INTRODUCTION TO tmQCD AND ITS APPLICATIONS to WMEs

STRONGnet Summer School
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Basic references

- Reviews

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Lesson 1: tmQCD action and symmetries

Generalities

- tmQCD is a relative newcomer in the family of lattice fermion regularizations

- it consists in modifying the standard Wilson fermion matrix by adding a mass term , which is "twisted" in chiral space

$$i\mu \ \bar{\psi} \ \tau^3 \gamma_5 \ \psi$$

Pauli matrix in SU(2) flavour space

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Pauli matrix in SU(2) flavour space

- there are several advantages in such a choice:

  - natural infrared cutoff enables a safer approach to the chiral limit (and keeps us safe from exceptional configurations in the quenched approximation)

  - in many cases the renormalization properties of WMEs are simplified

  - in most cases of interest observable quantities are improved "automatically" (i.e. without Symanzik counter-terms in the action and the operators)

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    - in many cases the renormalization properties of WMEs are simplified
    - in most cases of interest observable quantities are improved "automatically" (i.e. without Symanzik counter-terms in the action and the operators)
- there is a price to pay: flavour symmetry is lost and so are parity and time reversal (recovered in the continuum limit)

Classical tmQCD

Classical tmQCD

- for simplicity consider with two degenerate flavours $\bar{\psi} = (\bar{u} \quad \bar{d})$
- the classical QCD theory with SU(2) flavour symmetry is:

$$\mathcal{L} = \bar{\psi} \, [\, \not{D} + m + i \, \mu \, \tau^3 \, \gamma_5 \,] \, \psi$$

- **apparently** this is not QCD! (parity breaking? isospin braking? extra mass term?)
- **but** this theory is form invariant under chiral transformations in 3rd isospin direction, combined with spurionic transformations of the two mass parameters
- to see this, define first an **invariant mass** and a **twist angle**:

$$M = \sqrt{m^2 + \mu^2} \qquad \tan(\omega) = \frac{\mu}{m}$$

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twist angle

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standard mass squared

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- in terms of these the theory may be written as:

$$\mathcal{L} = \bar{\psi} \left[\not{D} + M \, \exp[i\omega\tau^3\gamma_5] \right] \psi$$

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- redefine fermionic fields through chiral rotations [$I_3(\alpha)$ - rotations]:

$$\psi \rightarrow \psi' = \exp\left[i\frac{\alpha}{2}\gamma_5\tau^3\right]\psi \qquad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}\exp\left[i\frac{\alpha}{2}\gamma_5\tau^3\right]$$

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Transformation angle

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- redefine mass parameters through spurionic transformations are:

$$m \rightarrow m' = m\cos(\alpha) + \mu\sin(\alpha)$$
$$\mu \rightarrow \mu' = \mu\cos(\alpha) - m\sin(\alpha)$$

Classical tmQCD

- the form invariance of the theory is:

$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\,\mu\,\tau^3\gamma_5\right]\psi = \bar{\psi}\left[\not{D} + M\ \exp[i\omega\tau^3\gamma_5]\right]\psi$$

$$\downarrow$$

$$\mathcal{L}' = \bar{\psi}'\left[\not{D} + m' + i\,\mu'\,\tau^3\gamma_5\right]\psi' = \bar{\psi}'\left[\not{D} + M\ \exp[i\omega'\tau^3\gamma_5]\right]\psi'$$

- with the **same** invariant mass and a **new** twist angle

$$M' = M \qquad \tan(\omega') = \frac{\mu'}{m'}$$

$$\omega' = \omega - \alpha$$

Classical tmQCD

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- with the **same** invariant mass and a **new** twist angle

$$M' = M \qquad \tan(\omega') = \frac{\mu'}{m'}$$

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- we have a family of theories, prametrised by their twist angle
- they are **equivalent**, as they are linked by field and mass redefinitions
- the quark mass is given by the invariant mass ***M* = √(*m*² + µ2)**

Classical tmQCD

- the form invariance of the theory is:

$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\,\mu\,\tau^3\,\gamma_5\right]\psi = \bar{\psi}\left[\not{D} + M\;\exp[i\omega\tau^3\gamma_5]\right]\psi$$

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- with the **same** invariant mass and a **new** twist angle

$$M' = M \qquad \tan(\omega') = \frac{\mu'}{m'}$$

$$\omega' = \omega - \alpha$$

- with $I_3(\alpha = \omega)$ - rotations we obtain $\boldsymbol{\omega' = 0 \Leftrightarrow \mu' = 0}$ and $\boldsymbol{m' = M}$

- i.e. the special case of zero twist angle is **QCD !!**

Classical tmQCD

- the form invariance of the theory is:

$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\,\mu\,\tau^3\,\gamma_5 \right]\psi = \bar{\psi}\left[\not{D} + M\ \exp[i\omega\tau^3\gamma_5] \right]\psi$$

$\downarrow$

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- with the **same** invariant mass and a **new** twist angle

$$M' = M \qquad \tan(\omega') = \frac{\mu'}{m'}$$

$$\omega' = \omega - \alpha$$

- with $I_3(\alpha = \omega - \pi/2)$ - rotations we obtain $\boldsymbol{\omega' = \pi/2 \Leftrightarrow m' = 0}$ and $\boldsymbol{\mu' = M}$

- this special case of interest is known as **fully twisted QCD or maximally twisted QCD!!**

Classical tmQCD

- symmetries are lost only **apparently**, since at the classical level QCD $\leftrightarrow$ tmQCD

- parity breaking? isospin braking?

$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\mu\tau^3\gamma_5\right]\psi = \bar{\psi}\left[\not{D} + M\exp[i\omega\tau^3\gamma_5]\right]\psi$$

- QCD is obtained from tmQCD (defined at fixed **ω**) with chiral transformations in 3rd isospin direction [$I_3(\omega)$-rotations], combined with spurionic transformations of the two mass parameters:

$$\tan(\omega) = \frac{\mu}{m}$$

$$\psi \to \psi' = \exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\psi \qquad \bar{\psi} \to \bar{\psi}' = \bar{\psi}\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]$$

$$m \to m' = M$$

$$\mu \to \mu' = 0$$

- the symmetry transformations of the fermion fields in the tmQCD formalism are obtained by performing the opposite $I_3(-\omega)$-rotations to the standard symmetry transformations of the fields in QCD

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- twisted parity is ***P*****$_\omega$**

$$\tan(\omega) = \frac{\mu}{m}$$

$$\begin{aligned}
x = (x^0, \mathbf{x}) \quad &\rightarrow \quad x' = (x^0, -\mathbf{x}) \\
A_0(x) \quad &\rightarrow \quad A_0(x') \\
A_k(x) \quad &\rightarrow \quad -A_k(x') \\
\psi(x) \quad &\rightarrow \quad \gamma_0 \exp\left[i\omega\gamma_5\tau^3\right]\psi(x') \\
\bar{\psi}(x) \quad &\rightarrow \quad \bar{\psi}(x') \exp\left[i\omega\gamma_5\tau^3\right]\gamma_0
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NB!

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$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\,\mu\,\tau^3\gamma_5\right]\psi = \bar{\psi}\left[\not{D} + M\ \exp[i\omega\tau^3\gamma_5]\right]\psi$$

- twisted time-reversal is similarly ***T*ω**

$$\tan(\omega) = \frac{\mu}{m}$$

$$\begin{aligned} x = (x^0, \mathbf{x}) &\rightarrow x' = (-x^0, \mathbf{x}) \\ A_0(x) &\rightarrow -A_0(x') \\ A_k(x) &\rightarrow A_k(x') \\ \psi(x) &\rightarrow i\,\gamma_0\gamma_5 \exp\left[i\omega\gamma_5\tau^3\right]\psi(x') \\ \bar{\psi}(x) &\rightarrow -i\,\bar{\psi}(x')\exp\left[i\omega\gamma_5\tau^3\right]\gamma_5\gamma_0 \end{aligned}$$

NB!

Classical tmQCD

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- parity breaking? isospin braking?

$$\mathcal{L} = \bar{\psi} \left[\not{D} + m + i \, \mu \, \tau^3 \gamma_5 \right] \psi = \bar{\psi} \left[\not{D} + M \exp[i\omega\tau^3\gamma_5] \right] \psi$$

- NB: instead of twisted parity $\boldsymbol{P_\omega}$ we may have standard parity $\boldsymbol{P_0}$, combined with (spurionic) twisted mass sign flip: $\boldsymbol{P_0} \otimes [\mu \to -\mu]$

$$\begin{aligned}
x = (x^0, \mathbf{x}) \quad &\to \quad x' = (x^0, -\mathbf{x}) \\
A_0(x) \quad &\to \quad A_0(x') \\
A_k(x) \quad &\to \quad -A_k(x') \\
\psi(x) \quad &\to \quad \gamma_0 \; \psi(x') \\
\bar{\psi}(x) \quad &\to \quad \bar{\psi}(x') \; \gamma_0 \\
\mu \quad &\to \quad -\mu
\end{aligned}$$

- similarly for time-reversal $\boldsymbol{T_0} \otimes [\mu \to -\mu]$

Classical tmQCD

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- parity breaking? isospin braking?

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- twisted vector symmetry (isospin) is ***SU$_V$(2)$_\omega$***

$$\tan(\omega) = \frac{\mu}{m}$$

$$\psi(x) \rightarrow \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]\ \exp\left[i\frac{\theta^a}{2}\tau^a\right]\ \exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\ \psi(x)$$

$$\bar{\psi}(x) \rightarrow \bar{\psi}(x)\ \exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\ \exp\left[-i\frac{\theta^a}{2}\tau^a\right]\ \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]$$

- vector symmetry transformation angles are θ^a

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$$\bar\psi(x) \rightarrow \bar\psi(x) \exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right] \exp\left[-i\frac{\theta^a}{2}\tau^a\right] \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]$$

NB!

- vector symmetry transformation angles are θ^a
- "extra" twist angle is ω

Classical tmQCD

- symmetries are lost only **apparently**, since at the classical level QCD $\leftrightarrow$ tmQCD
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- by analogy, twisted axial symmetry is ***SU$_A$(2)$_\omega$***

$$\tan(\omega) = \frac{\mu}{m}$$

$$\psi(x) \rightarrow \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right] \exp\left[i\frac{\theta^a}{2}\tau^a\gamma_5\right] \exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right] \psi(x)$$

$$\bar\psi(x) \rightarrow \bar\psi(x) \exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right] \exp\left[i\frac{\theta^a}{2}\tau^a\gamma_5\right] \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]$$

- axial symmetry transformation angles are θ^a
- "extra" twist angle is ω
- axial symmetry valid at $M = 0$

Classical tmQCD

- the $I_3(\omega)$-rotations relating QCD $\leftrightarrow$ tmQCD give operator correspondences

$$\mathcal{V}_\mu^a = \cos(\omega)\, V_\mu^a + \epsilon^{3ab} \sin(\omega)\, A_\mu^b \qquad a = 1,2$$
$$\mathcal{A}_\mu^a = \cos(\omega)\, A_\mu^a + \epsilon^{3ab} \sin(\omega)\, V_\mu^b \qquad a = 1,2$$
$$\mathcal{V}_\mu^3 = V_\mu^3$$
$$\mathcal{A}_\mu^3 = A_\mu^3$$

defined in QCD

defined in tmQCD

$$Q_\Gamma^a = \bar{\psi}\, \Gamma \frac{\tau^a}{2}\, \psi$$

$$S^0 = \bar{\psi}\, \psi$$

$$\tan(\omega) = \frac{\mu}{m}$$

Classical tmQCD

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defined in QCD

defined in tmQCD

$$S^0 = \bar{\psi}\, \psi$$

$$\mathcal{P}^a = P^a \qquad a = 1,2$$

$$\mathcal{P}^3 = \cos(\omega)\, P^3 + \frac{i}{2} \sin(\omega)\, S^0$$

$$\tan(\omega) = \frac{\mu}{m}$$

$$\mathcal{S}^0 = \cos(\omega)\, S^0 + 2i \sin(\omega)\, P^3$$

Classical tmQCD

- similar correspondences occur in Ward identities
- in tmQCD the "PCVC" is

$$\partial_\mu V_\mu^a \;=\; -2\mu\,\epsilon^{3ab}\,P^b$$

- in tmQCD the "PCAC" is

$$\partial_\mu A_\mu^a \;=\; 2m\,P^a \;+\; i\,\mu\,\delta^{3a}\,S^0$$

$$Q_\Gamma^a \;=\; \bar\psi\,\Gamma\,\frac{\tau^a}{2}\,\psi$$
$$S^0 \;=\; \bar\psi\,\psi$$

- in terms of the QCD currents and densities, they become the standard expressions

Lattice tmQCD

Alpha Collab., R. Frezzotti, P.A. Grassi, S. Sint and P.Weisz, JHEP08 (2001) 058

Lattice tmQCD

- QCD ↔ tmQCD equivalence carries over to the renormalized quantum level
- Ingredients:
    - chiral symmetry of Ginsparg-Wilson (GW) fermions
    - mass-independent renormalization scheme
    - universality of different lattice regularizations in the continuum limit
    - twist angle tuned to ratio of renormalized masses $\tan(\omega) = \mu_{\mathrm{R}} / m_{\mathrm{R}}$
- QCD ↔ tmQCD equivalence proceeds through linear mapping between renormalized Green functions
- regularize QCD and tmQCD with GW fermions

$$\mathcal{Z}_{\mathrm{GW}}^{\mathrm{QCD}} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}U \exp\left[-\mathcal{S}_{\mathrm{GW}}^{\mathrm{QCD}} \right]$$

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QCD: discretization of mass term

$$\mathcal{Z}_{\rm GW}^{\rm QCD} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}U \exp\left[-\mathcal{S}_{\rm GW}^{\rm QCD}\right]$$

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- QCD ↔ tmQCD equivalence proceeds through linear mapping between renormalized Green functions
- regularize QCD and tmQCD with GW fermions

$$\mathcal{Z}_{\rm GW}^{\rm QCD} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}U \exp\left[- \mathcal{S}_{\rm GW}^{\rm QCD} \right]$$

GW: discretization of kinetic term

Lattice tmQCD

- QCD ↔ tmQCD equivalence carries over to the renormalized quantum level
- Ingredients:
  - chiral symmetry of Ginsparg-Wilson (GW) fermions
  - mass-independent renormalization scheme
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$$\Big[\, \langle \, Q \, \rangle \, \Big]^{\mathrm{QCD}}_{\mathrm{GW}} \; = \; \frac{1}{\mathcal{Z}^{\mathrm{QCD}}_{\mathrm{GW}}} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}U \exp\left[-\, \mathcal{S}^{\mathrm{QCD}}_{\mathrm{GW}} \right] \, Q$$

Lattice tmQCD

- QCD ↔ tmQCD equivalence carries over to the renormalized quantum level
- Ingredients:
  - chiral symmetry of Ginsparg-Wilson (GW) fermions
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$$\mathcal{Z}_{\rm GW}^{\rm tmQCD} \; = \; \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}U \exp\left[- \mathcal{S}_{\rm GW}^{\rm tmQCD} \right]$$

$$\left[\langle \, Q \, \rangle \right]_{\rm GW}^{\rm tmQCD} \; = \; \frac{1}{\mathcal{Z}_{\rm GW}^{\rm tmQCD}} \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}U \exp\left[- \mathcal{S}_{\rm GW}^{\rm tmQCD} \right] Q$$

Lattice tmQCD

- QCD $\leftrightarrow$ tmQCD equivalence carries over to the renormalized quantum level
- Ingredients:
  - chiral symmetry of Ginsparg-Wilson (GW) fermions
  - mass-independent renormalization scheme
  - universality of different lattice regularizations in the continuum limit
  - twist angle tuned to ratio of renormalized masses $\tan(\omega) = \mu_R / m_R$
- QCD $\leftrightarrow$ tmQCD equivalence proceeds through linear mapping between renormalized Green functions
- regularize QCD and tmQCD with GW fermions
- GW chiral symmetry guarantees the same considerations of a trivial QCD $\leftrightarrow$ tmQCD equivalence as in the classical case are valid (with minor caveats)
- example: **bare** Green function of the scalar operator (chiral condensate)

$$\Big[< \cdots \mathcal{S}^0 \cdots > \Big]^{\rm QCD}_{\rm GW} = \Big[\cos(\omega) < \cdots S^0 \cdots > + i \; \sin(\omega) < \cdots P^3 \cdots > \Big]^{\rm tmQCD}_{\rm GW}$$

Lattice tmQCD

- QCD ↔ tmQCD equivalence between bare Green functions with GW regularization

$$\Big[< \cdots \mathcal{S}^0 \cdots > \Big]^{\rm QCD}_{\rm GW} = \Big[\cos(\omega) \; < \cdots S^0 \cdots > \; + i \; \sin(\omega) \; < \cdots P^3 \cdots > \Big]^{\rm tmQCD}_{\rm GW}$$

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- QCD $\leftrightarrow$ tmQCD equivalence carries over to renormalized quantities, due to mass independent renormalization schemes (i.e. S^0 and P^3 in both QCD and tmQCD have the same renormalization constant $Z_S = Z_P = Z$)

$$Z(a\tilde{\mu}) \Big[< \cdots \mathcal{S}^0 \cdots > \Big]^{\rm QCD}_{\rm GW} = Z(a\tilde{\mu}) \Big[\cos(\omega) < \cdots S^0 \cdots > + i \sin(\omega) < \cdots P^3 \cdots > \Big]^{\rm tmQCD}_{\rm GW}$$

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- since QCD ↔ tmQCD equivalence holds between renormalized (continuum) Green functions (proved through GW regularization), evoking universality we claim that this is also true for renormalized (continuum) Green functions computed with any other lattice regularization; e.g. tmQCD with Wilson fermions

$$\Big[< \cdots \mathcal{S}^0 \cdots >_{\rm R} \Big]^{\rm QCD} = \Big[\cos(\omega) < \cdots S^0 \cdots >_{\rm R} + i \sin(\omega) < \cdots P^3 \cdots >_{\rm R} \Big]^{\rm tmQCD}$$

Lattice tmQCD

- QCD ↔ tmQCD equivalence between bare Green functions with GW regularization

$$\Big[< \cdots \mathcal{S}^0 \cdots > \Big]^{\rm QCD}_{\rm GW} \; = \; \Big[\cos(\omega) \; < \cdots S^0 \cdots > \; + i \;\; \sin(\omega) \; < \cdots P^3 \cdots > \Big]^{\rm tmQCD}_{\rm GW}$$

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$$\Big[< \cdots \mathcal{S}^0 \cdots >_{\rm R} \Big]^{\rm QCD} \; = \; \Big[\cos(\omega) < \cdots S^0 \cdots >_{\rm R} \; + i \sin(\omega) < \cdots P^3 \cdots >_{\rm R} \Big]^{\rm tmQCD}$$

- QCD ↔ tmQCD equivalence amounts to operator transcriptions in lattice tmQCD

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) \;+\; \frac{1}{2}ar\nabla^*_\mu\nabla_\mu \;+\; m_0 \;+\; i\mu_q\tau^3\gamma_5\right]\psi \\ &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) \;+\; \frac{1}{2}ar\nabla^*_\mu\nabla_\mu \;+\; M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi\end{aligned}$$

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naive derivative

Lattice tmQCD

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Wilson term

- Wilson term cures the fermion doubling problem

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2} a r \nabla^*_\mu \nabla_\mu + m_0 + i\mu_q \tau^3 \gamma_5 \right]\psi \\
&= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2} a r \nabla^*_\mu \nabla_\mu + M_0 \exp[i\omega_0 \tau^3 \gamma_5] \right]\psi
\end{aligned}
$$

bare
standard
mass

- bare standard mass renormalizes as in standard Wilson fermions:

$$
m_R = Z_{m^0}\left[m_0 - m_{cr}\right] = Z_{S^0}^{-1}\left[m_0 - m_{cr}\right]
$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\mathcal{L} = \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + \frac{1}{2}ar\nabla_\mu^*\nabla_\mu + m_0 + i\mu_q\tau^3\gamma_5\right]\psi$$

$$= \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + \frac{1}{2}ar\nabla_\mu^*\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi$$

bare twisted mass

$\mu_q \leftrightarrow \mu$

- bare twisted mass renormalizes multiplicatively, due to the **exact** tmQCD vector Ward identity:

$$\nabla_\mu\langle\cdots V_\mu^{a,\mathrm{cons}}\cdots\rangle = -2\,\mu_q\,\epsilon^{3ab}\,\langle\cdots P^b\cdots\rangle$$

$$\mu_\mathrm{R} = Z_\mu\,\mu_q = Z_\mathrm{P}^{-1}\,\mu_q$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\mathcal{L} = \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi$$
$$= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi$$

- Wilson term causes loss of twisted symmetries: ***P***$_\omega$, ***T***$_\omega$

- ***P***$_\omega$:

$$\begin{aligned} x = (x^0, \mathbf{x}) &\rightarrow x' = (x^0, -\mathbf{x}) \\ A_0(x) &\rightarrow A_0(x') \\ A_k(x) &\rightarrow -A_k(x') \\ \psi(x) &\rightarrow \gamma_0 \exp\left[i\omega\gamma_5\tau^3\right]\psi(x') \\ \bar{\psi}(x) &\rightarrow \bar{\psi}(x')\exp\left[i\omega\gamma_5\tau^3\right]\gamma_0 \end{aligned}$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\mathcal{L} = \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi$$

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- Wilson term causes loss of twisted symmetries: $\boldsymbol{P_\omega}$, $\boldsymbol{T_\omega}$
- **Parity** survives if combined either with flavour exchange (defined as $\boldsymbol{P_F^1}$, $\boldsymbol{P_F^2}$) ...

$$\psi(x) \to i\,\gamma_0\,\tau^1\,\psi(x') \qquad \bar\psi(x) \to -i\,\bar\psi(x')\,\gamma_0\,\tau^1$$

$$\psi(x) \to i\,\gamma_0\,\tau^2\,\psi(x') \qquad \bar\psi(x) \to -i\,\bar\psi(x')\,\gamma_0\,\tau^2$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

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- Wilson term causes loss of twisted symmetries: ***P*ω, *T*ω**
- ... or with a sigh flip of the twisted mass (defined as ***P*** ⊗[μ → - μ])

$$\psi(x) \to i\,\gamma_0\,\psi(x') \qquad \bar\psi(x) \to -i\,\bar\psi(x')\,\gamma_0$$

$$\mu_q \to -\,\mu_q$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\mathcal{L} = \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi$$
$$= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi$$

- Wilson term causes loss of twisted symmetries: ***P***$_\omega$, ***T***$_\omega$
- ... or with a sigh flip of the twisted mass (defined as ***P*** ⊗[μ → - μ])

$$\psi(x) \to i\,\gamma_0\,\psi(x') \qquad \bar{\psi}(x) \to -i\,\bar{\psi}(x')\,\gamma_0$$
$$\mu_q \to -\mu_q$$

- The same holds for time reversal: ***T***$_F^1$, ***T***$_F^2$, ***T*** ⊗[μ → - μ] are symmetries

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi \\
&= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi
\end{aligned}
$$

- Wilson term causes loss of twisted vector symmetry: ***$SU_V(2)_\omega$***; i.e. flavour symmetry is **hard**-broken (Wilson term) in tmQCD
- However it is not completely broken; upon setting $\theta^1 = \theta^2 = 0$, the subgroup ***$U_V^3(1)$*** survives (NB: ω-dependence drops out!)

$$
\begin{aligned}
\psi(x) &\rightarrow \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[i\frac{\theta^a}{2}\tau^a\right]\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\psi(x) \\
\bar{\psi}(x) &\rightarrow \bar{\psi}(x)\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[-i\frac{\theta^a}{2}\tau^a\right]\exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]
\end{aligned}
$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\mathcal{L} = \bar\psi \Big[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + \boxed{\frac{1}{2} a r \nabla_\mu^* \nabla_\mu} + m_0 + i\mu_q \tau^3 \gamma_5 \Big]\psi$$
$$= \bar\psi \Big[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + \frac{1}{2} a r \nabla_\mu^* \nabla_\mu + M_0 \exp[i\omega_0 \tau^3 \gamma_5] \Big]\psi$$

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$$\psi \rightarrow \exp\Big[i\frac{\theta^3}{2}\tau^3\Big]\,\psi$$
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- this hard ***$SU_V(2)_\omega$ → $U_V^3(1)$*** breaking causes a lack of degeneracy between the neutral pion π^0 and the two charged pions $\pi^\pm$
- It is a discretization effect which vanishes in the continuum limit (***$SU_V(2)$*** restoration)

$$\psi \rightarrow \exp\left[i\frac{\theta^3}{2}\tau^3\right] \psi$$

$$\bar\psi \rightarrow \bar\psi \exp\left[-i\frac{\theta^3}{2}\tau^3\right]$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

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- axial symmetry ***$SU_A(2)_\omega$***, broken **softly** by mass term M_0, also **hard**-broken by Wilson term in standard fashion
- also this symmetry is restored in the continuum and chiral limits

$$\psi(x) \rightarrow \exp\left[- i\frac{\omega}{2}\gamma_5\tau^3 \right] \exp\left[i\frac{\theta^a}{2}\tau^a\gamma_5 \right] \exp\left[i\frac{\omega}{2}\gamma_5\tau^3 \right] \psi(x)$$

$$\bar\psi(x) \rightarrow \bar\psi(x) \exp\left[i\frac{\omega}{2}\gamma_5\tau^3 \right] \exp\left[i\frac{\theta^a}{2}\tau^a\gamma_5 \right] \exp\left[- i\frac{\omega}{2}\gamma_5\tau^3 \right]$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\mathcal{L} = \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi$$
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- However it is not completely broken in maximally tmQCD ($\omega=\pi/2$); upon setting $\theta^2 = \theta^3 = 0$, the subgroup ***$U_A'(1)$*** survives

$$\psi(x) \rightarrow \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[i\frac{\theta^a}{2}\tau^a\gamma_5\right]\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\psi(x)$$
$$\bar\psi(x) \rightarrow \bar\psi(x)\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[i\frac{\theta^a}{2}\tau^a\gamma_5\right]\exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

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$$\psi \rightarrow \exp\left[i\frac{\theta^1}{2}\tau^2\right]\psi$$
$$\bar\psi \rightarrow \bar\psi \exp\left[-i\frac{\theta^1}{2}\tau^2\right]$$

NB: this is called an **axial** symmetry, 'though it has a **vector** form, because it is softly broken by the twisted mass term!

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

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$$= \bar\psi \Big[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2} a r \nabla^*_\mu \nabla_\mu + M_0 \exp[i\omega_0 \tau^3 \gamma_5] \Big] \psi$$

- axial symmetry ***$SU_A(2)_\omega$***, broken **softly** by mass term M_0, also **hard**-broken by Wilson term in standard fashion
- also this symmetry is restored in the continuum and chiral limits
- However it is not completely broken in maximally tmQCD ($\omega=\pi/2$); upon setting $\theta^2 = \theta^3 = 0$, the subgroup ***$U_A^1(1)$*** survives

$$\psi \rightarrow \exp\Big[i\frac{\theta^1}{2}\tau^2\Big]\psi$$
$$\bar\psi \rightarrow \bar\psi \exp\Big[-i\frac{\theta^1}{2}\tau^2\Big]$$

NB: analogously, for $\theta^1 = \theta^3 = 0$, obtain the subgroup ***$U_A^2(1)$***

Lattice tmQCD

- Aftermath: we are left with the following symmetry structure:

$$U_V^3(1) \otimes U_A^1(1) \otimes U_A^2(1) \approx SU(2)$$

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Vector symmetry of all tmQCD regularizations

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Axial symmetries of maximally tmQCD, broken by twisted mass term

Lattice tmQCD

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Vector symmetry of all tmQCD regularizations

Axial symmetries of maximally tmQCD, broken by twisted mass term

- This is ***SU(2)*** with one "vector" and two "axial" generators.
- Not surprising, since in the chiral limit standard and tmQCD Wilson regularizations coincide.
- Interpretation of symmetry as "vector" or "axial" depends on how the mass term is introduced in the action.

Shortcoming: loss of flavour symmetry

Flavour symmetry violation

- it generates mass splittings between, say, neutral and charged pions
- it is a discretization effect, which should vanish in the continuum
- preliminary studies suggest that this is a big effect in the quenched approximation, but diminishes significantly in the $N_f = 2$ unquenched case
- the quenched case offers an interesting playing-field for explorative studies of flavour breaking, as it is possible (and cheap) to compare, on the same ensemble, mesons with the following valence quark content:
    - 2 twisted flavours from same isospin multiplet
    - 2 twisted flavours from different isospin multiplets
    - 2 untwisted flavours (the standard Wilson case, without flavour breaking)
    - 1 twisted and 1 untwisted flavour

Flavour symmetry violation

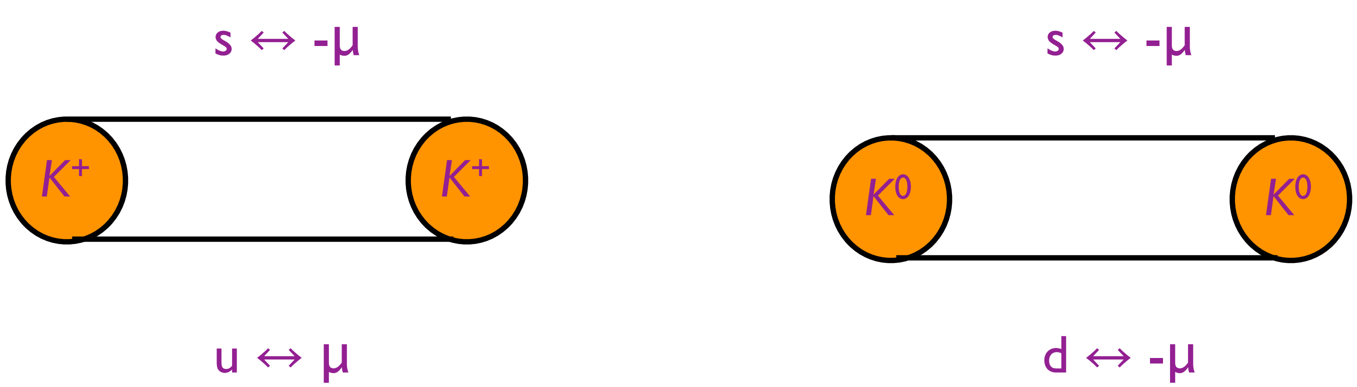
- mass splittings between, say, neutral and charged Kaons
- 4 quenched flavours organized in two maximally twisted, mass degenerate doublets
- K^+ made of an "up" and a "strange" twisted flavour
- K^0 made of a "down" and a "strange" twisted flavour
- at the smallest lattice spacing, neutral-charged Kaon splitting is $m_{K0} - m_{K+} \sim 50$ MeV

$$\psi_l = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \psi_h = \begin{pmatrix} c \\ s \end{pmatrix}$$

spectator

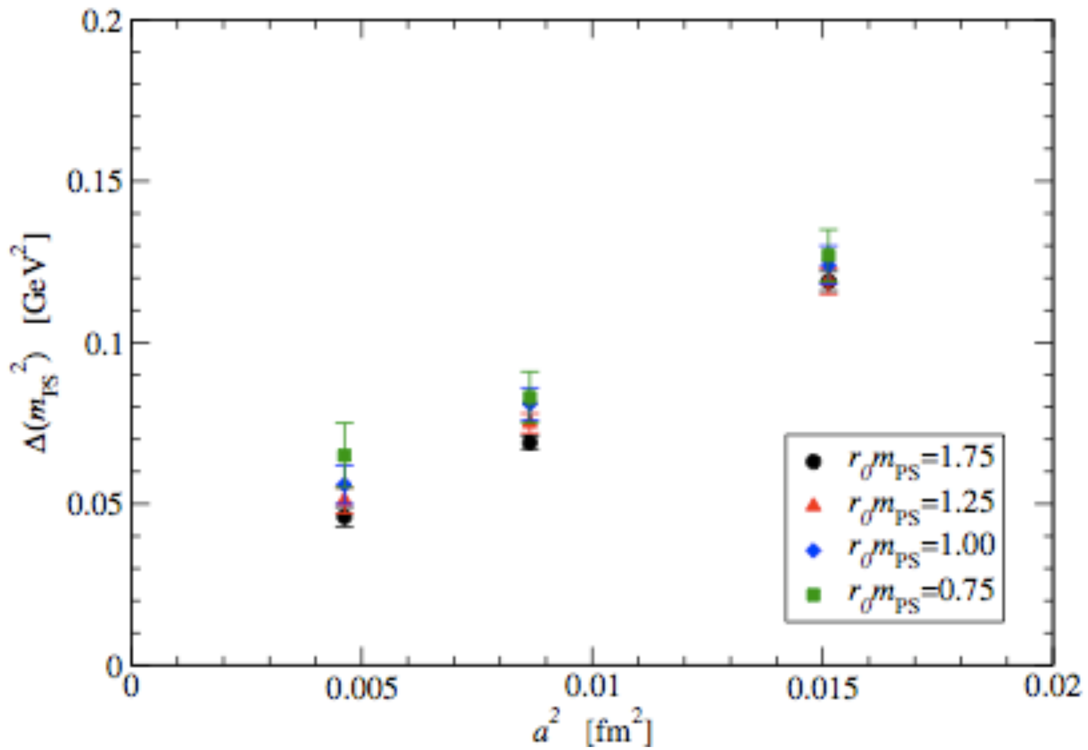
MtmQCD doublets

A.M. Abdel-Rehim, et al., Phys.Rev.D74(2006)014507



Flavour symmetry violation

- mass splittings between, say, neutral and charged Kaons
- 4 quenched flavours organized in two maximally twisted, mass degenerate doublets
- K^+ made of an "up" and a "strange" twisted flavour
- K^0 made of a "down" and a "strange" twisted flavour
- at the smallest lattice spacing, neutral-charged Kaon splitting is m_{K0} - m_{K+} ~ ~ 50 MeV



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Flavour symmetry violation

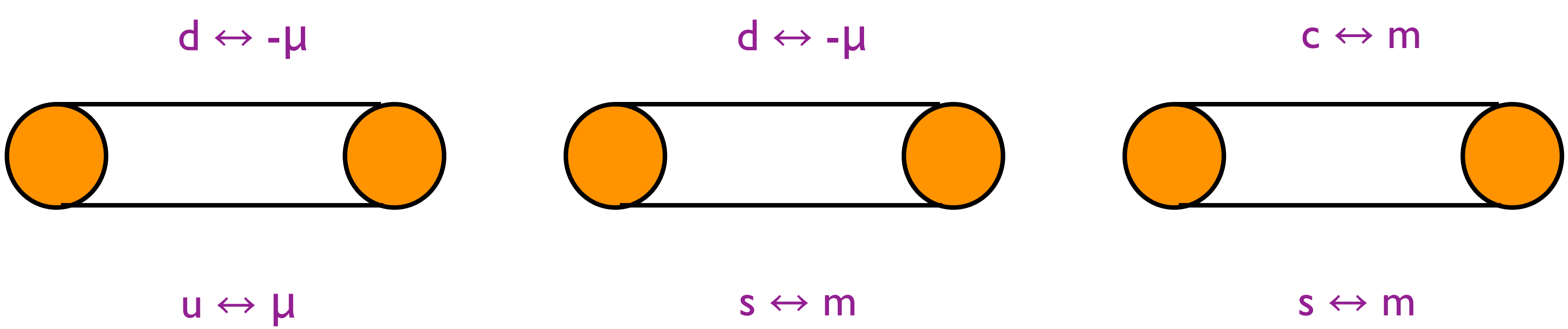
- mass splittings between, say, neutral and charged Kaons
- 4 quenched flavours organized in a maximally twisted and an untwisted doublet
- pseudoscalars made of twisted (u-d), untwisted (s-c) and mixed (s-d) valence quarks
- NB: the untwisted (s-c) case is the standard Wilson one, without flavour breaking
- comparison at a several lattice spacings, for quark masses "above" strangeness

$$\psi_l = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \psi_h = \begin{pmatrix} c \\ s \end{pmatrix}$$

MtmQCD doublet

QCD doublet

ALPHA P.Dimopoulos, et al., NPB 776 (2007) 258



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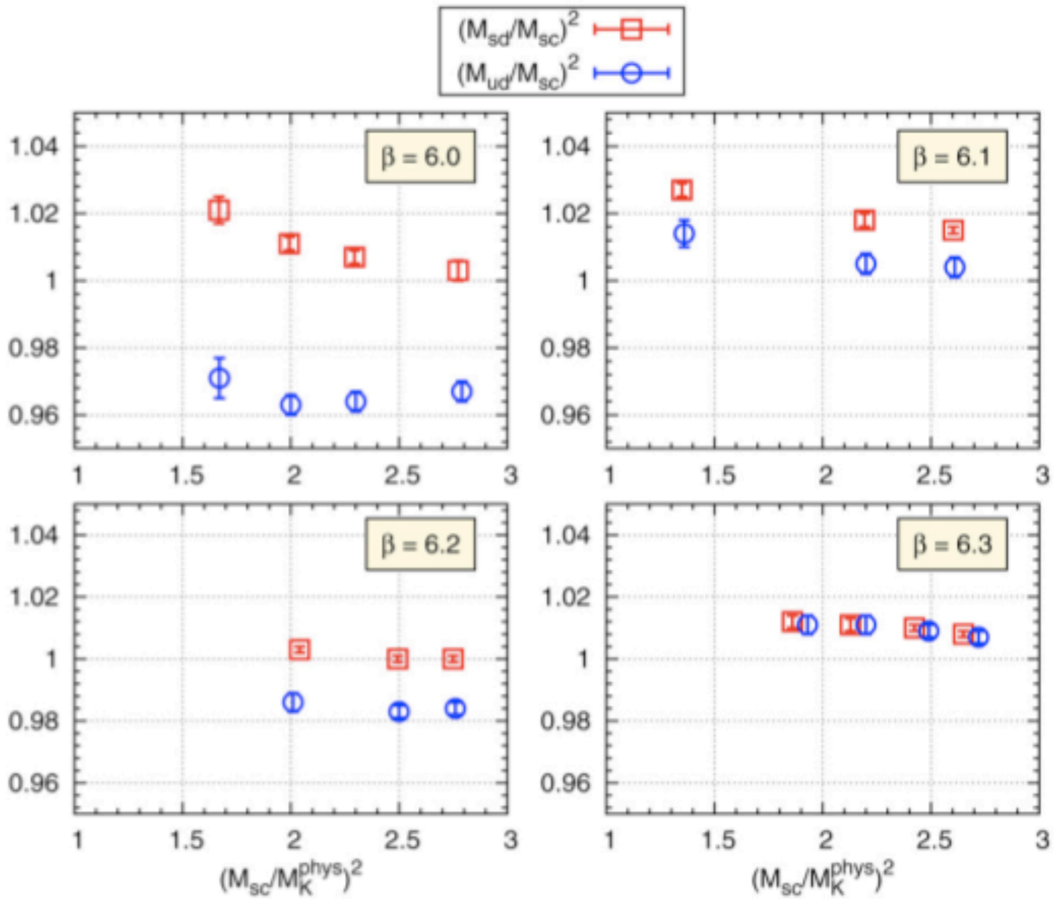

ALPHA P.Dimopoulos, et al.,
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Flavour symmetry violation

- further detailed unquenched studies are required
- the main difficulty is an accurate and efficient computation of disconnected diagrams for neutral pions; cf. C. Michael et al. in various papers and conference proceedings
- result so far for $N_f = 2$ unquenched case is:

$$1 - \frac{m_{\pi^0}}{m_{\pi^+}} \sim 0.2 \qquad @ \qquad a = 0.09\mathrm{fm}$$

ETMC Ph. Boucaud et al. Phys.Lett. B650 (2007) 304

ETMC Ph. Boucaud et al. Comp. Phys. Comm. 179 (2008) 695