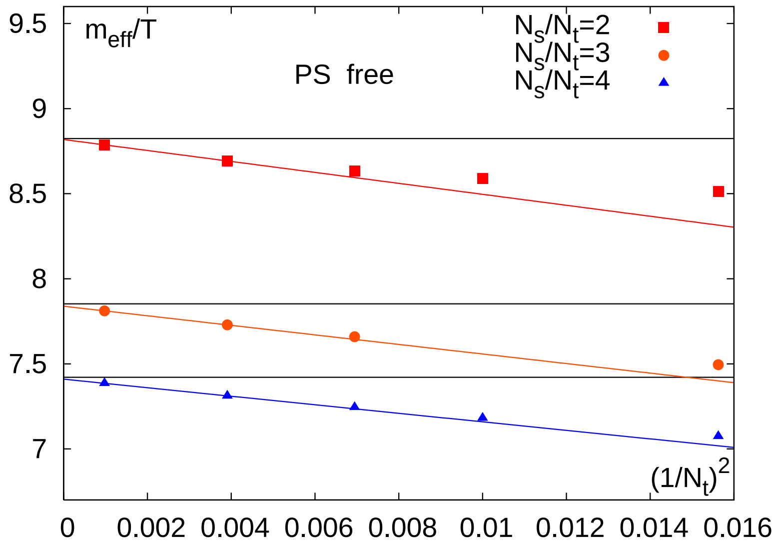
$$G_M^{S,subtr}(z) = G_M^S(z) - G_M^S(z = N_\sigma/2)$$

and solved for $m^{\text{eff}}(z)$

$$\frac{G_M^{S,subtr}(z)}{G_M^{S,subtr}(z+1)} = \frac{\sinh^2[(m^{\text{eff}}/2)(N_\sigma/2 - z)]}{\sinh^2[(m^{\text{eff}}/2)(N_\sigma/2 - z - 1)]}$$

free Wilson results   in the following, to be definite, $z = N_\sigma/4 \to zT = \frac{1}{4} \times N_\sigma/N_\tau$



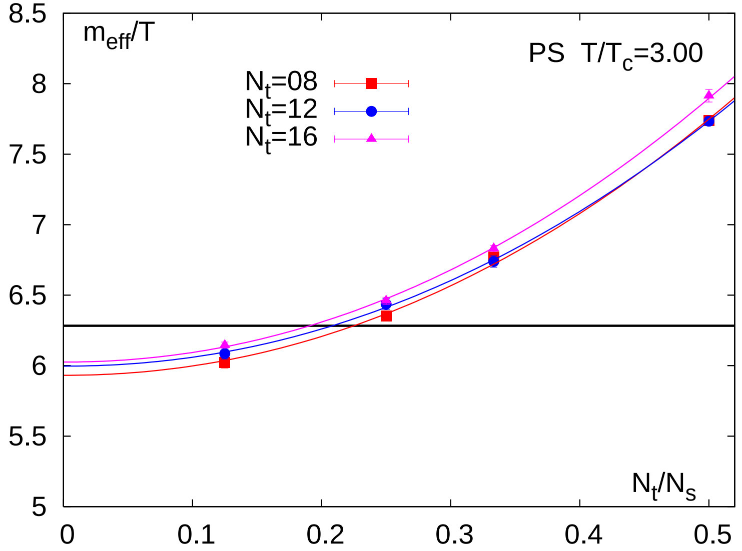


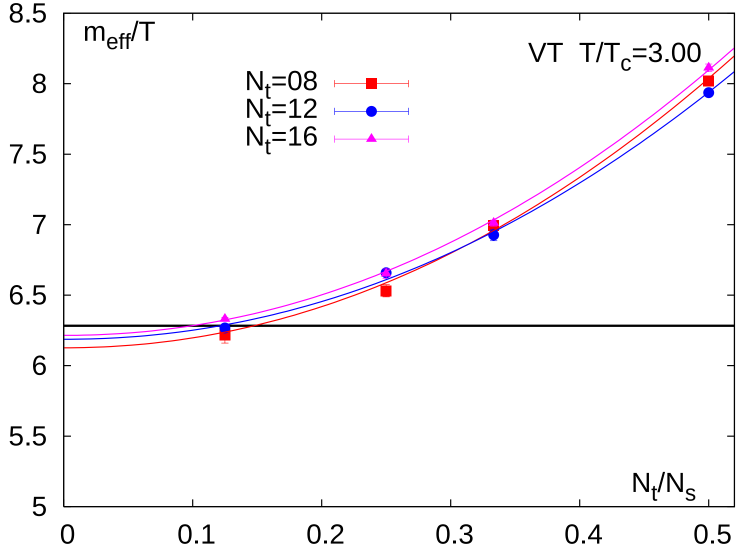
- free case very sensitive to finite $zT \sim LT = N_\sigma/N_\tau$ and finite lattice spacing $aT = 1/N_\tau$ effects
- finite $zT \sim LT = N_\sigma/N_\tau$ drive masses up
- finite lattice spacings $aT = 1/N_\tau$ drive them down

- leading behavior (free case) is $\sim 1/LT$

  and $\sim (aT)^2$

interacting case: thermodynamic limit



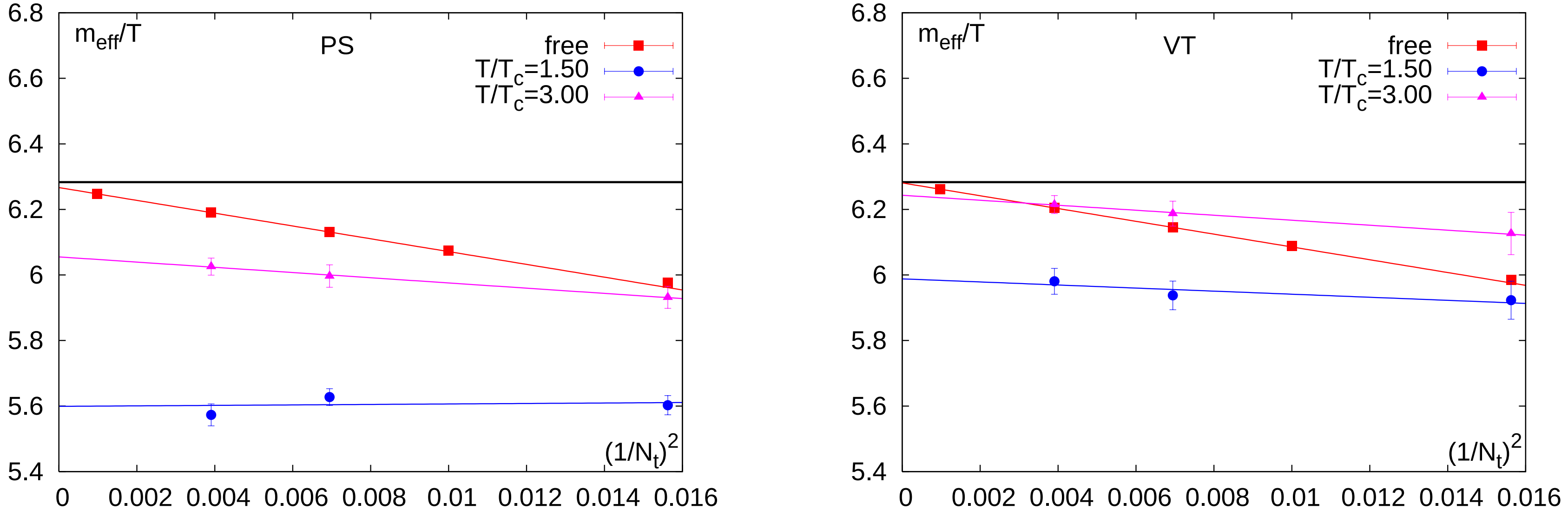


- finite volumes $LT = N_\sigma / N_\tau$ drive masses up
- assume power behavior

$$m/T = a + b \left(\frac{N_\tau}{N_\sigma} \right)^p$$

p	PS	VT
$1.5T_c$	2.22(10)	2.18(13)
$3.0T_c$	2.05(08)	2.05(10)

continuum limit



- also in interacting case: finite lattice spacings $aT = 1/N_\tau$ drive masses down $\sim a^2$ as expected
- slopes less than in free case
- T dependent deviations from free case
- up to $T = 3T_c \simeq 900\text{MeV}$ sizeable differences between π and ρ $\Rightarrow$ not free theory

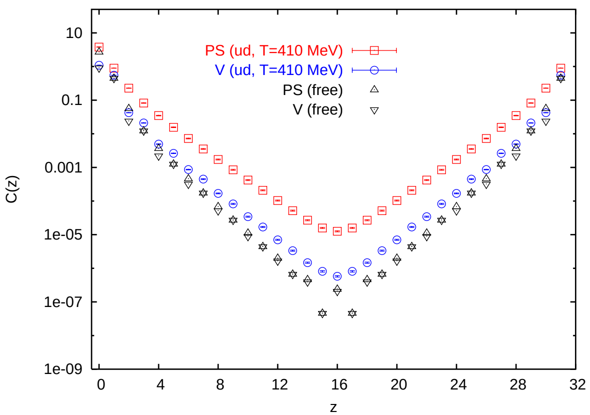
free lattice staggered fermions

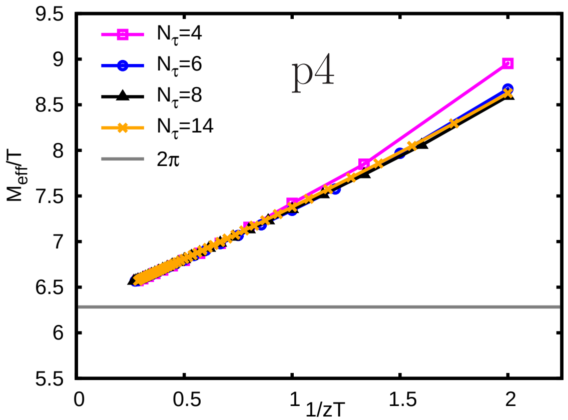
$$G_M^S(z) = \frac{1}{N_\sigma^2 N_\tau} \sum_{k_1,k_2,k_4} \frac{1}{\cosh^2 E} \frac{1}{\sinh^2(E N_\sigma/2)} \left\{ b_M \cosh\left[2E\left(\frac{N_\sigma}{2} - z\right)\right] + d_M \right\}$$

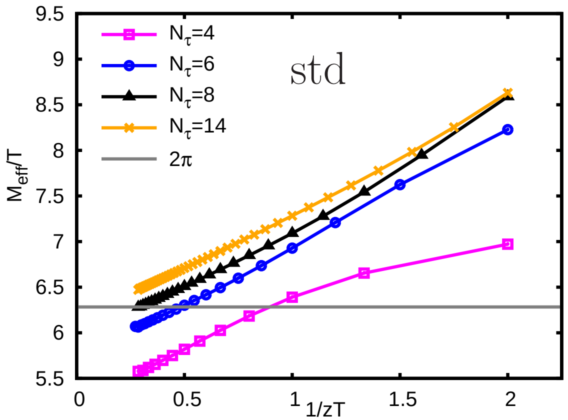
where $\sinh^2 E = m^2 + \sum_{1,2,4} \sin^2 k_i$

	b^{odd}	d^{odd}	b^{even}	d^{even}
P	1	-1	1	+1
$\frac{1}{2}(V_1+V_2)$	1	-1	$\frac{m^2+\sin^2 k_4}{\sinh^2(E)}$	$\frac{m^2+\sin^2 k_4}{\sinh^2(E)}$
S	1	-1	$\frac{2m^2}{\sinh^2(E)} - 1$	$\frac{2m^2}{\sinh^2(E)} - 1$
$\frac{1}{2}(A_1+A_2)$	1	-1	$\frac{m^2-\sin^2 k_4}{\sinh^2(E)}$	$\frac{m^2-\sin^2 k_4}{\sinh^2(E)}$

standard staggered; similar for p4

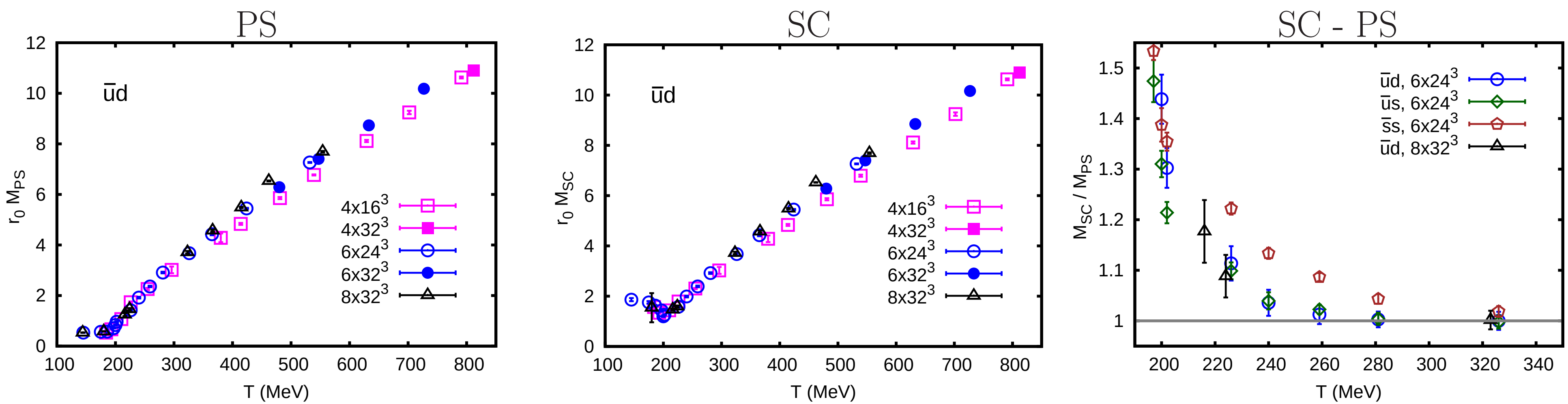






Pseudoscalar - Scalar (connected a_0/δ) sector

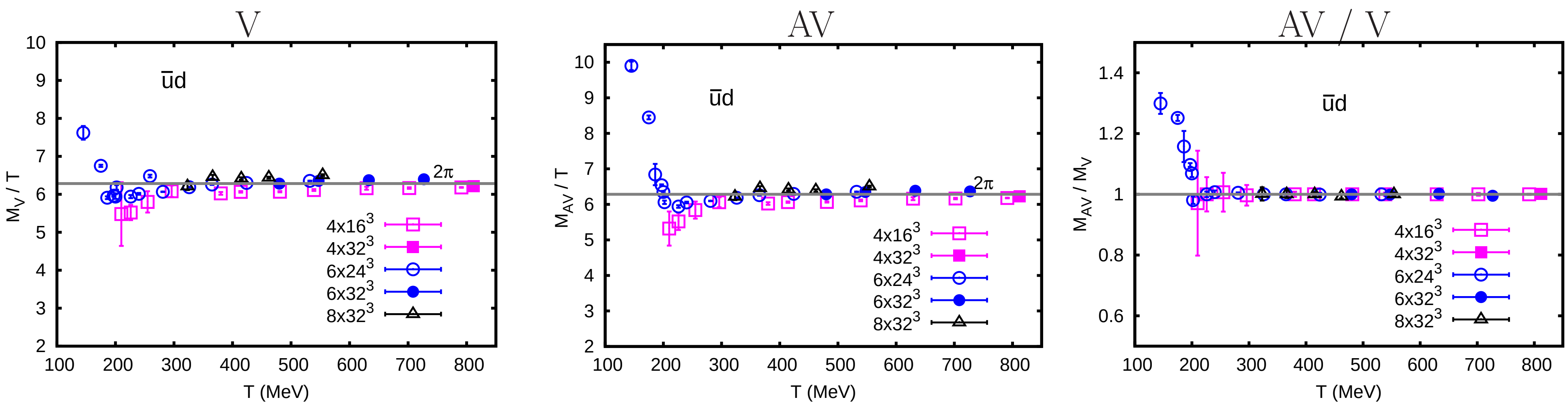
data mostly from $N_\sigma/N_\tau = 4$ but some points at larger aspect ratio $\longrightarrow$ larger zT



data shifted to a common $T_c(N_\tau = 6) \simeq 200$ MeV

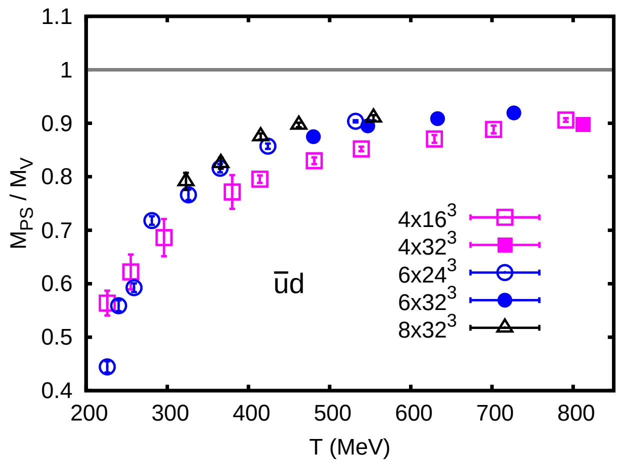
- PS starts rising at about T_c
- SC drops towards T_c, then rises
- π and a_0 do not become degenerate up to $T \simeq 240$ MeV

(transverse) Vector - Axialvector channel



- $m_V \gtrsim 2\pi T$ but no thermodynamic limit yet
- V_T and A_T are degenerate at $T \geq T_c$, signalling chiral symmetry restoration

PS / V



- PS and V are not degenerate up to $T \simeq 3T_c$

  i.e. not free theory

Summary

- scaling of f_{sing} provides powerful relations to study the behavior in the vicinity of a phase transition
- evidence that QCD with physical values of the quark masses m_l, m_s is in

  the scaling region of a critical point at $m_l = 0$ in the $O(N)$ universality class

  $\Rightarrow$ a.o. prediction for $T_p(m_l)$
- in agreement with $T_p(m_l)$ from χ_{disc}
- important to take continuum limit to reach agreement on T_p between various discretizations
- symmetry pattern of spatial meson correlators corroborates $O(N)$