INTRODUCTION TO tmQCD AND ITS APPLICATIONS to WMEs

STRONGnet Summer School
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Bielefeld
13-26/6/2011

tmQCD for WMEs: the B_K paradigm

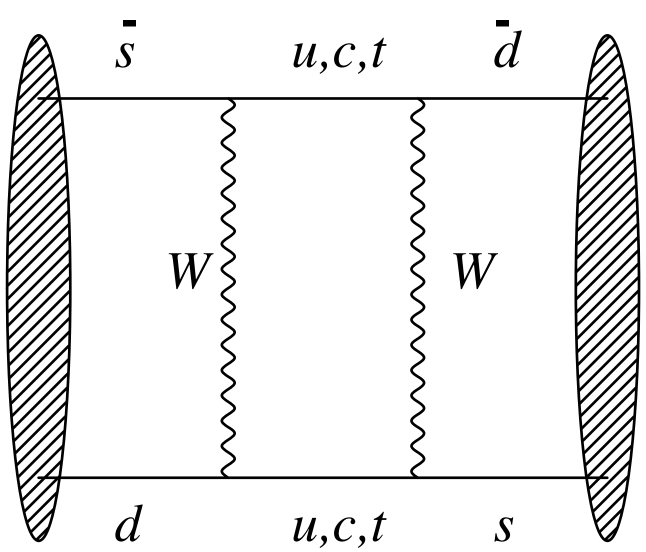
ε_K, B_K and the Unitarity Triangle

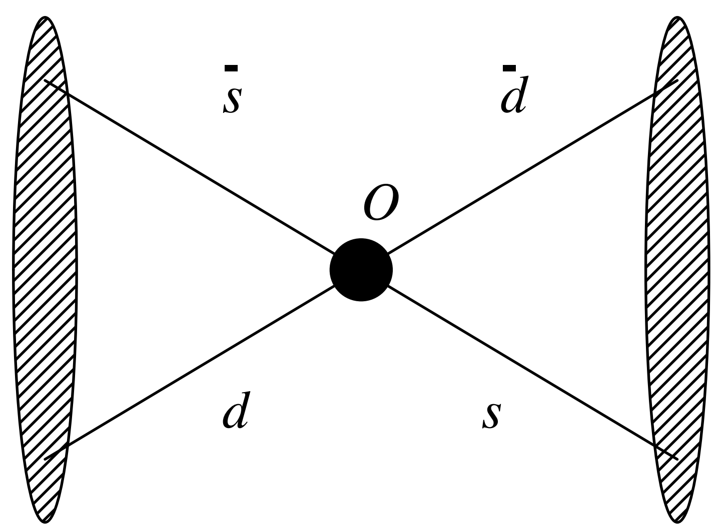
ΔS=2 transitions: ε_K

indirect CP-violation

$$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}]\,\exp(i\pi/4)$$

can also be expressed in terms of neutral meson oscillations: $K^0 - \bar{K}^0$
dominant EW process is FCNC (2 W exchange)





lowest order EW contribution (no QCD)

can perform loop integration exactly (for small quark external momenta and masses)

ΔS=2 transitions: ε_K

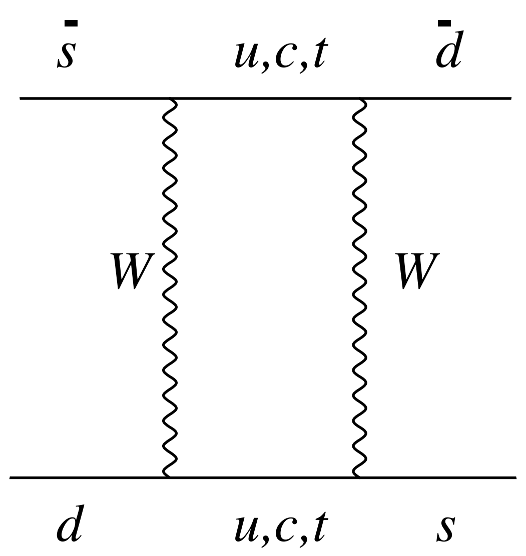
start with the EW theory (no QCD yet)

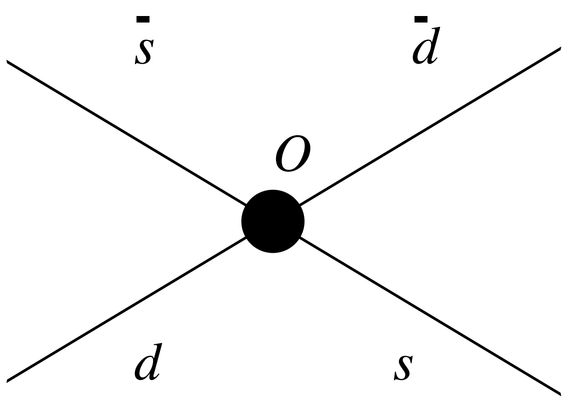
effective Hamiltonian

$$\mathcal{H}_{\rm eff}^{\Delta S=2} = \frac{G_F^2 M_{\rm W}^2}{16\pi^2} \mathcal{F}^0 Q^{\Delta S=2} + \text{h.c.}$$

4-fermion operator

$$Q^{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d] \equiv O_{\rm VV+AA} - O_{\rm VA+AV}$$





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$$\mathcal{F}^0 = \lambda_c^2 S_0(x_c) + \lambda_t^2 S_0(x_t) + 2\lambda_c\lambda_t S_0(x_c, x_t)$$

$$\lambda_a = V_{as}^* V_{ad} \qquad a = c\,,t$$

ΔS=2 transitions: ε_K

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$x_c = m_c^2/M_{\rm W}^2$

$x_t = m_t^2/M_{\rm W}^2$

ΔS=2 transitions: ε_K

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$S_0(x_c)$ $\quad$ $S_0(x_t)$ $\quad$ $S_0(x_c, x_t)$

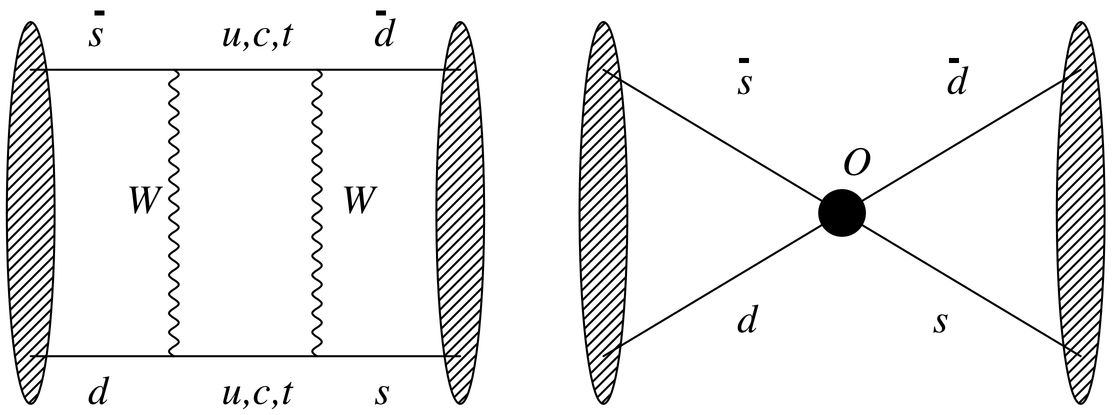
Inami-Lin functions

ΔS=2 transitions: ε_K

- add QCD interactions: <u>must</u> consider MEs between hadronic (K-meson) states
- cannot calculate ME in perturbation theory (at hadronic scales QCD coupling is large)
- OPE factorizes long- and short-distance effects; below charm scale we have:

$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle \;=\; \frac{G_F^2 M_{\rm W}^2}{16\pi^2}\Big[\lambda_c^2 S_0(x_c)\eta_1 \;+\; \lambda_t^2 S_0(x_t)\eta_2 \;+\; 2\lambda_c\lambda_t S_0(x_c,x_t)\eta_3\Big]$$
$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \left\{1+\frac{\bar{g}(\mu)^2}{(4\pi)^2}\left[\frac{\beta_1\gamma_0-\beta_0\gamma_1}{2\beta_0^2}\right]\right\} \langle \bar{K}^0 | Q_{\rm R}^{\Delta S=2}(\mu) | K^0 \rangle \;+\; {\rm h.c.}$$

- η_1, η_2 and η_3 are functions of the various thresholds m_t, m_c, m_b, and M_W
- $g(\mu)$ is the renormalized QCD coupling
- $Q_R^{\Delta S=2}(\mu)$ is the renormalized 4-fermion operator



- β_0, β_1 are NLO RG-running coefficients of Callan-Symanzik beta function
- γ_0, γ_1 are NLO RG-running coefficients of 4-fermion operator anomalous dimension

ΔS=2 transitions: ε_K

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$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right] \right\} \langle \bar{K}^0 | Q_{\rm R}^{\Delta S=2}(\mu) | K^0 \rangle \; + \; {\rm h.c.}$$

- quark mass dependence cancels out in product of η-functions and S_0- functions
- renormalization scale μ-dependence cancels out between WME and RG-coefficient
- the WME $<K|Q_R^{\Delta S=2}(\mu)|K>$ between K-meson states is the long-distance NP-quantity which must be computed on the lattice
- the rest is the OPE Wilson coefficient (short-distance, perturbative object)

ΔS=2 transitions: ε_K

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$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \left\{1+\frac{\bar{g}(\mu)^2}{(4\pi)^2}\left[\frac{\beta_1\gamma_0-\beta_0\gamma_1}{2\beta_0^2}\right]\right\} \langle \bar{K}^0 | Q_{\rm R}^{\Delta S=2}(\mu) | K^0 \rangle \;+\; {\rm h.c.}$$

- for historical and technical reasons, instead of $\langle K|Q_R^{\Delta S=2}(\mu)|K\rangle$, the bag parameter is used:

$$B_K(\mu) = \frac{\left\langle \bar{K}^0 \left| Q_{\rm R}^{\Delta S=2}(\mu) \right| K^0 \right\rangle}{\frac{8}{3} f_{\rm K}^2 m_{\rm K}^2}$$

- its RGI version is:

$$\hat{B}_K = \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \exp\left\{\int_0^{\bar{g}(\mu)} dg \left(\frac{\gamma(g)}{\beta(g)} + \frac{\gamma_0}{\beta_0 g}\right)\right\} B_K(\mu)$$

- at NLO this is:

$$\hat{B}_K = \left(\frac{\bar{g}(\mu)^2}{4\pi}\right)^{-\gamma_0/(2\beta_0)} \left\{1+\frac{\bar{g}(\mu)^2}{(4\pi)^2}\left[\frac{\beta_1\gamma_0-\beta_0\gamma_1}{2\beta_0^2}\right]\right\} B_K(\mu)$$

ΔS=2 transitions: ε_K

- How is B_K connected with ε_K?

$$\epsilon_K = \exp(i\phi_\epsilon)\ \sin(\phi_\epsilon)\ \left[\frac{\Im[\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle]}{\Delta M_K} + \frac{\Im(A_0)}{\Re(A_0)}\right]$$

- the phase of ε_K is given by:

$$\phi_\epsilon = \arctan\frac{\Delta M_K}{\Delta\Gamma_K/2}$$

- ΔM_K : mass difference between long- and short-lived neutral Kaons
- $\Delta\Gamma_K$: decay width difference between long- and short-lived neutral Kaons
- A_0 : amplitude of K → ππ(I=0) decay
- experimentally:

$$\begin{aligned} |\epsilon_K| &= 2.280(13)\times 10^{-3}\ , \\ \phi_\epsilon &= 43.51(5)^\circ\ , \\ \Delta M_K &= 3.491(9)\times 10^{-12}\,\mathrm{MeV}\ , \\ \Delta\Gamma_K &= 7.335(4)\times 10^{-15}\,\mathrm{GeV}\ , \end{aligned}$$

ΔS=2 transitions: ε_K

$$\epsilon_K = \exp(i\phi_\epsilon)\ \sin(\phi_\epsilon)\left[\frac{\Im[\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle]}{\Delta M_K} + \frac{\Im(A_0)}{\Re(A_0)}\right]$$

$$\langle \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0\rangle = \frac{G_F^2 M_{\rm W}^2}{16\pi^2}\left[\lambda_c^2 S_0(x_c)\eta_1 + \lambda_t^2 S_0(x_t)\eta_2 + 2\lambda_c\lambda_t S_0(x_c,x_t)\eta_3\right]$$
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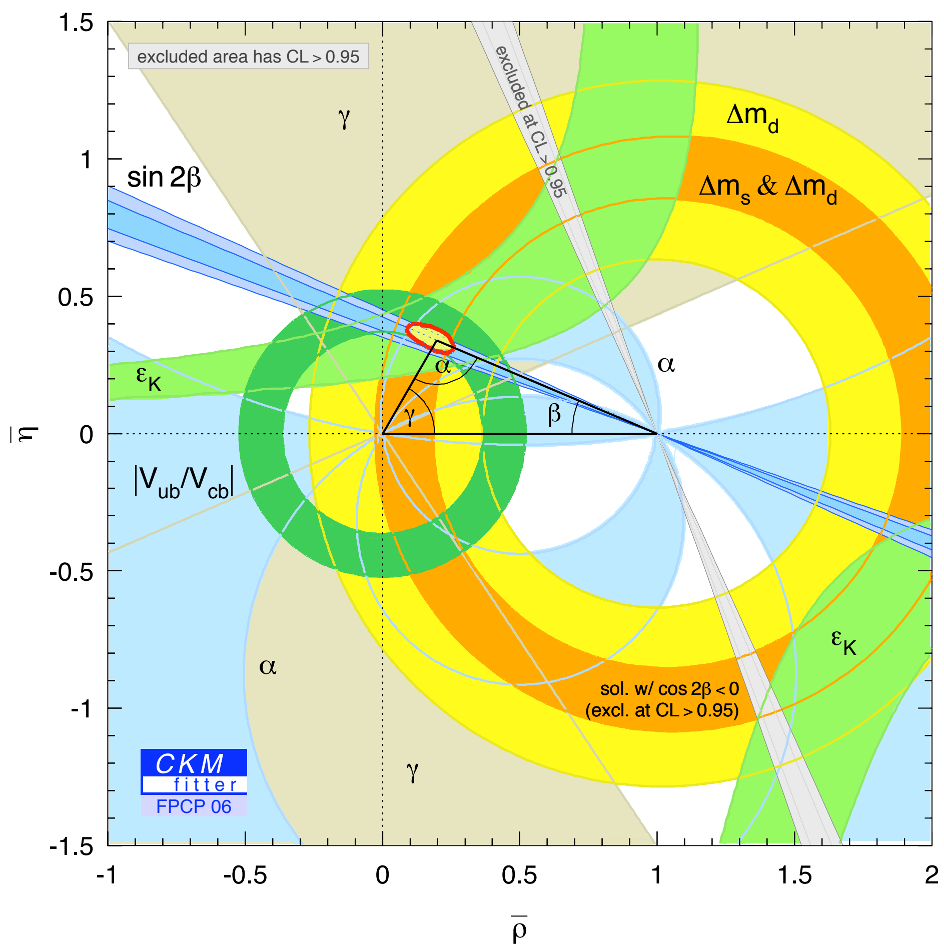

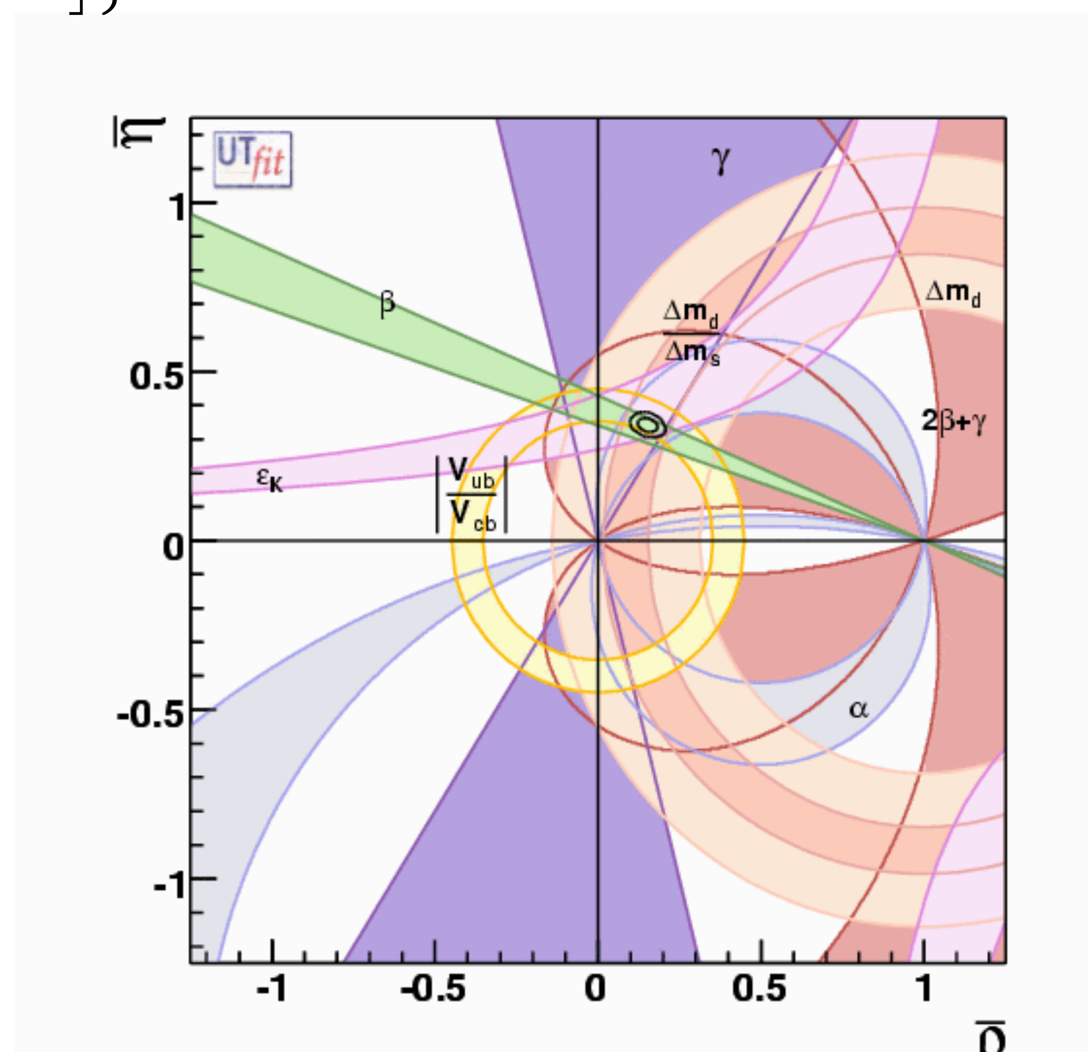

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B_K basics

B_K – basics

$$\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2 \, m_K^2}$$

operator

$$\begin{aligned} \hat{O}^{\Delta S=2} &= [\bar{s}(x)\, \gamma_\mu^L \, d(x)]\; [\bar{s}(x)\, \gamma_\mu^L \, d(x)] \\ &= O_{VV+AA} + O_{VA+AV} \end{aligned}$$

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$$O_{VV+AA} = [\bar{s}(x)\,\gamma_\mu\, d(x)]\ [\bar{s}(x)\,\gamma_\mu\, d(x)] + [\bar{s}(x)\,\gamma_\mu\gamma_5\, d(x)]\ [\bar{s}(x)\,\gamma_\mu\gamma_5\, d(x)]$$

P-even, contributes to B_K

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↑ P-even, contributes to B_K

$$O_{VA+AV} = [\bar{s}(x)\,\gamma_\mu\, d(x)]\,[\bar{s}(x)\,\gamma_\mu\gamma_5\, d(x)] + [\bar{s}(x)\,\gamma_\mu\gamma_5\, d(x)]\,[\bar{s}(x)\,\gamma_\mu\, d(x)]$$

↑ P-odd, no B_K contribution

B_K – a renormalisation classic

In the presence of explicit chiral symmetry breaking four-fermion operators of different chiralities mix under renormalisation.

Martinelli 1984; Bernard, Draper, (Hockney), Soni 1987, 1998;
Gupta et al. 1993; Donini et al. 1999

$$O^{\Delta S=2} = \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)]}_{O_{\mathrm{VV+AA}}} - \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu\gamma_5 d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu d)]}_{O_{\mathrm{VA+AV}}}$$

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$$\bar{O}_{\mathrm{VV+AA}} = \lim_{a\to 0} Z_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O_{\mathrm{VV+AA}}(a) + \sum_{k=1}^{4}\Delta_k(g_0^2)O_k(a)\right]$$

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Vanishes if chiral symmetry is preserved (at least partially)

Vanishes for staggered, GW, DW fermions

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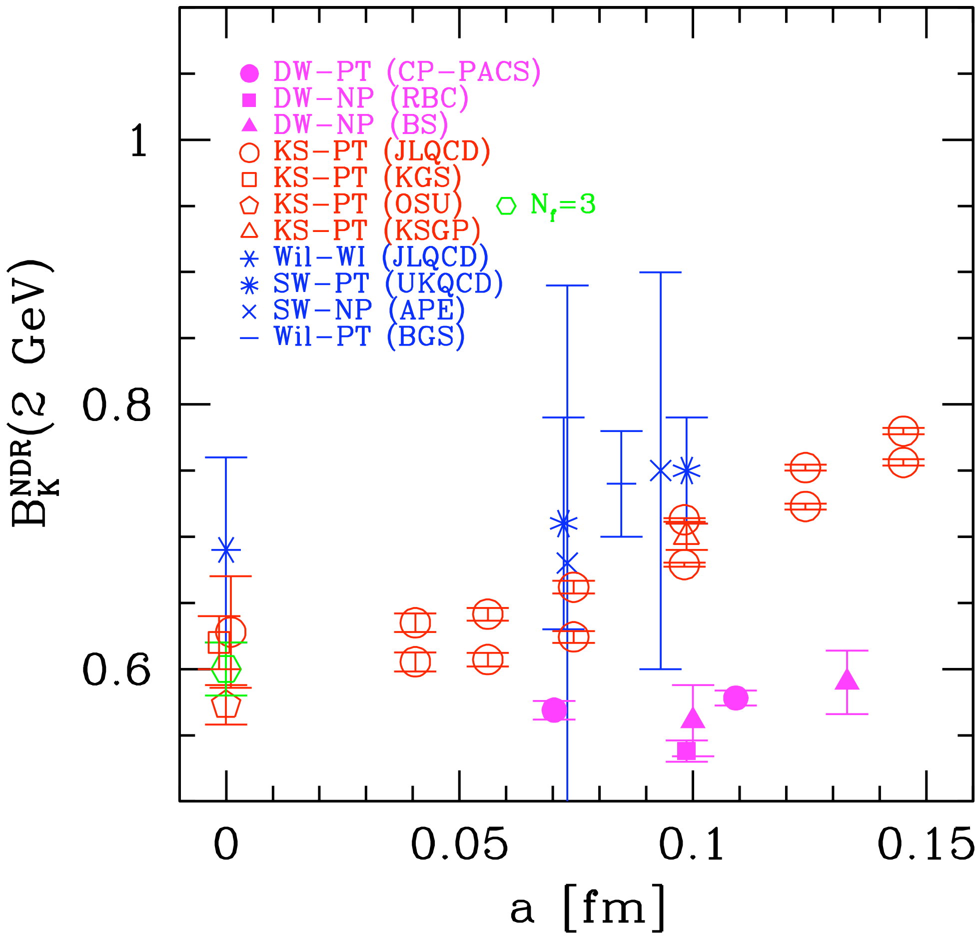
$$\bar{O}_{\rm VA+AV} = \lim_{a\to 0} Z_{\rm VA+AV}(g_0^2, a\mu)\, O_{\rm VA+AV}(a)$$

Protected from mixing by discrete symmetries *C P S(s ↔ d)*

B_K – a renormalisation classic

Subtractions flaw the quality of Wilson fermion results



L. Lellouch Nucl.Phys.Proc.Suppl.94(2001)142

There are two important sources of systematic error which would better be removed if Wilson fermion B_K determinations are to be on the same footing as the others:

1. Additive renormalization;
2. $O(a)$ discretization errors

Getting rid of mixing

- Straightforward option: preserve chiral symmetry — possibly exactly.

- Wilson 1: axial Ward identity (3-point function with O_{VV+AA} → 4-point function with O_{VA+AV}).

D.Becirevic et al. Phys.Lett.B487(2000)74; Eur.Phys.J.C37(2004)315

- Wilson 2: tmQCD (3-point function with O_{VA+AV}).

ALPHA Frezzotti, Grassi, Sint & Weisz, JHEP08(2001)058

Palombi, Pena, Sint JHEP 03 (2006) 089

ALPHA Guagnelli, Heitger, Pena, Sint, A.V. JHEP 03 (2006) 088

ALPHA Dimopoulos, Heitger, Palombi, Pena, Sint, A.V. NPB 749 (2006) 69

- tmQCD bonus: push safely towards low quark masses in quenched simulations.

B_K and tmQCD

B_K renormalization: four flavours

- four-fermion operator renormalization is best studied in general terms: we start with an operator with four distinct flavours, work out its renormalization properties and in the end identify the four flavours with physical ones.

$$Q_{\rm VV+AA} \; = \; [\bar\psi_1 \, \gamma_\mu \, \psi_2][\bar\psi_3 \, \gamma_\mu \, \psi_4] \; + \; [\bar\psi_1 \, \gamma_\mu \gamma_5 \, \psi_2][\bar\psi_3 \, \gamma_\mu \gamma_5 \, \psi_4] \; + \; (2 \leftrightarrow 4)$$

$$P_{21} \; = \; \bar\psi_2 \, \gamma_5 \, \psi_1 \qquad\qquad P_{43} \; = \; \bar\psi_4 \, \gamma_5 \, \psi_3$$

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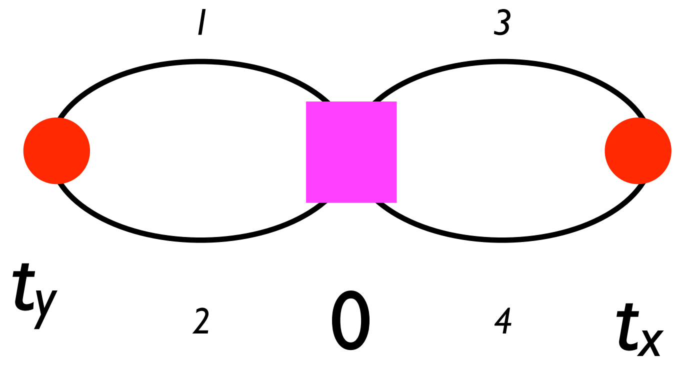
- with these definitions we can construct a correlation function with the same contractions as the 3-pt. function of B_K



$$\langle\, P_{21}\; Q_{\rm VV+AA}\; P_{43}\,\rangle$$

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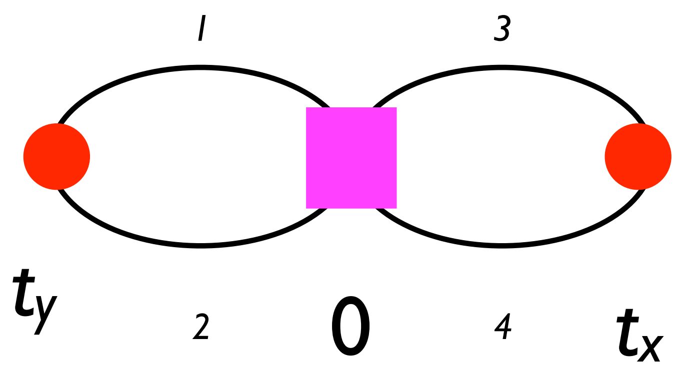
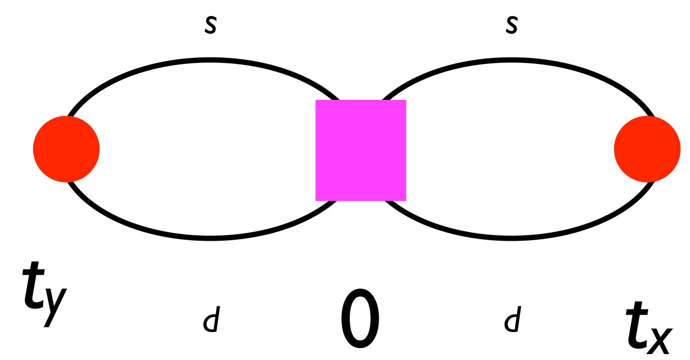


$$\langle\, P_{21}\; Q_{\mathrm{VV+AA}}\; P_{43} \,\rangle$$

$$\langle P_{sd}\; Q^{\Delta S=2}_{VV+AA}\; P_{sd} \rangle$$



- this object can be worked out, say, in tmQCD and then fields 1 and 3 are "identified" with the strange quark while fields 2 and 4 are "identified" with the down quark
- the flavour exchange $2 \leftrightarrow 4$ in the operator ensures that all Wick contractions are reproduced correctly

B_K renormalization and tmQCD

- we must now establish the relation between tmQCD and standard QCD operators
- consider chiral rotations which are **independent for each flavour**

$$\psi_f \;\rightarrow\; \exp\left[i\gamma_5 \frac{\alpha_f}{2}\right] \psi_f \qquad\qquad \bar{\psi}_f \;\rightarrow\; \bar{\psi}_f \; \exp\left[i\gamma_5 \frac{\alpha_f}{2}\right] \qquad\qquad f \;=\; 1, \cdots, 4$$

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- NB: these are not standard tmQCD rotations; the latter have an isospin τ^3 matrix
- they are called Osterwalder-Seiler rotations

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- under this change of basis, the relation between standard QCD and tmQCD operators is

$$\left[Q_{\mathrm{VV+AA}}\right]_{\mathrm{R}}^{\mathrm{QCD}} = \cos\left(\frac{\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4}{2}\right) \left[Q_{\mathrm{VV+AA}}\right]_{\mathrm{R}}^{\mathrm{tmQCD}} - i\sin\left(\frac{\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4}{2}\right) \left[Q_{\mathrm{VA+AV}}\right]_{\mathrm{R}}^{\mathrm{tmQCD}}$$

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- NB: these are not standard tmQCD rotations; the latter have an isospin τ^3 matrix
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- under this change of basis, the relation between standard QCD and tmQCD operators is

$$\left[Q_{\rm VV+AA}\right]_{\rm R}^{\rm QCD} = \cos\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\left[Q_{\rm VV+AA}\right]_{\rm R}^{\rm tmQCD} - i\sin\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\left[Q_{\rm VA+AV}\right]_{\rm R}^{\rm tmQCD}$$

- we must chose the angles so as to kill the cosine term on the rhs
- in this way the QCD quantity is computed in terms of a parity-odd tmQCD operator
- this ensures multiplicative renormalizability
- can such choices be made in a way that preserves the QCD-tmQCD equivalence?
- can we also ensure automatic improvement?

B_K renormalization and tmQCD

- we must now establish the relation between tmQCD and standard QCD operators
- consider chiral rotations which are **independent for each flavour**

$$\psi_f \;\rightarrow\; \exp\left[i\gamma_5\frac{\alpha_f}{2}\right]\,\psi_f \qquad\qquad \bar{\psi}_f \;\rightarrow\; \bar{\psi}_f\,\exp\left[i\gamma_5\frac{\alpha_f}{2}\right] \qquad\qquad f \;=\; 1,\cdots,4$$

- NB: these are not standard tmQCD rotations; the latter have an isospin τ^3 matrix
- they are called Osterwalder-Seiler rotations
- under this change of basis, the relation between standard QCD and tmQCD operators is

$$\left[Q_{\rm VV+AA}\right]_{\rm R}^{\rm QCD} = \cos\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\left[Q_{\rm VV+AA}\right]_{\rm R}^{\rm tmQCD} - i\sin\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\left[Q_{\rm VA+AV}\right]_{\rm R}^{\rm tmQCD}$$

- we would like to satisfy all 3 requirements:
  - "kill" the cosine; i.e. all four rotations angles sum to $\pm\pi/2$
  - ensure automatic improvement; i.e. each rotation angle is $\pm\pi/2$
  - angles have $\pm$ relative signs so that they can be organized in τ^3 isospin doublets
- this is impossible! C.Pena, S.Sint, A.V. , JHEP09(2004)069

B_K renormalization and tmQCD

- the QCD-tmQCD equivalence is preserved only with non singlet rotations; i.e. we need an isospin τ^3 matrix in the mass term of the action and isospin rotations
- the four flavours must be organized in tmQCD doublets

$$Q_{\rm VV+AA} = [\bar\psi_1\,\gamma_\mu\,\psi_2][\bar\psi_3\,\gamma_\mu\,\psi_4] + [\bar\psi_1\,\gamma_\mu\gamma_5\,\psi_2][\bar\psi_3\,\gamma_\mu\gamma_5\,\psi_4] + (2\leftrightarrow 4)$$

$$\Big[Q_{\rm VV+AA}\Big]_{\rm R}^{\rm QCD} = \cos\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\Big[Q_{\rm VV+AA}\Big]_{\rm R}^{\rm tmQCD} - i\sin\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\Big[Q_{\rm VA+AV}\Big]_{\rm R}^{\rm tmQCD}$$

- FIRST POSSIBILITY: $\alpha_1 = \alpha_3 = 0$ and $\alpha_2 = \alpha_4 = \pi/2$
- this corresponds to standard lattice QCD for strange quark and tmQCD for up/down quark

$$\mathcal{L} = \bar\psi\Big[D_W + i\mu_q\tau^3\gamma_5\Big]\psi + \bar s\Big[D_W + m_0\Big]s$$

$$\bar\psi = \begin{pmatrix}\bar u & \bar d\end{pmatrix}$$

- problem: no automatic improvement

ALPHA R.Frezzotti, P.Grassi, S.Sint & P.Weisz, JHEP08(2001)058

ALPHA P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 749 (2006) 69

B_K renormalization and tmQCD

- the QCD-tmQCD equivalence is preserved only with non singlet rotations; i.e. we need an isospin τ^3 matrix in the mass term of the action and isospin rotations
- the four flavours must be organized in tmQCD doublets

$$Q_{\mathrm{VV+AA}} = [\bar\psi_1\,\gamma_\mu\,\psi_2][\bar\psi_3\,\gamma_\mu\,\psi_4] + [\bar\psi_1\,\gamma_\mu\gamma_5\,\psi_2][\bar\psi_3\,\gamma_\mu\gamma_5\,\psi_4] + (2 \leftrightarrow 4)$$

$$\left[Q_{\mathrm{VV+AA}}\right]_{\mathrm{R}}^{\mathrm{QCD}} = \cos\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\left[Q_{\mathrm{VV+AA}}\right]_{\mathrm{R}}^{\mathrm{tmQCD}} - i\sin\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\left[Q_{\mathrm{VA+AV}}\right]_{\mathrm{R}}^{\mathrm{tmQCD}}$$

- SECOND POSSIBILITY: $\alpha_1 = \alpha_3 = -\pi/4$ and $\alpha_2 = \alpha_4 = \pi/4$
- this corresponds to tmQCD for strange/down quark (up may be added untwisted)

$$\mathcal{L} = \bar\psi\left[D_W + m_0 + i\mu_q\tau^3\gamma_5\right]\psi$$

$$\bar\psi = \begin{pmatrix}\bar s & \bar d\end{pmatrix}$$

- problem: no automatic improvement
- problem: only good for quenched-QCD with down/strange degeneracy or mixed actions with different sea/valence quark actions

ALPHA P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 749 (2006) 69

B_K renormalization and tmQCD

- the QCD-tmQCD equivalence is preserved only with non singlet rotations; i.e. we need an isospin τ^3 matrix in the mass term of the action and isospin rotations
- the four flavours must be organized in tmQCD doublets

$$Q_{\mathrm{VV+AA}} = [\bar{\psi}_1 \gamma_\mu \psi_2][\bar{\psi}_3 \gamma_\mu \psi_4] + [\bar{\psi}_1 \gamma_\mu\gamma_5 \psi_2][\bar{\psi}_3 \gamma_\mu\gamma_5 \psi_4] + (2 \leftrightarrow 4)$$

$$\Big[Q_{\mathrm{VV+AA}}\Big]_{\mathrm{R}}^{\mathrm{QCD}} = \cos\left(\frac{\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4}{2}\right)\Big[Q_{\mathrm{VV+AA}}\Big]_{\mathrm{R}}^{\mathrm{tmQCD}} - i\sin\left(\frac{\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4}{2}\right)\Big[Q_{\mathrm{VA+AV}}\Big]_{\mathrm{R}}^{\mathrm{tmQCD}}$$

- THIRD POSSIBILITY: treat sea quarks in standard tmQCD fashion and valence quarks in OS fashion (i.e. use a mixed action formulation) R. Frezzotti, G.C. Rossi, JHEP10 (2004) 070
- The **sea** quark flavours are regularized by the standard Wilson fermion action with a fully twisted mass term (standard tmQCD); e.g. for $N_f = 2$

$$\bar{\psi} = (\,\bar{u} \quad \bar{d}\,) \qquad\qquad \mathcal{L}_{tm} = \bar{\psi}\left[D_W + i\mu_q\tau^3\gamma_5\right]\psi$$

B_K renormalization and tmQCD

- the QCD-tmQCD equivalence is preserved only with non singlet rotations; i.e. we need an isospin τ^3 matrix in the mass term of the action and isospin rotations
- the four flavours must be organized in tmQCD doublets

$$Q_{\rm VV+AA} = [\bar\psi_1\,\gamma_\mu\,\psi_2][\bar\psi_3\,\gamma_\mu\,\psi_4] + [\bar\psi_1\,\gamma_\mu\gamma_5\,\psi_2][\bar\psi_3\,\gamma_\mu\gamma_5\,\psi_4] + (2\leftrightarrow 4)$$

$$\Big[Q_{\rm VV+AA}\Big]_{\rm R}^{\rm QCD} = \cos\Big(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\Big)\Big[Q_{\rm VV+AA}\Big]_{\rm R}^{\rm tmQCD} - i\sin\Big(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\Big)\Big[Q_{\rm VA+AV}\Big]_{\rm R}^{\rm tmQCD}$$

- THIRD POSSIBILITY: treat sea quarks in standard tmQCD fashion and valence quarks in OS fashion (i.e. use a mixed action formulation) R. Frezzotti, G.C. Rossi, JHEP10 (2004) 070
- Each **valence** quark flavour is regularized by the Osterwalder-Seiler (OS) variant of tmQCD
- Valence quarks enter with a distinct action for each flavour, which is **fully twisted**.
    - quark fields are not organized in isospin doublets (i.e. no τ^3)
    - there is a separate mass term for each flavour, μ_f may be negative, corresponding to twist angle $\alpha = -\pi/2$

$$\mathcal{L}_{OS} = \bar\psi_f\Big[D_W + i\mu_f\gamma_5\Big]\psi_f \qquad f = u,\,d,\,s\,\cdots$$

B_K renormalization and tmQCD

- the QCD-tmQCD equivalence is preserved only with non singlet rotations; i.e. we need an isospin τ^3 matrix in the mass term of the action and isospin rotations
- the four flavours must be organized in tmQCD doublets

$$Q_{\rm VV+AA} \;=\; [\bar\psi_1\,\gamma_\mu\,\psi_2][\bar\psi_3\,\gamma_\mu\,\psi_4] \;+\; [\bar\psi_1\,\gamma_\mu\gamma_5\,\psi_2][\bar\psi_3\,\gamma_\mu\gamma_5\,\psi_4] \;+\; (2\leftrightarrow 4)$$

$$\Big[Q_{\rm VV+AA}\Big]_{\rm R}^{\rm QCD} = \cos\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\Big[Q_{\rm VV+AA}\Big]_{\rm R}^{\rm tmQCD} - i\sin\left(\frac{\alpha_1-\alpha_2+\alpha_3-\alpha_4}{2}\right)\Big[Q_{\rm VA+AV}\Big]_{\rm R}^{\rm tmQCD}$$

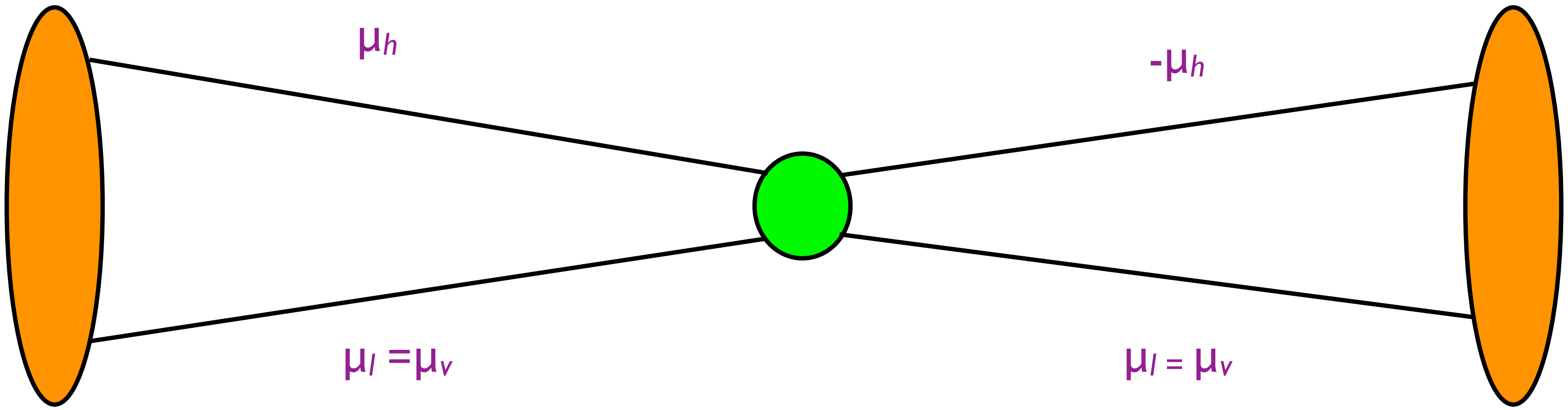
- THIRD POSSIBILITY: treat sea quarks in standard tmQCD fashion and valence quarks in IS fashion (i.e. use a mixed action formulation) R. Frezzotti, G.C. Rossi, JHEP10 (2004) 070
- for valence OS quarks set $\alpha_1 = \alpha_2 = \alpha_3 = \pi/2$ and $\alpha_4 = -\pi/2$
- operator is multiplicatively renormalizable
- improvement is automatic
- problem: unitarity is lost; recovered in the continuum limit
- problem: we have two types of pseudoscalar states (tmQCD and an OS) which are non-degenerate by $O(a^2)$ effects
- this calls for a full investigation

B_K renormalization and tmQCD

- mixed action formulation:

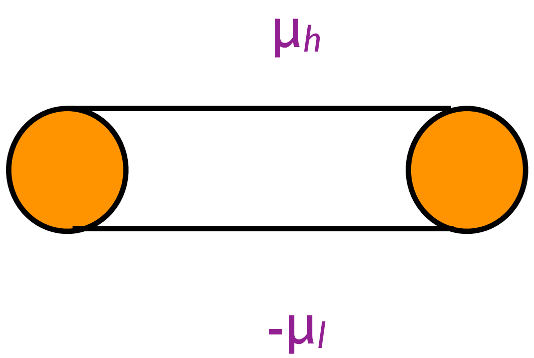
$$\mathcal{L}_{OS} = \bar{\psi}_f \left[D_W + i\mu_f \gamma_5 \right] \psi_f \qquad f = u, d, s \cdots$$

- suitable combinations of μ_f signs for each flavour ensure automatic improvement **and** multiplicative renormalization for say, B_K
- this is a compromise (unitarity issues arise when sea and valence flavours are treated differently)
- in a quenched or partially quenched setup ($N_f = 0,2$ sea quark flavours and a valence strange quark) this is unavoidable for any regularization

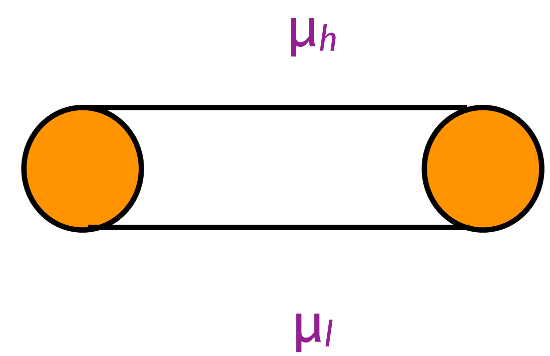


B_K renormalization and tmQCD

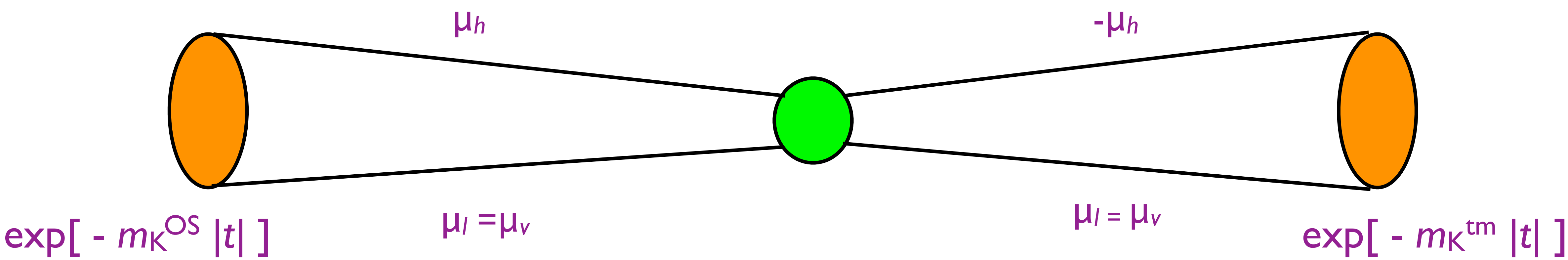
- Kaons and other mesons may be of the standard tmQCD variety or of OS type



$$m_K^{\rm tm} \;=\; m_K^{\rm OS} \;+\; O(a^2)$$



- B_K has a mixed tm-OS structure

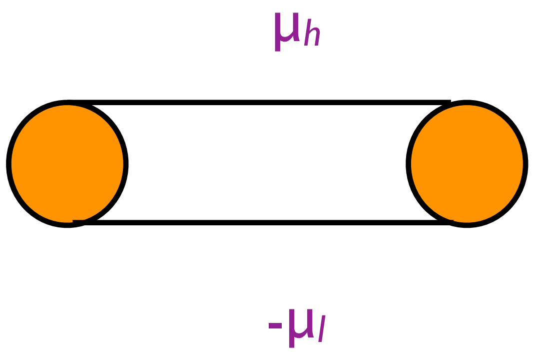


$$\frac{\langle W_K^{\rm OS}\; Q\; W_K^{\rm tm}\rangle}{\langle W_{0,K}^{\rm tm}\;\; A_{0,K}^{\rm tm}\rangle\;\langle A_{0,K}^{\rm OS}\;\; W_{0,K}^{\rm OS}\rangle} \;\;\rightarrow\;\; B_{\rm K} \;=\; \frac{3}{8}\,\frac{\langle \bar{K}^0 \mid Q \mid K^0\rangle}{[f_K^{\rm tm}\;\; m_K^{\rm tm}]\;[f_K^{\rm OS}\;\; m_K^{\rm OS}]}$$

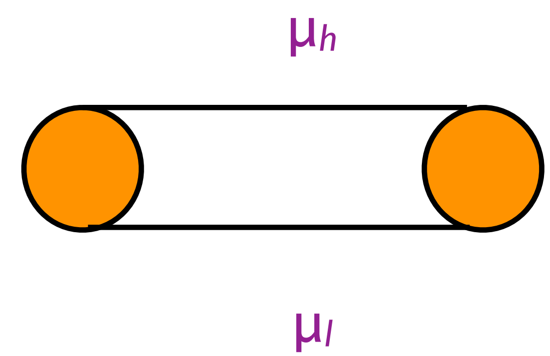
- the two exponential decays exp[- m_K^{tm} |t|] and exp[- m_K^{OS} |t|] cancel in the ratio but we are still left with an operator which injects an $O(a^2)$ energy

B_K renormalization and tmQCD

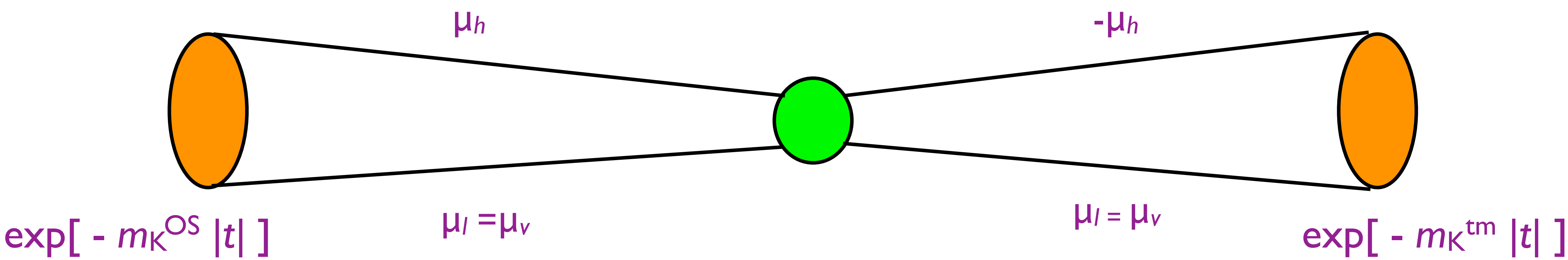
- Kaons and other mesons may be of the standard tmQCD variety or of OS type



$$m_K^{\rm tm} = m_K^{\rm OS} + O(a^2)$$



- B_K has a mixed tm-OS structure



$$\frac{\langle W_K^{\rm OS}\; Q\; W_K^{\rm tm}\rangle}{\langle W_{0,K}^{\rm tm}\; A_{0,K}^{\rm tm}\rangle\,\langle A_{0,K}^{\rm OS}\; W_{0,K}^{\rm OS}\rangle} \;\rightarrow\; B_{\rm K} = \frac{3}{8}\,\frac{\langle \bar{K}^0 \mid Q \mid K^0\rangle}{[f_K^{\rm tm}\; m_K^{\rm tm}]\;[f_K^{\rm OS}\; m_K^{\rm OS}]}$$

- NB: all operators above are renormalized (and thus continuum notation is used)
- the superscripts tm and OS denote the regularization that continuum quantities come from

B_K: quenched and twisted

M.Guagnelli, J.Heitger, C.Pena, S.Sint, A.V. JHEP 03 (2006) 088

F.Palombi, C.Pena, S.Sint JHEP 03 (2006) 089

P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 749 (2006) 69

P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 776 (2007) 258

ALPHA Collaboration

Quenched computation of B_K

M.Guagnelli, J.Heitger, C.Pena, S.Sint, A.V. JHEP 03 (2006) 088

F.Palombi, C.Pena, S.Sint JHEP 03 (2006) 089

P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 749 (2006) 69

P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 776 (2007) 258

- tmQCD → no operator mixing (no exceptional configurations).
- SF non-perturbative renormalisation.
- Various physical volumes: check control of finite volume effects.
- Two different regularisations: check control of the continuum limit.
- N.B.: action is O(a) improved, but four-fermion operator is *not* ⇒ continuum limit approached linearly in a.

Quenched computation of B_K

π/2 strategy:

$$S = \sum_{x,y} \{\bar{\psi}_\ell(x)\,[D_{\rm w,sw} + m_\ell + i\mu_\ell\gamma_5\tau_3]\,(x,y)\psi_\ell(y) \;+\; \bar{s}(x)\,[D_{\rm w,sw} + m_s]\,(x,y)s(y)\}$$

$m_\ell,\ \mu_\ell$ tuned to have $m_{\ell,\rm R} = 0$

π/4 strategy (specially devised for quenched case):

$$S = \sum_{x,y} \{\bar{\psi}(x)\,[D_{\rm w,sw} + m_{\hat{0}} + i\mu_{\hat{q}}\gamma_5\tau_3]\,(x,y)\psi(y)\}$$

$m_{\hat{0}},\ \mu_q$ tuned to have $m_{\rm R} = \mu_{\rm R}$

in both cases: $O_{\rm VV+AA} \xrightarrow{\text{twist}} O_{\rm VA+AV}$

NB: we never have *only* fully twisted quarks → “automatic” O(a) improvement argument does not apply.

Approach to continuum: non-perturbative renormalisation

- SF technique via finite size scaling: split renormalisation into
  - Renormalisation at a low, hadronic scale where contact with typical large-volume values of β is made.
  - NP running to very high scales (~100 GeV) where contact with PT is made.

$$\hat{B}_K = (\alpha_s(\mu))^{-\gamma_0/2b_0} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}g \left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\} \left[\lim_{a\to 0} Z(g_0^2, a\mu)\, B_K(a)\right]$$

Approach to continuum: non-perturbative renormalisation

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  - Renormalisation at a low, hadronic scale where contact with typical large-volume values of β is made.
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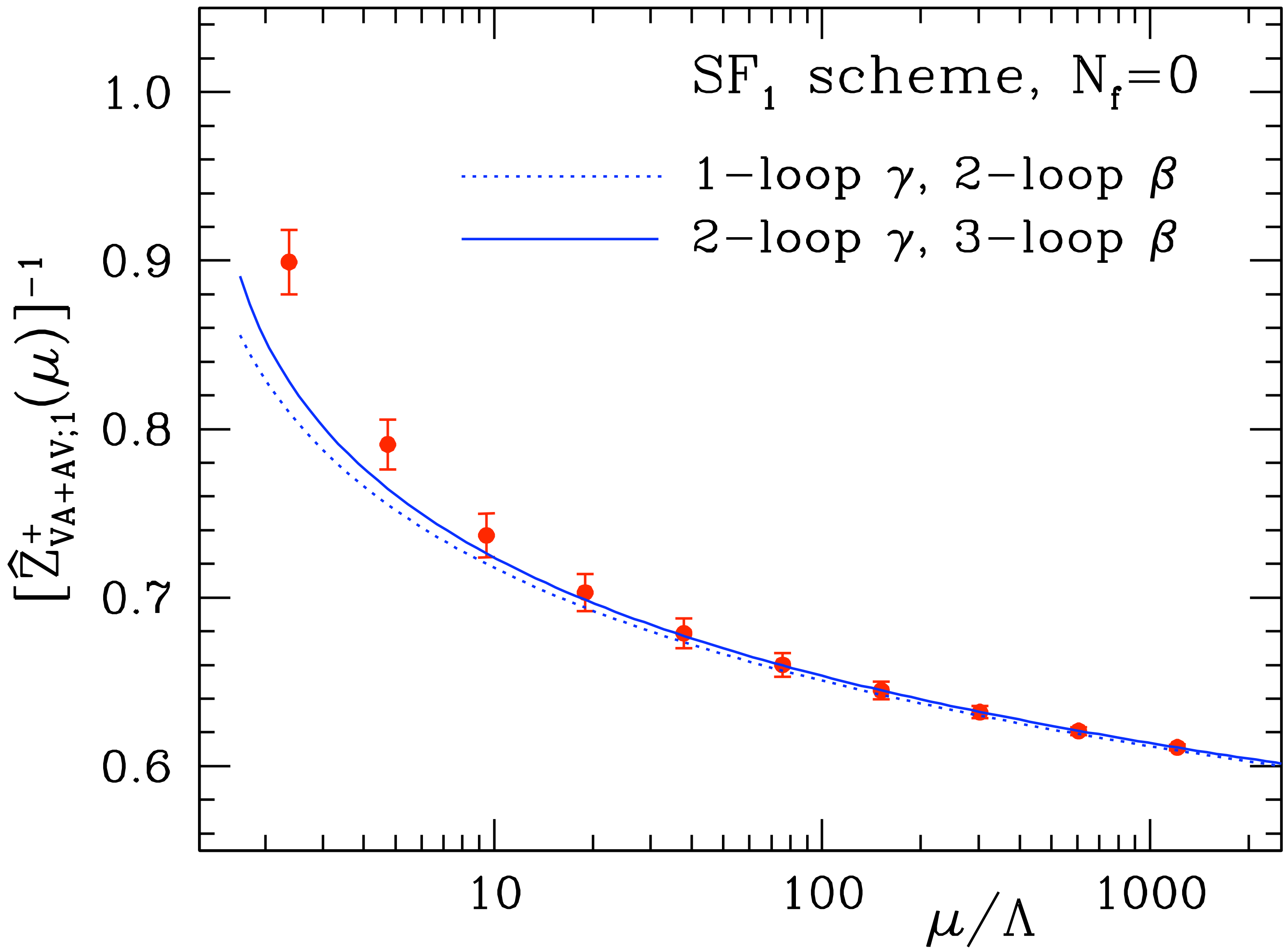
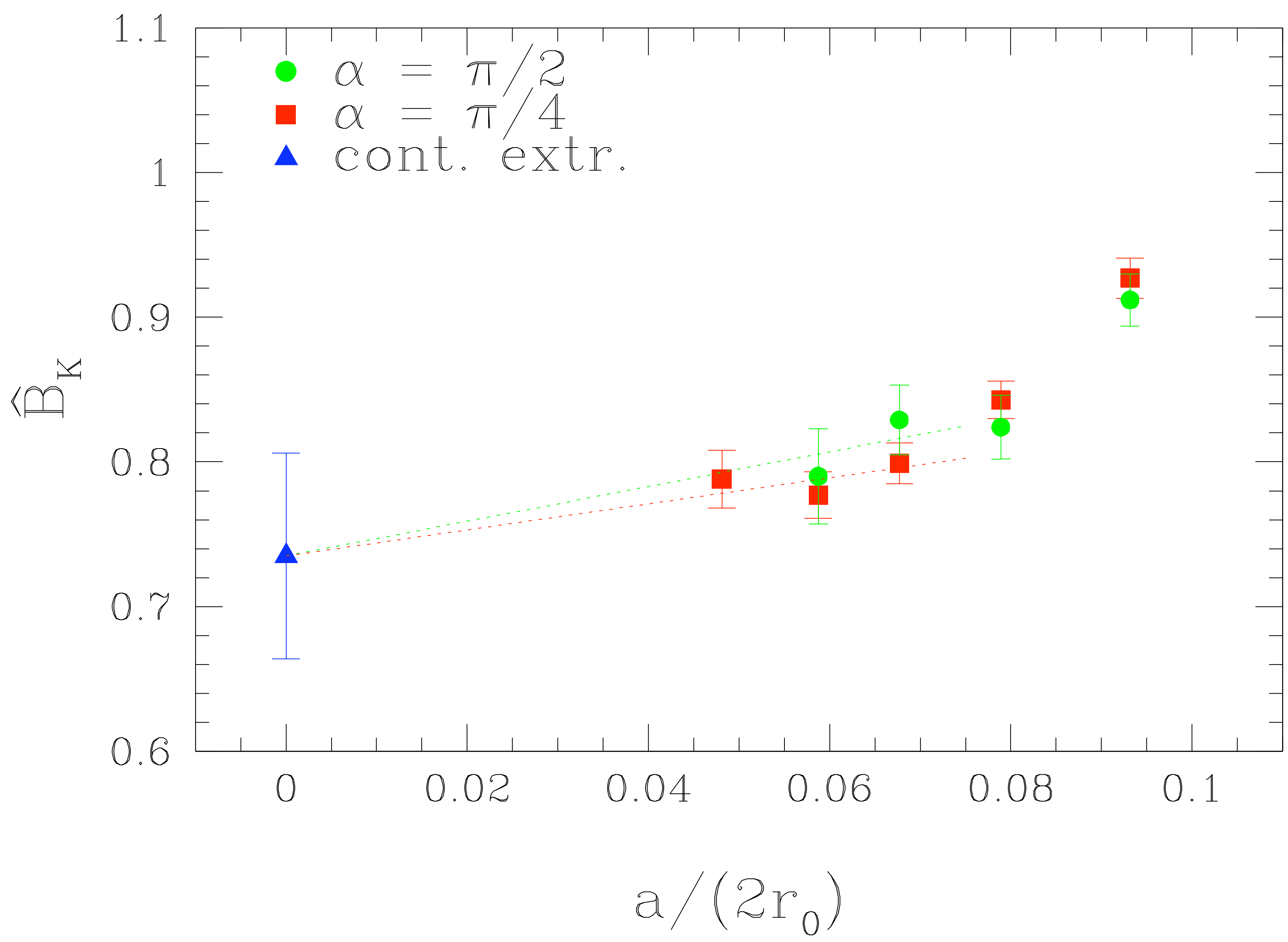


Continuum limit

- Combined linear extrapolation of the two regularisations, using ALPHA determination of current normalisations and improvement coefficients.

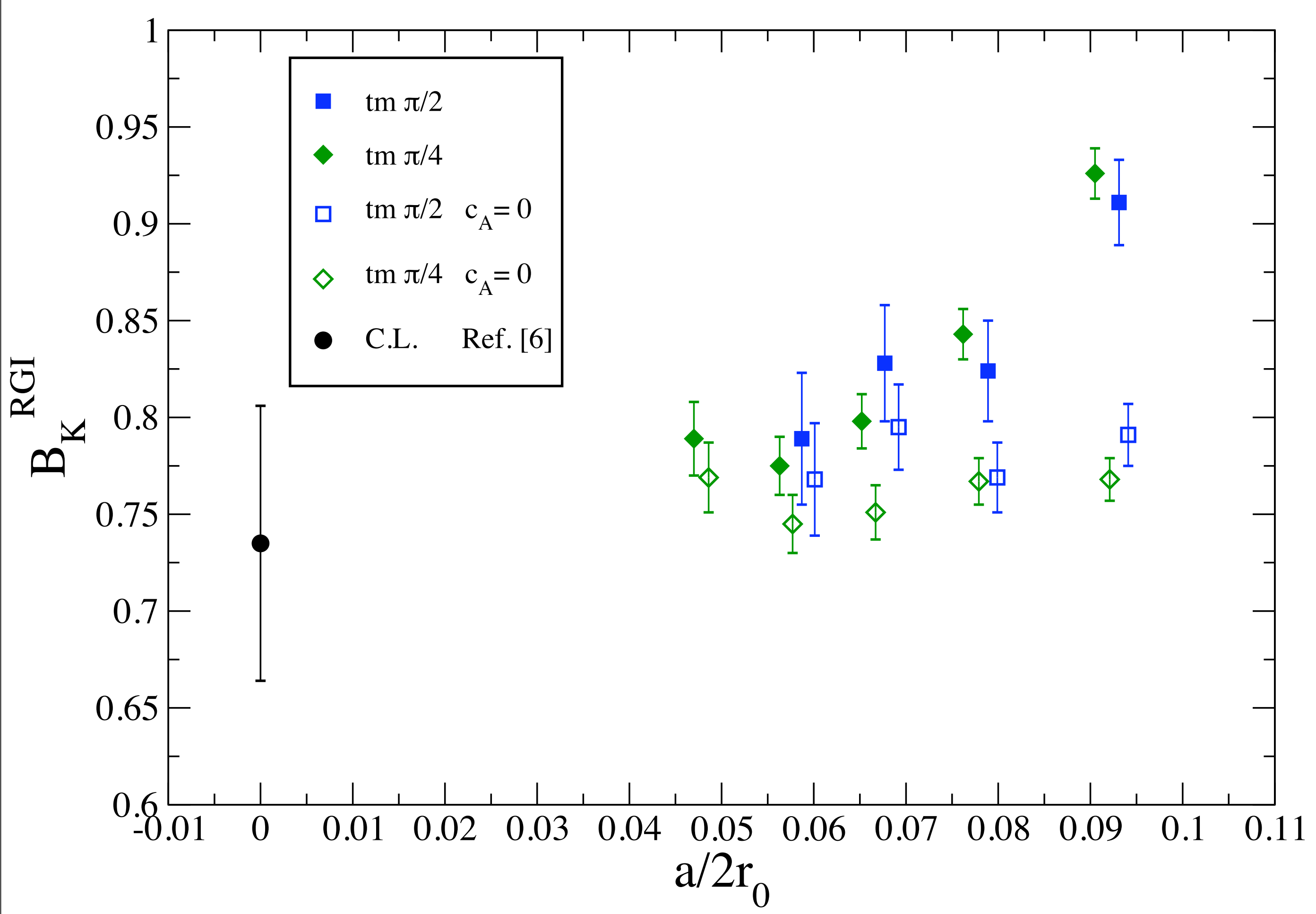


$$\hat{B}_K = 0.735(71)$$

$$\bar{B}_K^{\overline{\rm MS}}(2\,{\rm GeV}) = 0.534(52)$$

Continuum limit

- why the curvature? This $O(a)$ effect can be tampered with, through one Symanzik counter-term of the axial current



- π/2

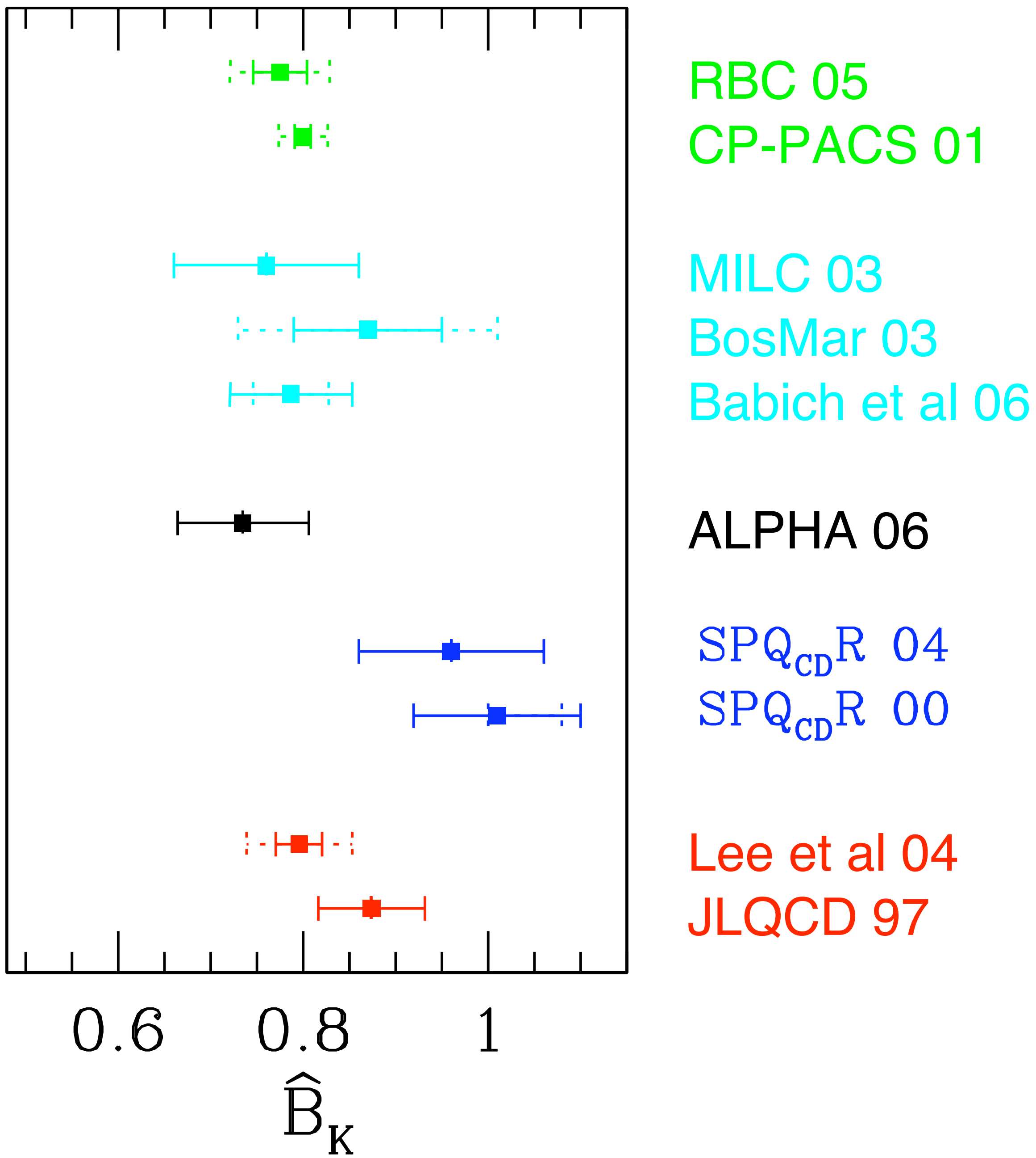
$$(A_R)_{\mu,sd} = Z_A\left[1+\frac{1}{2}b_A a m_{q,s}\right]\left[A_{\mu,sd} + c_A a\tilde{\partial}_\mu P_{sd} - i\frac{1}{2}a\mu_l \tilde{b}_A V_{\mu,sd}\right],$$
$$(V_R)_{\mu,sd} = Z_V\left[1+\frac{1}{2}b_V a m_{q,s}\right]\left[V_{\mu,sd} + c_V a\tilde{\partial}_\nu T_{\mu\nu,sd} - i\frac{1}{2}a\mu_l \tilde{b}_V A_{\mu,sd}\right]$$

- π/4

$$(A_R)_{\mu,sd} = Z_A[1+b_A a m_q][A_{\mu,sd} + c_A a\tilde{\partial}_\mu P_{sd} - ia\mu_l \tilde{b}_A V_{\mu,sd}],$$
$$(V_R)_{\mu,sd} = Z_V[1+b_V a m_q][V_{\mu,sd} + c_V a\tilde{\partial}_\nu T_{\mu\nu,sd} - ia\mu_l \tilde{b}_V A_{\mu,sd}]$$

- there seems to be a cancellation effect between $O(a)$ effects in numerator and denominator of B_K

Comparison with quenched literature



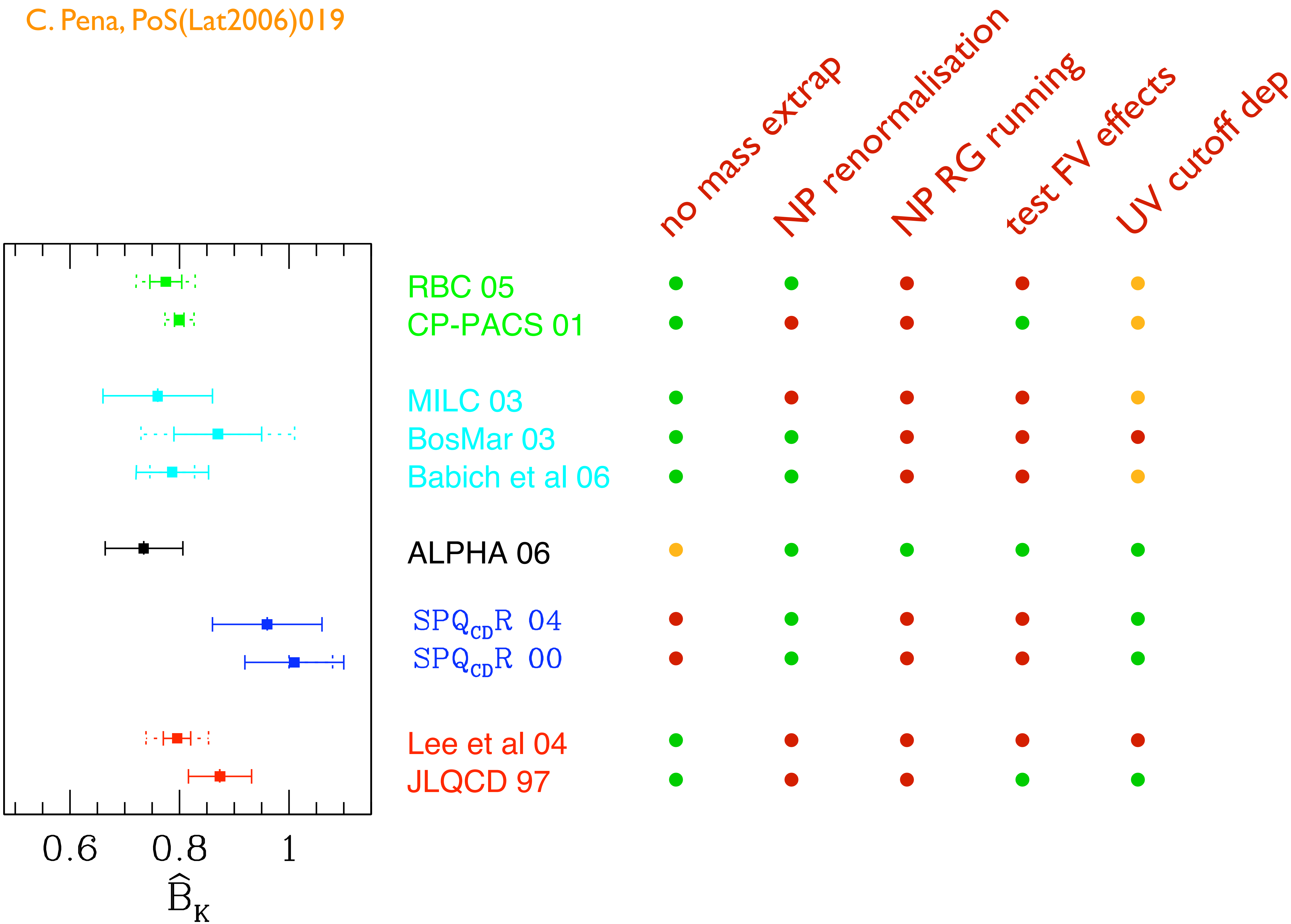
$$\hat{B}_K = 0.735(71)$$

$$\bar{B}_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.534(52)$$

Difference with other Wilson fermion computations mainly due to method employed to extract B_K.

Comparison with quenched literature

C. Pena, PoS(Lat2006)019



B_K: quenched, twisted and "improved"

ALPHA Collaboration

P. Dimopoulos, H.Simma, A.V., JHEP07 (2009) 007

tm - OS pseudoscalar meson mass splitting

- the tm-OS mass splitting may be due to two factors:
  - the presence of the Clover term in the action
  - the way maximal twist is imposed (i.e. the way κ_{crit} is determined)
- different groups made different choices, so comparison is possible

K. Jansen et al., Phys.Lett.B624(2005)334

- no Clover

- "optimal" κ_{crit}

$$m_{\rm PCAC} = \frac{\partial_0 \langle A_0 \; P \rangle}{\langle P \; P \rangle} \propto \left[\frac{\partial_0 \langle V_0 \; P \rangle}{\langle P \; P \rangle} \right]^{\rm cont}$$

A.M.Abdel-Rehim, R.Lewis, R.M.Woloshyn, J.M.S.Wu, Phys.Rev.D74(2006)014507

- no Clover

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$$\langle A_0 \; P \rangle \propto \left[\langle V_0 \; P \rangle \right]^{\rm cont}$$

D. Becirevic et al., Phys.Rev.D74(2006)034501

- Clover

- Wilson (untwisted) κ_{crit} from PCAC with pbc's

P. Dimopoulos, H.Simma, A.V. JHEP; JHEP07 (2009) 007

- Clover

- Wilson (untwisted) κ_{crit} from PCAC with SFbc's

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A.M.Abdel-Rehim, R.Lewis, R.M.Woloshyn, J.M.S.Wu, Phys.Rev.D74(2006)014507

- no Clover

- "optimal" κ_{crit}

$$\langle A_0 \; P \rangle \propto \left[\langle V_0 \; P \rangle \right]^{\mathrm{cont}}$$

NB: κ_{crit} unimproved; no Symanizk counterterms used
(but automatic improvement OK for hadron masses, WMEs etc)
NB: renormalization worked out from scratch in tmQCD

tm - OS pseudoscalar meson mass splitting

- the tm-OS mass splitting may be due to two factors:
  - the presence of the Clover term in the action
  - the way maximal twist is imposed (i.e. the way κ_{crit} is determined)
- different groups made different choices, so comparison is possible

NB: κ_{crit} improved; need two Symanzik counterterms, c_{SW}, c_A
NB: all renormalizations taken from untwisted theory

D. Becirevic et al., Phys.Rev.D74(2006)034501

- Clover
- Wilson (untwisted) κ_{crit} from PCAC with pbc's

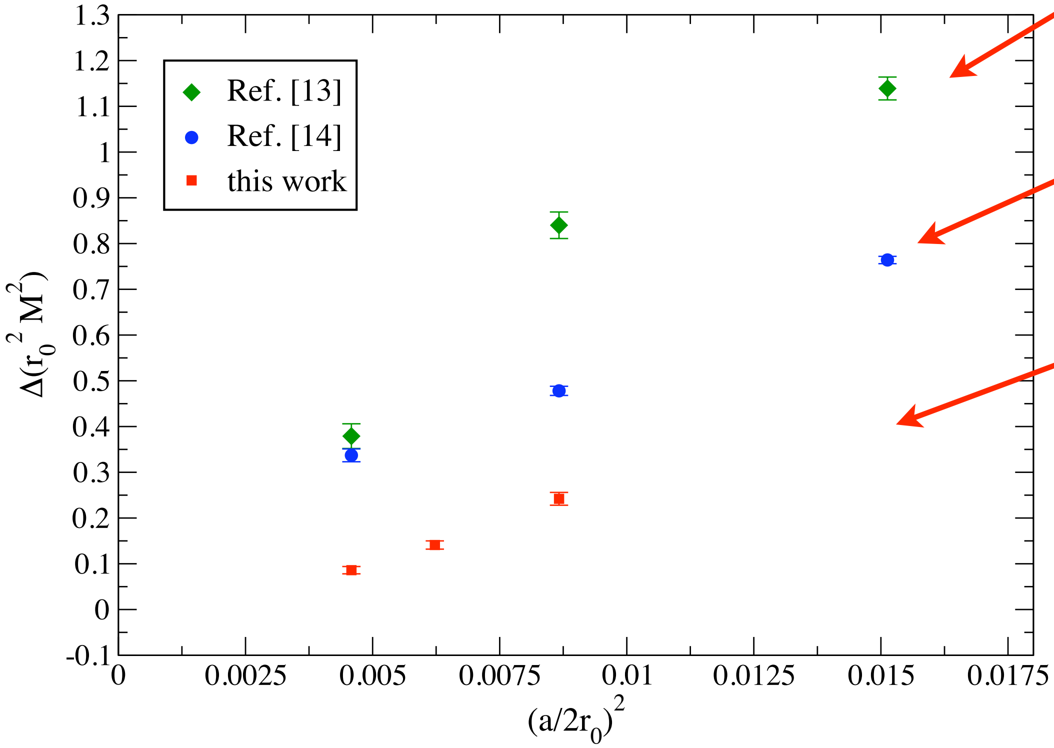
P. Dimopoulos, H.Simma, A.V. JHEP; JHEP07 (2009) 007

- Clover
- Wilson (untwisted) κ_{crit} from PCAC with SFbc's

tm - OS pseudoscalar meson mass splitting

- Comparison of these works for M_{PS}^{tm} and M_{PS}^{OS} in the Kaon mass range
- all determinations of M_{PS}^{tm} are compatible, discrepancies are found between the various M_{PS}^{OS}
- the M_{PS}^{tm} and M_{PS}^{OS} mass splitting is expressed in terms of the quantity:

$$\Delta(r_0^2 M^2) \;\equiv\; [r_0 M^{\rm OS}]^2 \;-\; [r_0 M^{\rm tm}]^2$$



K. Jansen et al., Phys.Lett.B624(2005)334

A.M.Abdel-Rehim, R.Lewis, R.M.Woloshyn, J.M.S.Wu, Phys.Rev.D74(2006)014507

P. Dimopoulos, H.Simma, A.V. JHEP

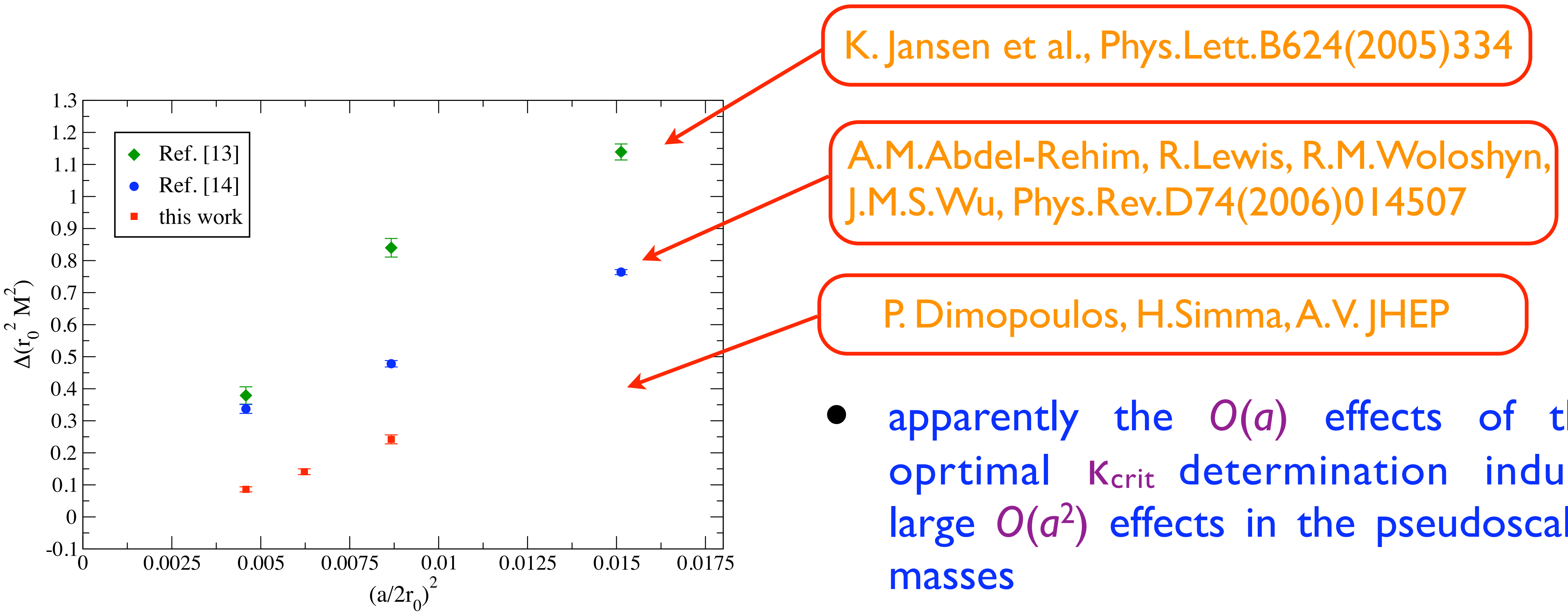
- non-Clover results with different "optimal" κ_{crit} determinations show discrepancies at larger lattice spacings

- Clover results are not sensitive to the details of the κ_{crit} determination: at β=6.0, D. Becirevic et al find $\Delta(r_0^2 M^2) \approx 0.27$ against our $\Delta(r_0^2 M^2) \approx 0.25$

tm - OS pseudoscalar meson mass splitting

- Comparison of these works for M_{PS}^{tm} and M_{PS}^{OS} in the Kaon mass range
- all determinations of M_{PS}^{tm} are compatible, discrepancies are found between the various M_{PS}^{OS}
- the M_{PS}^{tm} and M_{PS}^{OS} mass splitting is expressed in terms of the quantity:

$$\Delta(r_0^2 M^2) \equiv [r_0 M^{\mathrm{OS}}]^2 - [r_0 M^{\mathrm{tm}}]^2$$

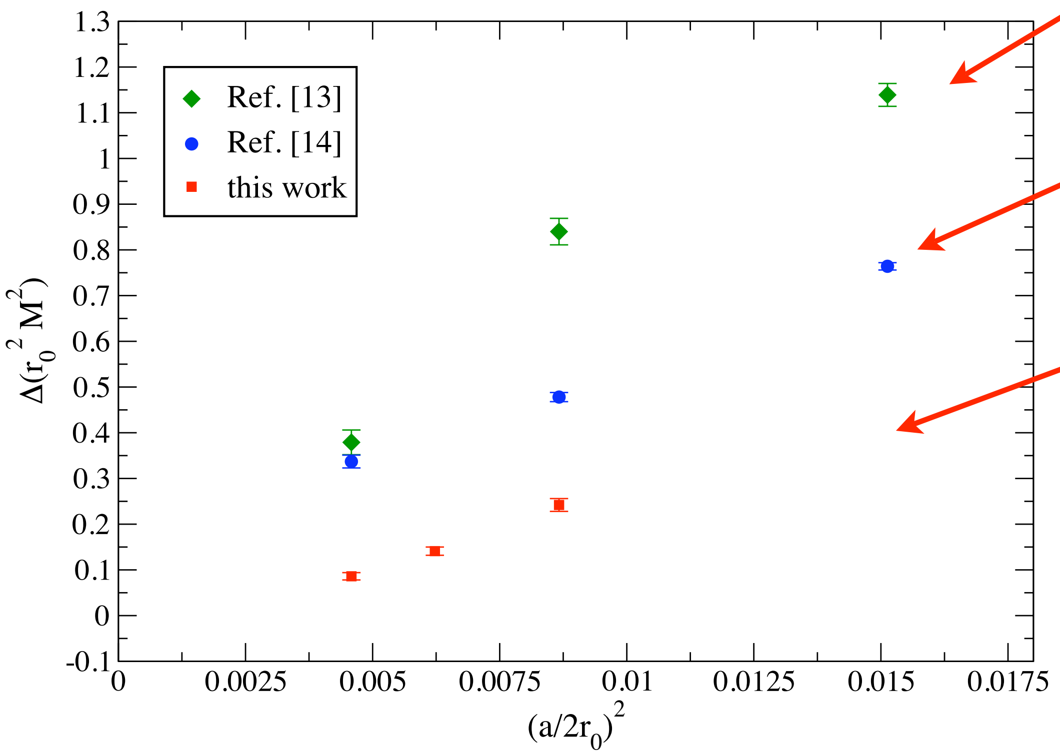


- apparently the $O(a)$ effects of the oprtimal κ_{crit} determination induce large $O(a^2)$ effects in the pseudoscalar masses
- these are probably milder once the Clover term is introduced

tm - OS pseudoscalar meson mass splitting

- Comparison of these works for M_{PS}^{tm} and M_{PS}^{OS} in the Kaon mass range
- all determinations of M_{PS}^{tm} are compatible, discrepancies are found between the various M_{PS}^{OS}
- the M_{PS}^{tm} and M_{PS}^{OS} mass splitting is expressed in terms of the quantity:

$$\Delta(r_0^2 M^2) \;\equiv\; [r_0 M^{\rm OS}]^2 \;-\; [r_0 M^{\rm tm}]^2$$



K. Jansen et al., Phys.Lett.B624(2005)334

A.M.Abdel-Rehim, R.Lewis, R.M.Woloshyn, J.M.S.Wu, Phys.Rev.D74(2006)014507

P. Dimopoulos, H.Simma, A.V. JHEP

- clearly a cutoff effect (not enough resolution to see it vanish in C.L.)
- <u>Clover data have significantly reduced mass splitting</u>

- NB: unquenched non-Clover ETMC data also show large splitting (see later)

tm - OS B_K parameter

- B_K is obtained in standard fashion as ratio of 2-pt. to 2-pt. correlation functions (SF variety)
- the novelty is in the tm -OS combination of valence quark propagators

$$R_{B_{\rm K}} = \frac{i\ Z_{VA+AV}\ \langle \bar{K}^0 | Q_{VA+AV} | K^0 \rangle}{(8/3)\ iZ_V\ [\langle 0 | V_0 | K^0 \rangle]^{\rm tm}\ Z_A\ [\langle K^0 | A_0 | 0 \rangle]^{\rm OS}}$$

- all Z 's from previous SF computations in (untwisted) Wilson theory
- the same holds for the continuum anomalous dimension of the 4-fermion operator
- all matrix elements are fully twisted, therefore $O(a)$ improved (automatically)
- Z_{VA+AV} has $O(a\,\Lambda_{\rm QCD})$ which are expected to be subdominant
- at each gauge coupling, simulations are carried out at a couple of degenerate quark mass values which give a K-meson close to its physical value

tm - OS B_K parameter

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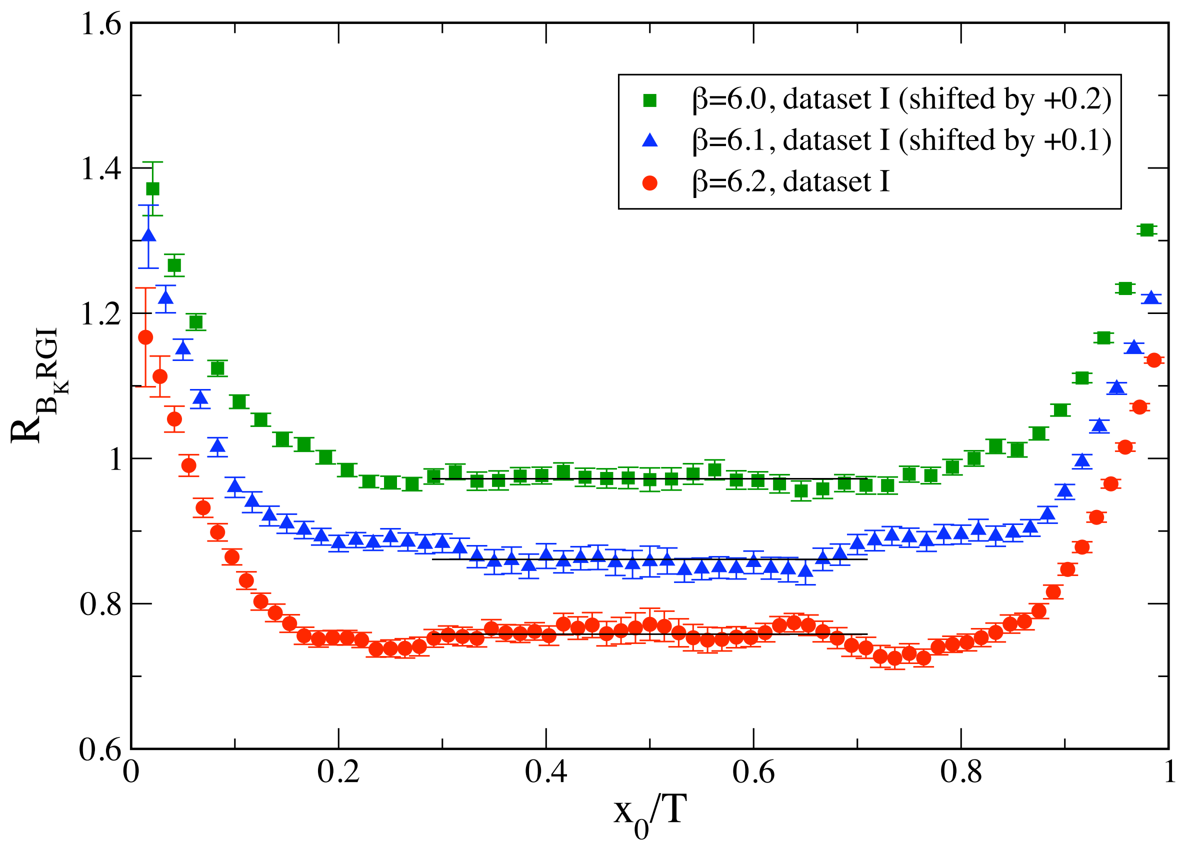
- the novelty is in the tm -OS combination of valence quark propagators

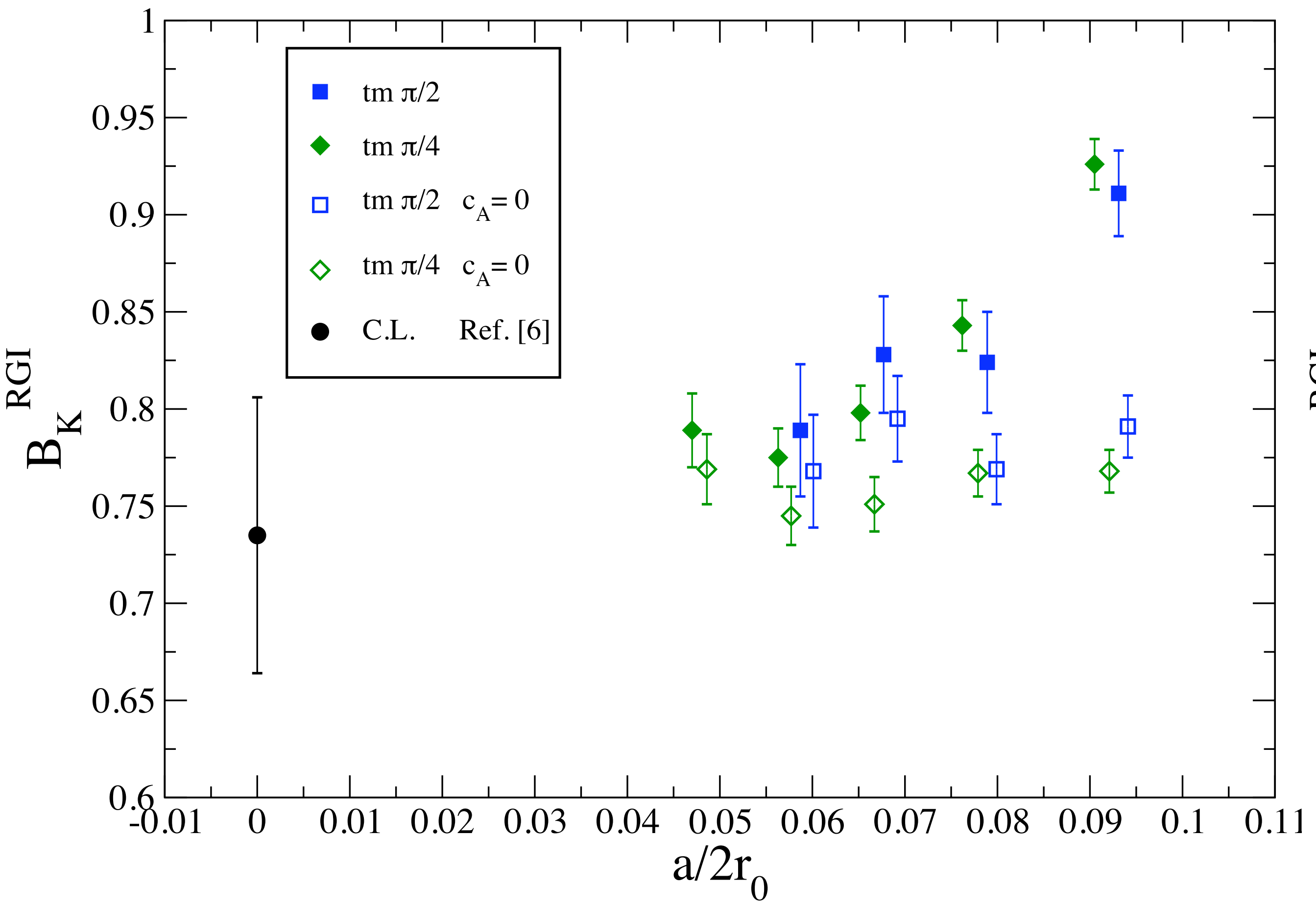
$$R_{B_{\mathrm{K}}} \quad = \quad \frac{i\ Z_{VA+AV}\ \langle \bar{K}^0 | Q_{VA+AV} | K^0 \rangle}{(8/3)\ iZ_V\ [\langle 0|V_0|K^0\rangle]^{\mathrm{tm}}\ Z_A\ [\langle K^0|A_0|0\rangle]^{\mathrm{OS}}}$$

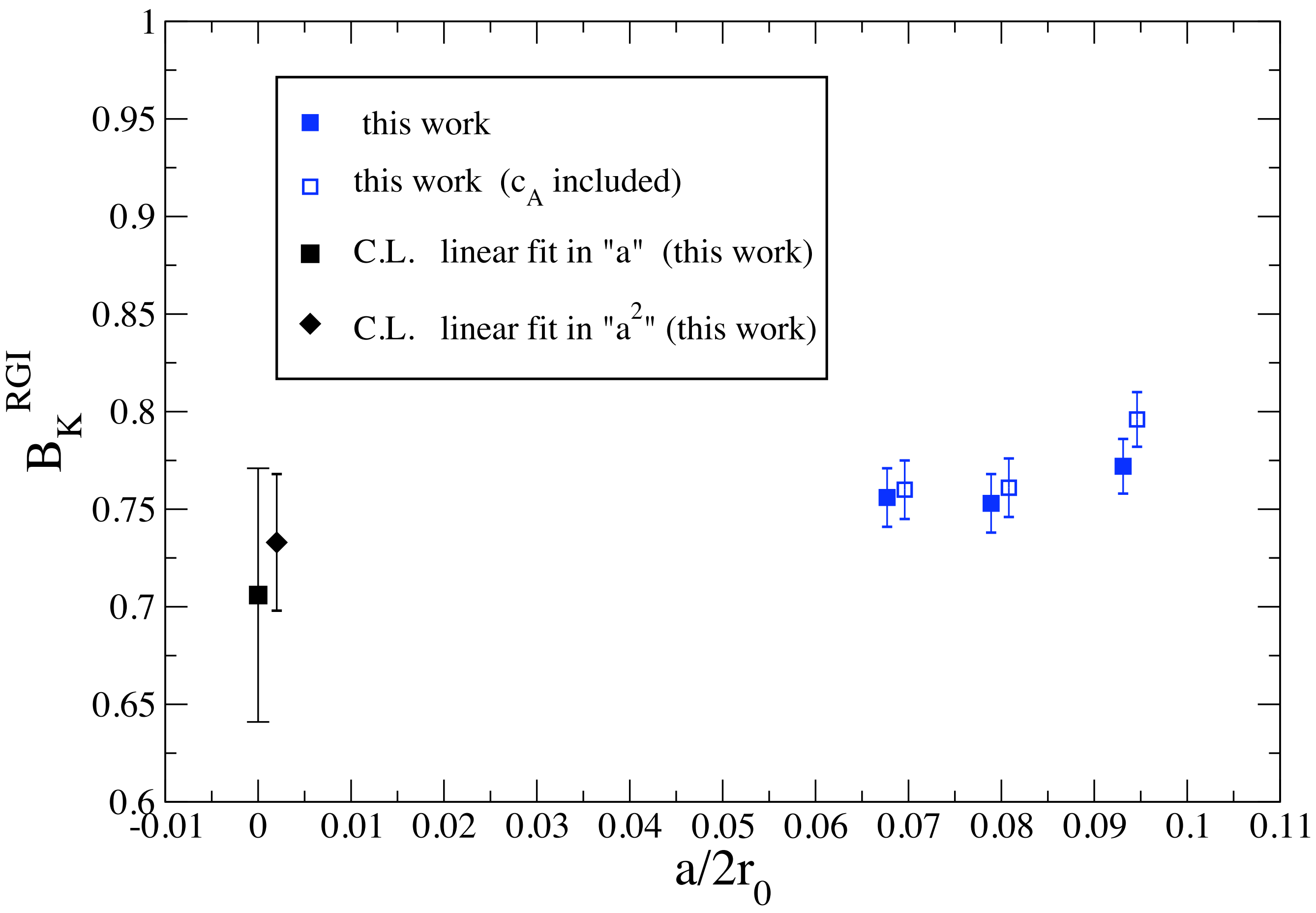
- as this is the first simulation of its kind, the question arises naturally: do we have a signal?



tm - OS B_K parameter

- comparison with our earlier quenched, unimproved B_K results



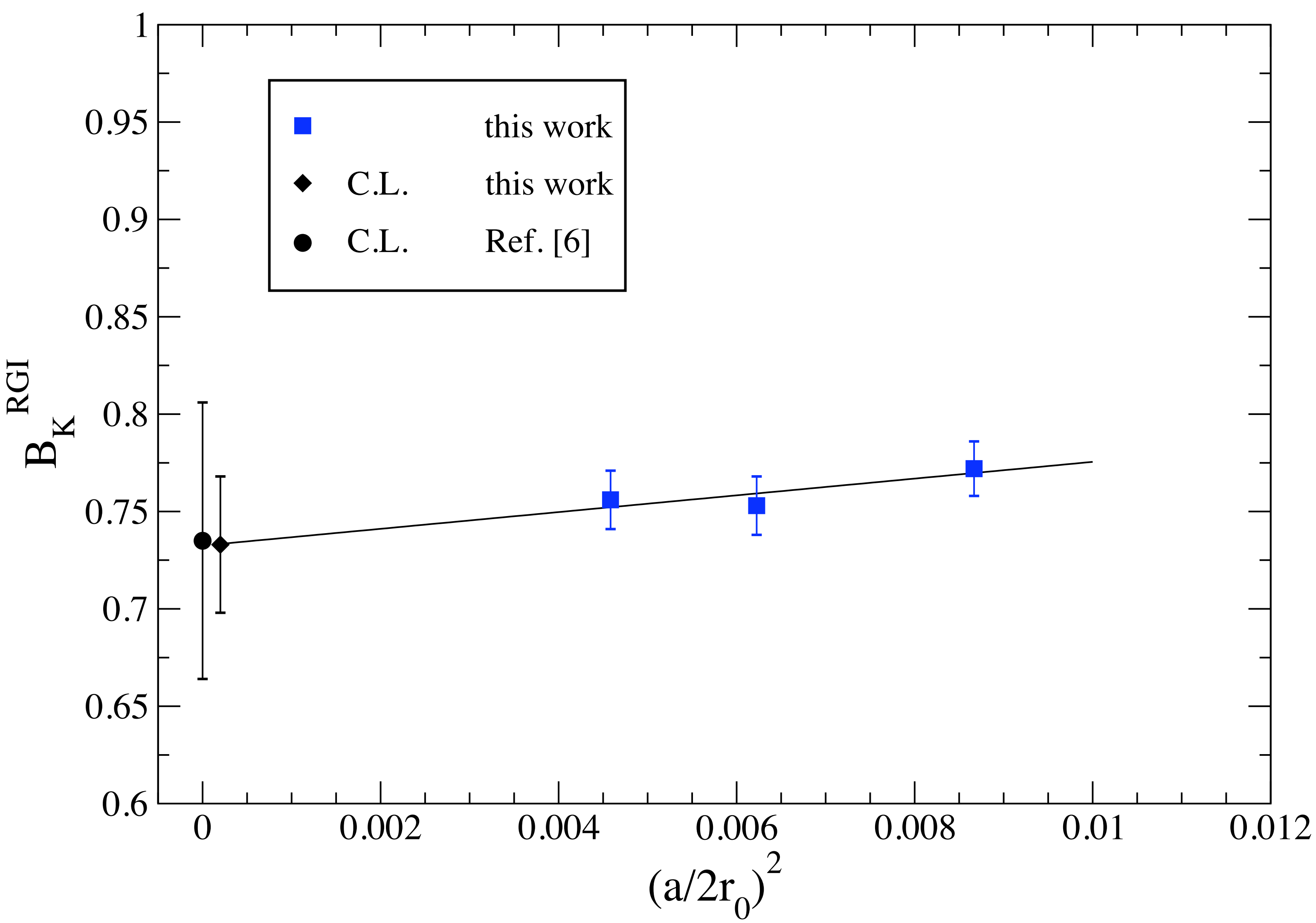


- c_A influences scaling properties; it is an $O(a)$ correcting term, which somehow spoils a cancellation mechanism between discretization errors in B_K numerator and denominator

- c_A has negligible influence; it is an $O(a^2)$ term

tm - OS B_K parameter

- continuum extrapolation of B_K results



- this behaviour suggests that $O(a\ \Lambda_{QCD})$ effects of Z_{VA+AV} are presumably less dominant than $O(a^2\ \mu^2)$ effects

- linear in a $\quad B_K^{RGI} = 0.706(65) \quad (\chi^2/dof = 0.30)$
- linear in a^2 $\quad B_K^{RGI} = 0.733(34) \quad (\chi^2/dof = 0.26)$

P.Dimopoulos et al.,NPB 776 (2007) 258 $\quad B_K^{RGI} = 0.735(71)$

B_K: partially quenched (N_f=2), twisted and improved

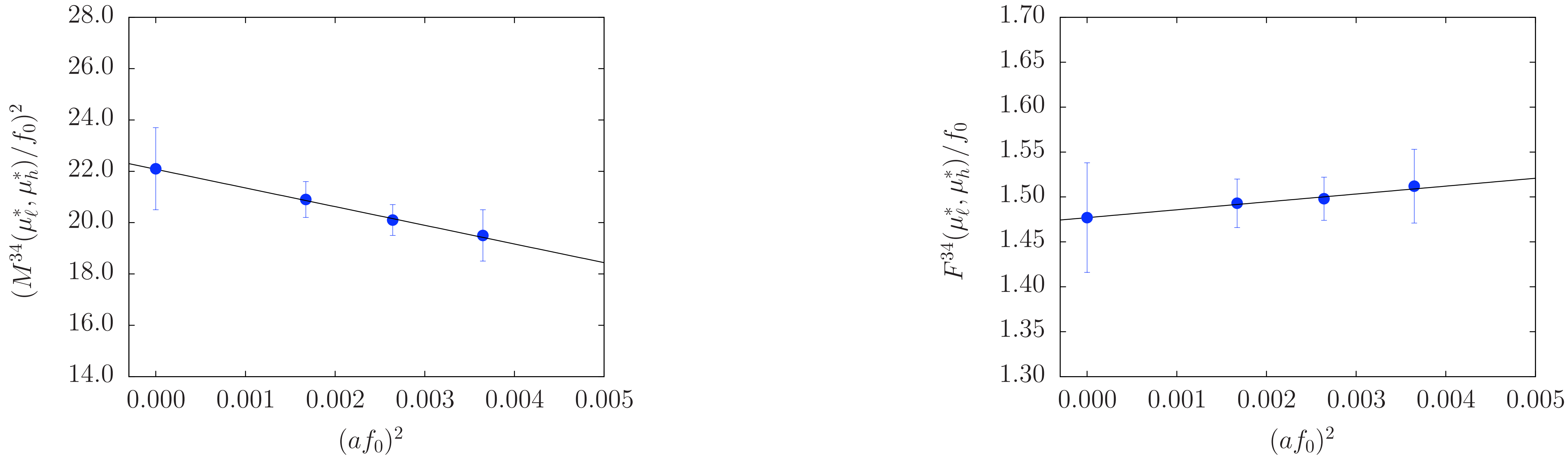
M. Constantinou et al., PRD83 (2011) 014505

The Simulation

- The $N_f = 2$ ETMC runs are performed at three gauge couplings β.
    - $\beta = 3.80$, corresponding to $a \approx 0.10$ fm [i.e. $1/a \approx 2.0$ GeV] $V = 24^3 \times 48$
    - $\beta = 3.90$, corresponding to $a \approx 0.09$ fm [i.e. $1/a \approx 2.2$ GeV] $V = 24^3 \times 48$ & $32^3 \times 64$
    - $\beta = 4.05$, corresponding to $a \approx 0.07$ fm [i.e. $1/a \approx 2.8$ GeV] $V = 32^3 \times 64$
- 3-4 sea quark masses = light valence quark masses in the range 280 MeV $\leq m_{PS} \leq$ 550 MeV
- @ each light quark, 3 heavy (strange) quark masses in the range 450 MeV $\leq m_{PS} \leq$ 700 MeV

Pseudoscalar meson mass splitting

- scaling tests performed at fixed quark masses $\mu_l \sim 40$ MeV and $\mu_h \sim 90$ MeV

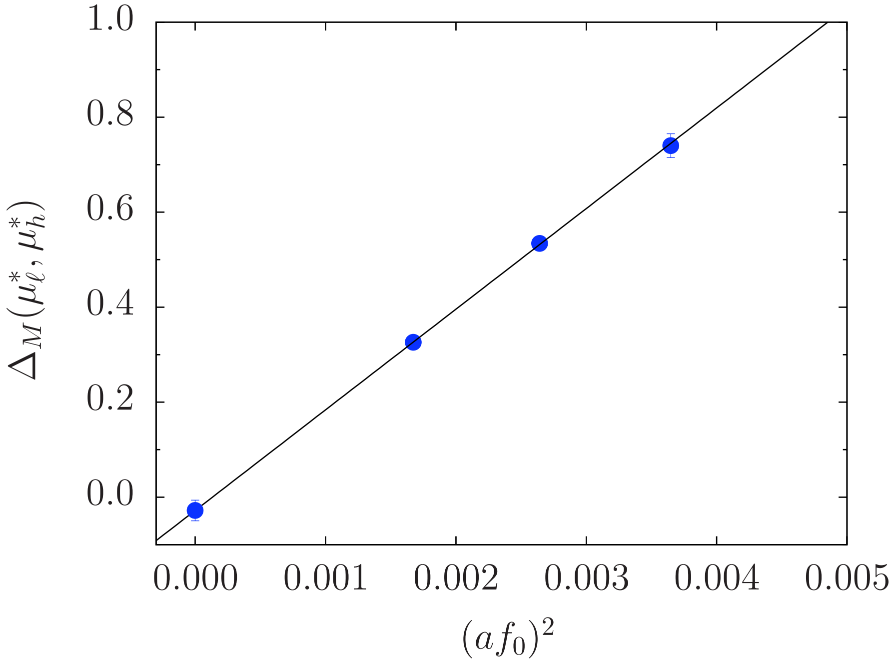


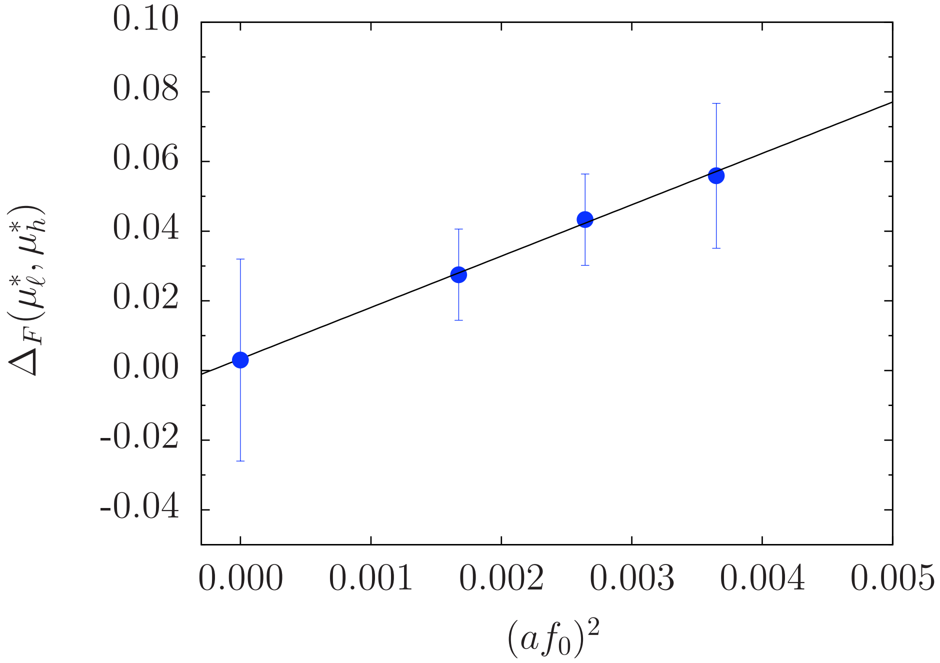
- small (a few %) scaling violations of the ("true") tmQCD Kaon mass and decay constant,

- compatible with O(a) automatic improvement

Pseudoscalar meson mass splitting

- scaling tests performed at fixed quark masses μ_l ~ 40 MeV and μ_h ~ 90 MeV





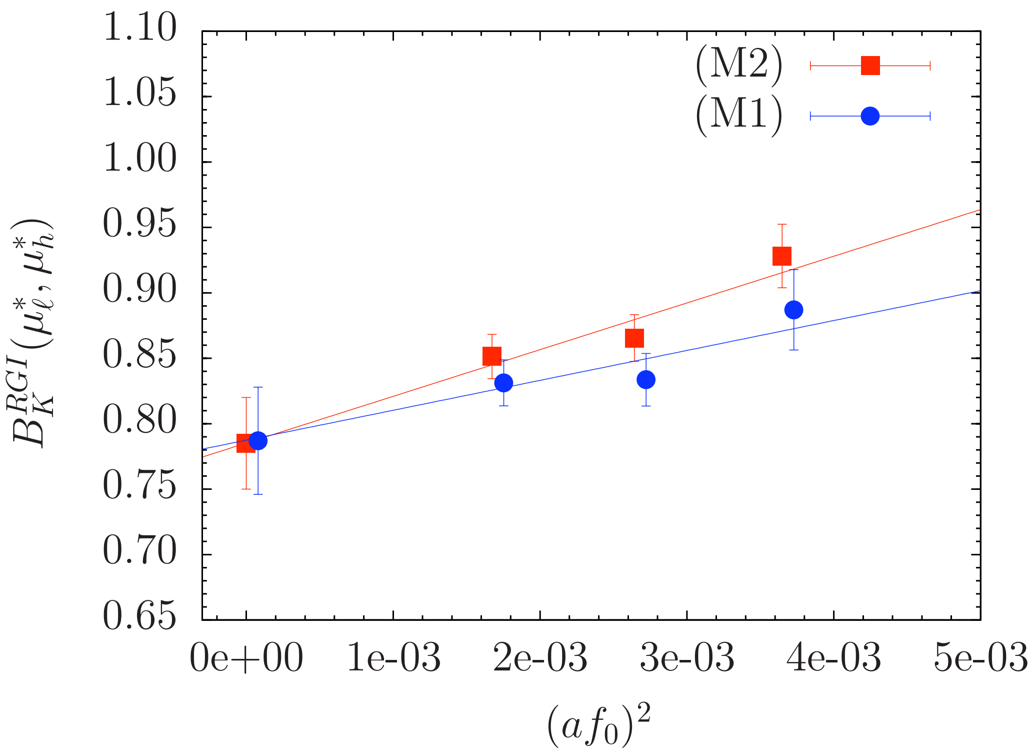
$$\Delta_M = \frac{(M^{12})^2 - (M^{34})^2}{(M^{34})^2}$$

$$\Delta_F = -\frac{F^{12} - F^{34}}{F^{34}}$$

- compatible with O(a) automatic improvement
- OS-Kaon mass shows 30% discretization error - vanishes in the continuum limit

Pseudoscalar meson mass splitting

- scaling tests performed at fixed quark masses $\mu_l \sim 40$ MeV and $\mu_h \sim 90$ MeV



$$B_K = \frac{3}{8}\,\frac{Z_{VA+AV}}{Z_V\;Z_A}\,\frac{\langle P^{34}|Q_{VA+AV}|P^{21}\rangle}{\langle P^{34}|V_0(0)|0\rangle\;\langle 0|A_0(0)|P^{21}\rangle}$$

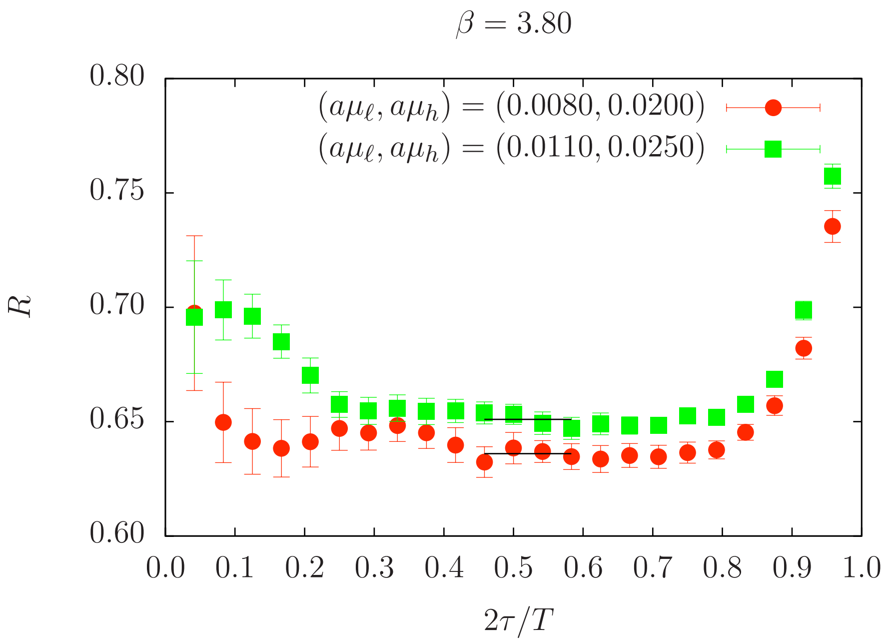
$$Z_V\;\langle P^{34}|V_0(0)|0\rangle = M^{34}\;F^{34}$$

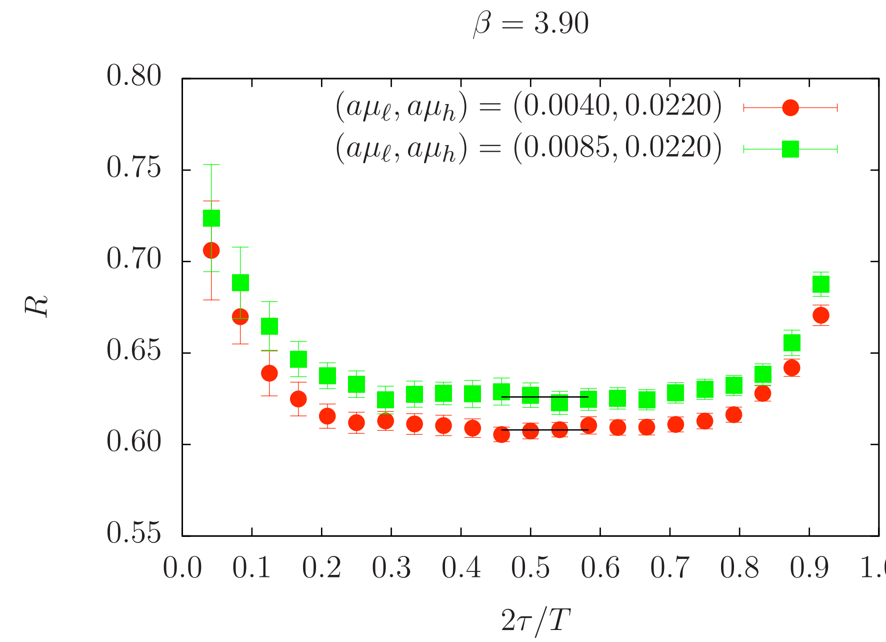
$$Z_A\;\langle 0|A_0(0)|P^{12}\rangle = M^{12}\;F^{12}$$

- benevolent cancellation mechanism between large $O(a^2)$ effects in numerator and denominator

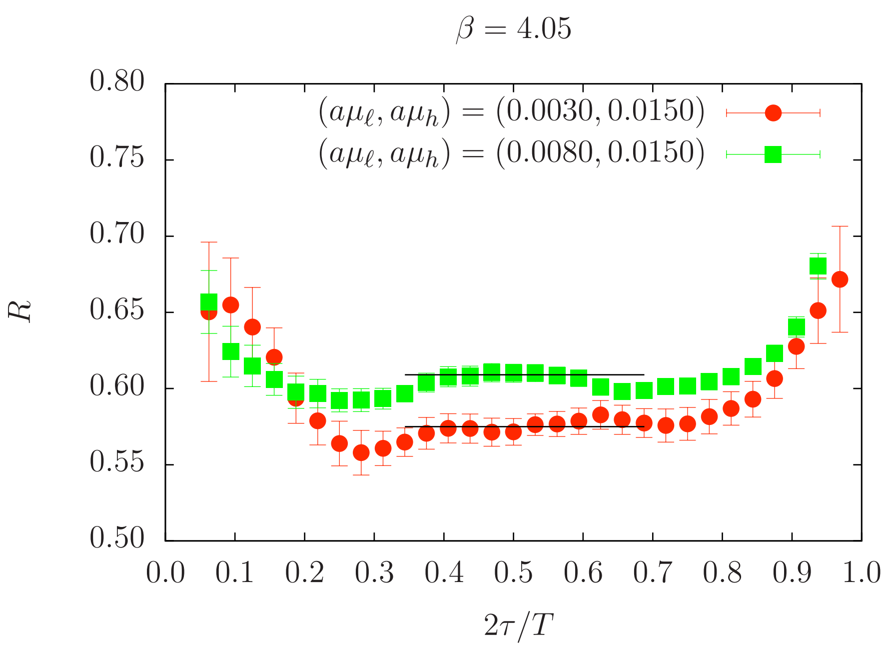
- two methods for the determination of Z_{VA+AV} (RI/MOM scheme) give compatible results

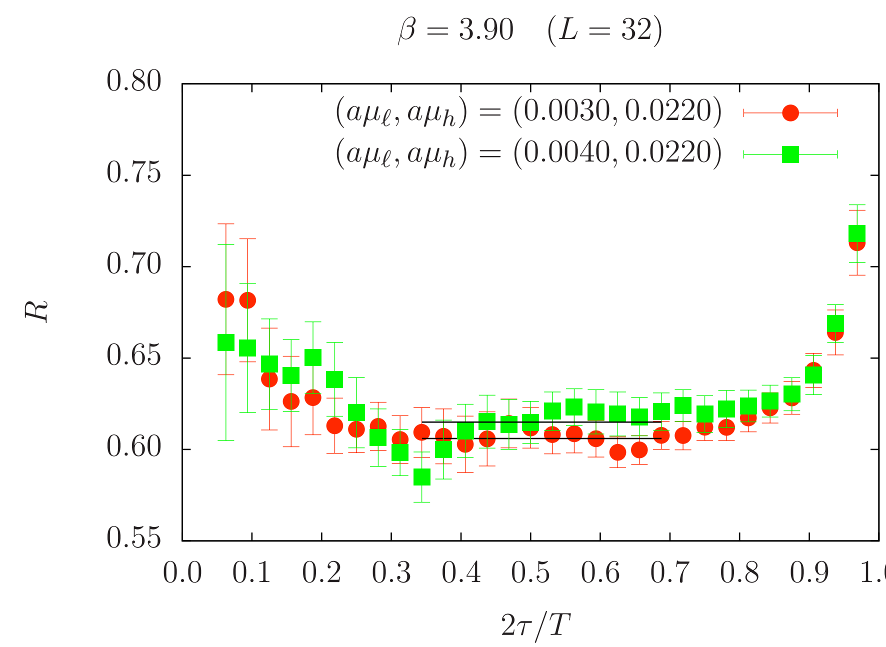
B_K: signal quality





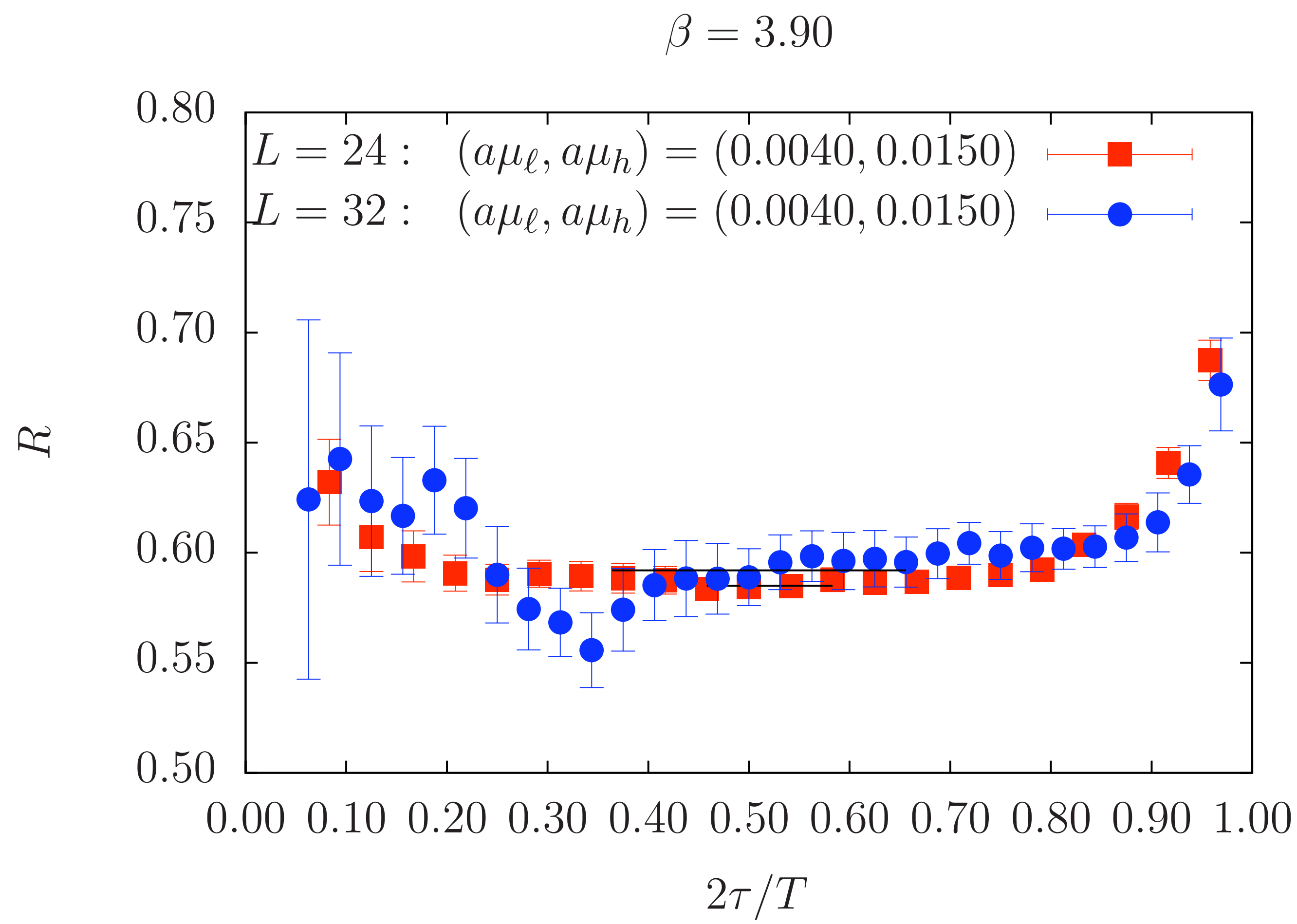
$24^3 \times 48$





$32^3 \times 64$

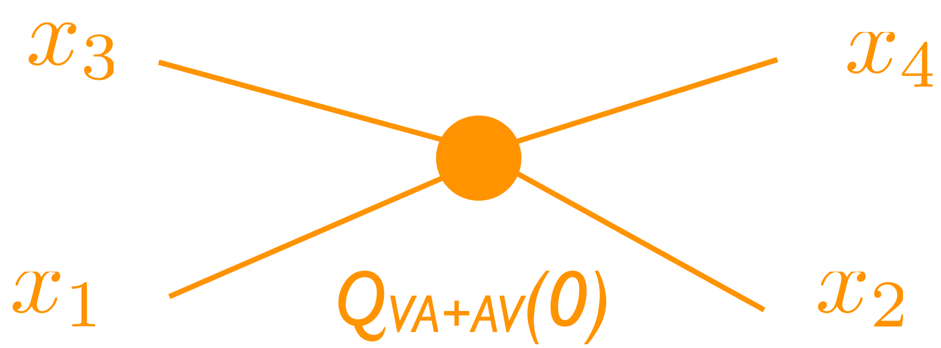
B_K: finite volume effects



B_K: renormalization (RI/MOM)

- Opt for *RI/MOM* scheme
- the correlation function of interest, in coordinate space, is obtained by inserting the quark bilinear operator in 4-point fermionic Green function (the quark propagator)

$$G_{VA+AV}(x_1,x_2,x_3,x_4) \; = \; \langle \psi_1(x_1)\,\bar{\psi}_2(x_2)\;Q_{VA+AV}(0)\;\psi_3(x_3)\,\bar{\psi}_4(x_4)\rangle$$



B_K: renormalization (RI/MOM)

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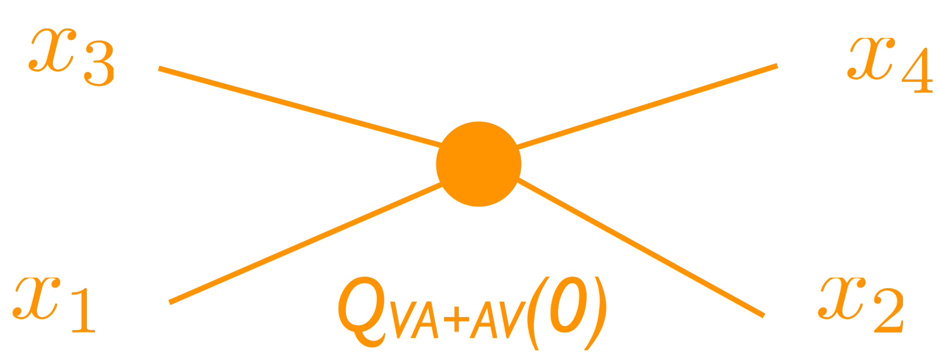
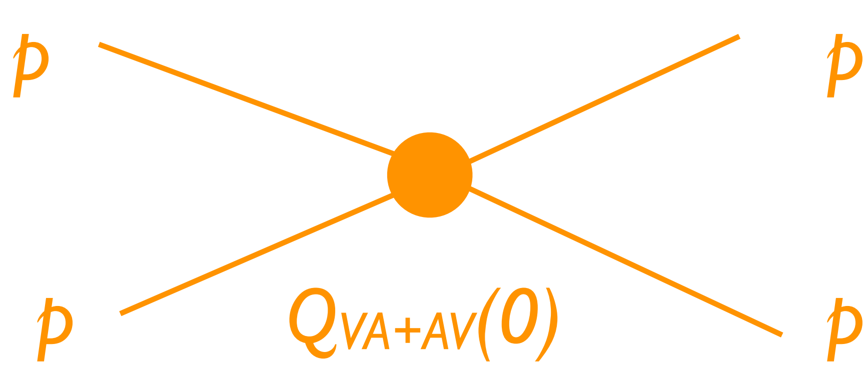
$$G_{VA+AV}(x_1,x_2,x_3,x_4) \;=\; \langle \psi_1(x_1)\,\bar{\psi}_2(x_2)\; Q_{VA+AV}(0)\; \psi_3(x_3)\,\bar{\psi}_4(x_4)\rangle$$

- Fourier transform it to obtain the correlation function in momentum space

$$G_{VA+AV}(p) \;=\; \int dx_1 dx_2 dx_3 dx_4\; G_{AV+VA}\,(x_1,x_2,x_3,x_4)\;\exp[-ip(x_1-x_2+x_3-x_4)]$$

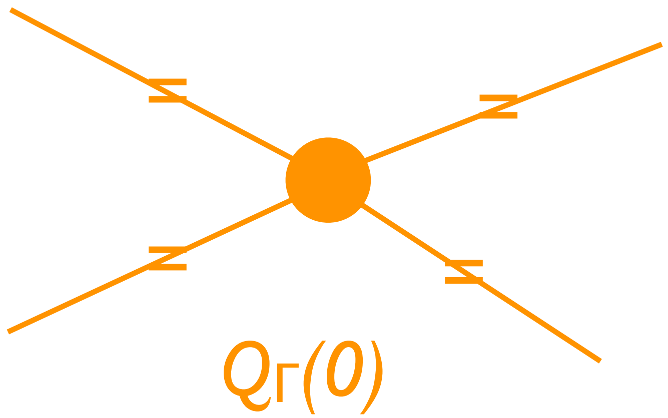




B_K: renormalization (RI/MOM)

- amputate the momentum space correlation function

$$\Lambda_{VA+AV}(p) \;=\; \mathcal{S}_1^{-1}(p)\; \mathcal{S}_2^{-1}(p)\; G_{AV+VA}(p)\; \mathcal{S}_3^{-1}(p)\; \mathcal{S}_4^{-1}(p)$$



- NB: exceptional momentum configuration (optional)

- NB: all manipulations are in the Landau gauge

- the amputated correlation function is a matrix in Dirac-colour space; its tree level value is (VA+AV)⊗*I*

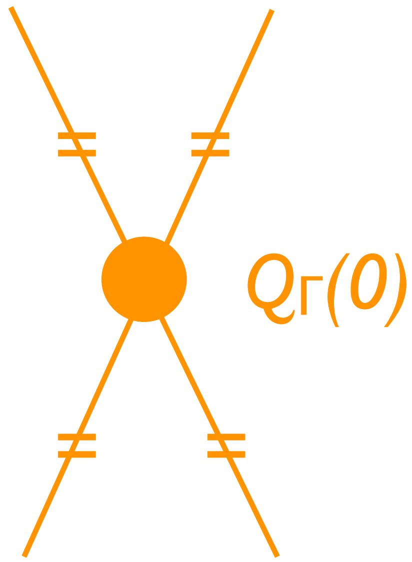
Basic definitions

- it is convenient to impose the renormalization condition on a function of momenta (rather than on a Dirac-colour matrix)

- we thus "project" the amputated correlation Dirac-colour Green function by suitable traces

- this consists in defining the projected-amputated Green function

$$\Gamma_{VA+AV}(p) \; = \; \mathrm{Tr}\left[P_{AV+VA}\, G_{AV+VA}(p)\right]$$



- the trace is over colour and spin indices

- the trace over colours is trivial

- the trace over spin is conditioned by the choice of the Dirac projectors P_Q, chosen so that the tree-level value of Γ_{VA+AV} is unity (recall that the tree level value of is Γ_{VA+AV} is $(VA+AV)\otimes I$).

RI/MOM renormalization scheme

- so far we only defined a convenient projected-amputated correlation function $\Gamma_{VA+AV}(p)$, in terms of the bilinear operator Q_{VA+AV} and the fermion fields ψ

- this bare quantity, regularized by the lattice, is computed non-perturbatively (i.e. numerically, at fixed UV cutoff)

- the renormalized $\Gamma_{VA+AV}(p)$ is formally given by:

$$\left[\Gamma_{VA+AV}(p)\right]_{\mathrm{R}} = \lim_{a\to 0} \left[Z_\psi^{-2}(a\mu)\; Z_{AV+VA}(a\mu)\; G_{AV+VA}(p)\right]$$

quark field renormalization

operator renormalization

- RI/MOM renormalization scheme: impose the following renormalization condition on $\Gamma_{VA+AV}(p)$

$$\left[\Gamma_{VA+AV}(p)\right]_{\mathrm{R}}\Big|_{p^2=\mu^2} = \left[Z_\psi^{-2}(a\mu)\; Z_{AV+VA}(a\mu)\; G_{AV+VA}(p)\right] = 1$$

- i.e. the renormalized amputated-projected correlation function $[\Gamma_{VA+AV}(p)]_{\mathrm{R}}$, at scale μ, is set to its tree level value. From it the product Z_Q / Z_ψ^2 is determined

RI/MOM renormalization scheme

$$\left[\Gamma_{VA+AV}(p)\right]_{\mathrm{R}}\Big|_{p^2=\mu^2} = \left[Z_\psi^{-2}(a\mu)\, Z_{AV+VA}(a\mu)\, G_{AV+VA}(p)\right] = 1$$

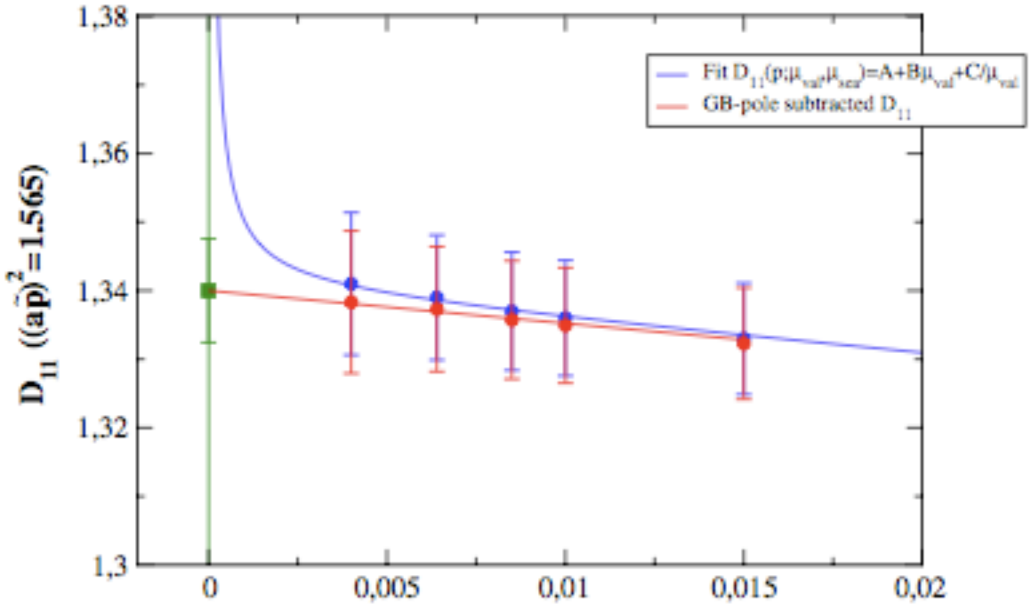
- in practice the bare $\Gamma_{VA+AV}(p)$ is computed at fixed UV cutoff (lattice spacing) for several quark masses μ_Q and renormalization scales μ
- being a mass-independent scheme, the chiral extrapolation $\mu_Q \to 0$ must be performed
- we must disentangle Z_Q from Z_ψ; conceptually the simplest way is by using the lattice conserved vector current V^C, which has $Z_{V^C} = 1$
- for this current, the RI/MOM condition gives a way to compute non-perturbatively Z_ψ

$$\left[\Gamma_{V^C}(p^2)\right]_{\mathrm{R}}\Big|_{p^2=\mu^2} = Z_\psi^{-1}(a\mu)\Gamma_{V^C}(\mu) = 1$$

- in practice this method is not applied because the conserved current is point split and somewhat intricate and costly to implement (in reality these are superable problems...)
- instead of V^C, we use $Z_V V = V^C$, with Z_V taken from a tmQCD Ward identity

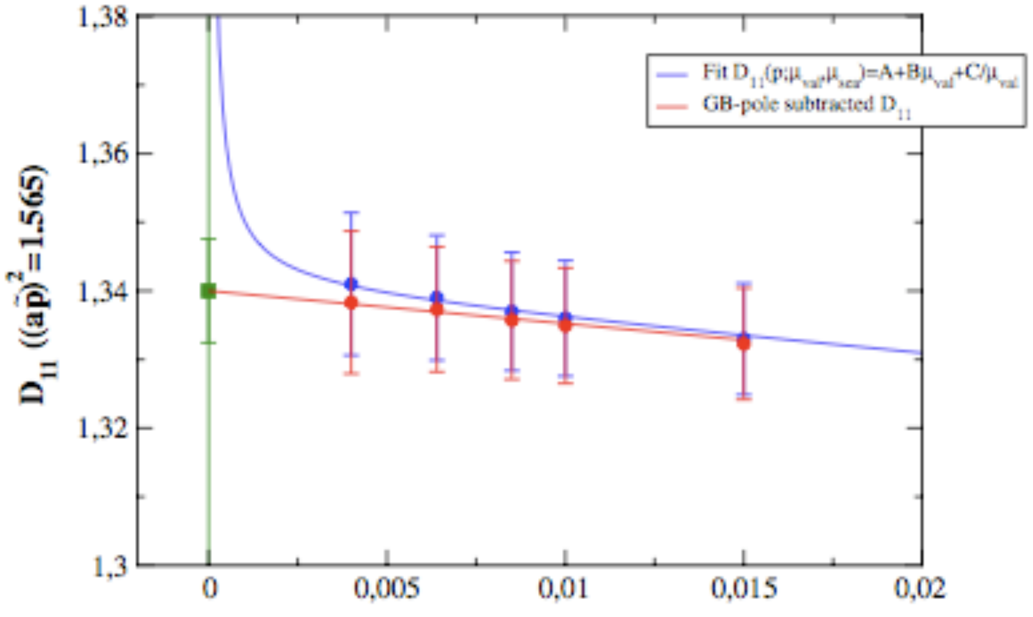
B_K: renormalization (RI/MOM)

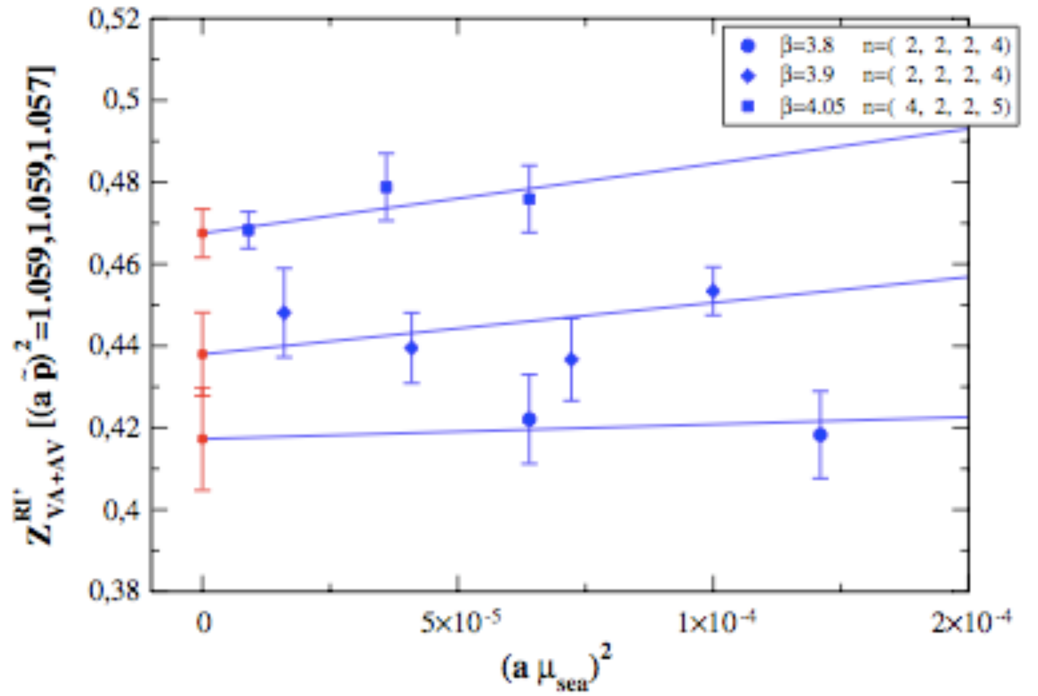
- a few "massages" are necessary:
- Goldstone pole subtraction



B_K: renormalization (RI/MOM)

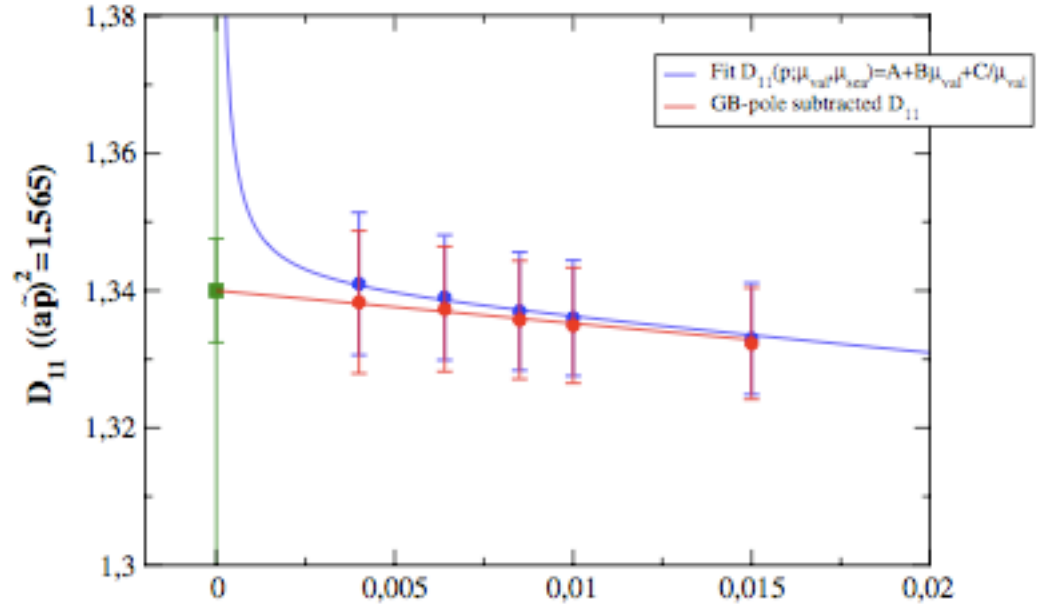
- a few "massages" are necessary:
- Goldstone pole subtraction
- extrapolation to sea-quark chiral limit

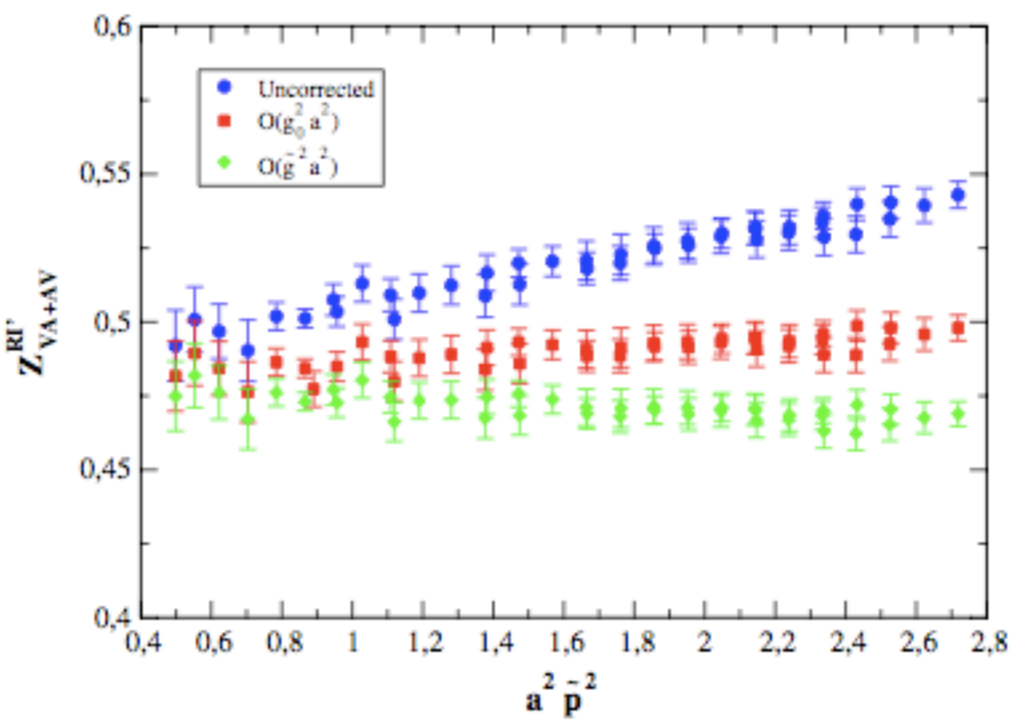


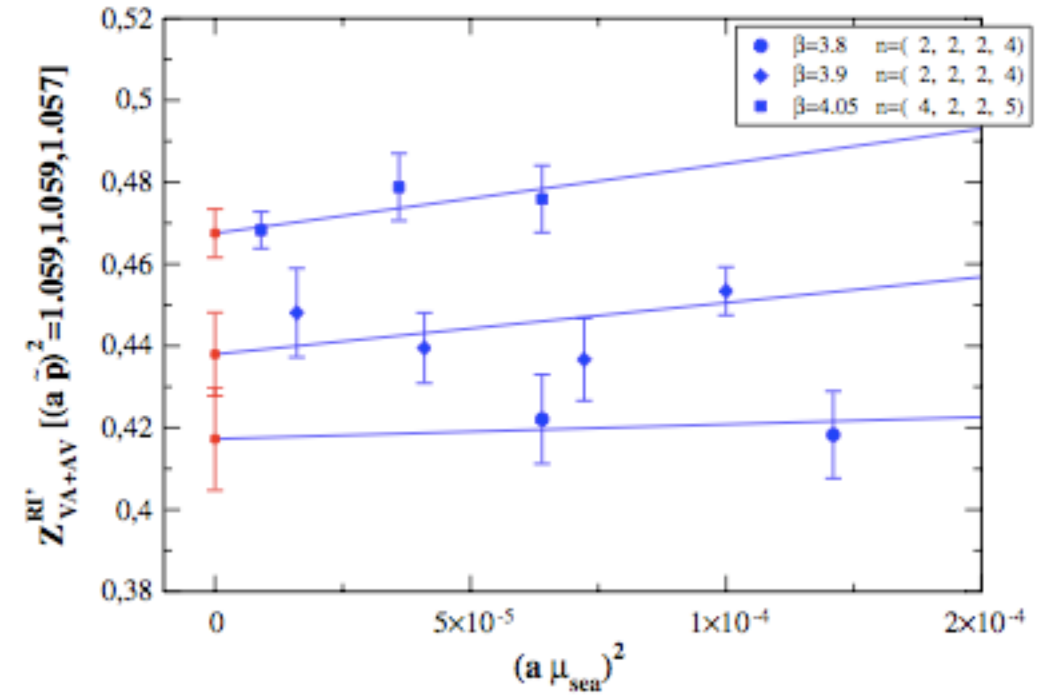


B_K: renormalization (RI/MOM)

- a few “massages” are necessary:
- Goldstone pole subtraction
- extrapolation to sea-quark chiral limit
- discretization effects calculated in 1-loop PT

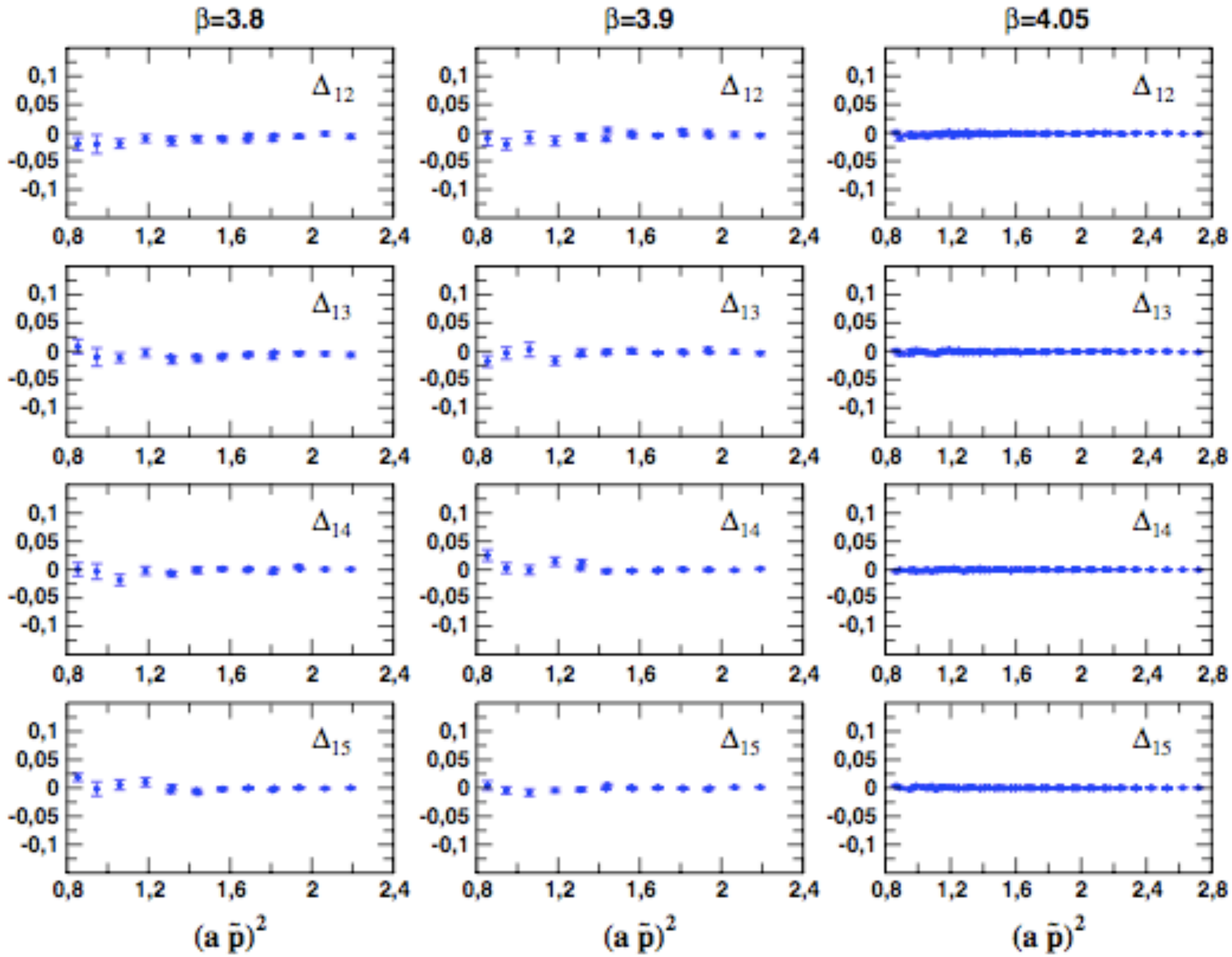






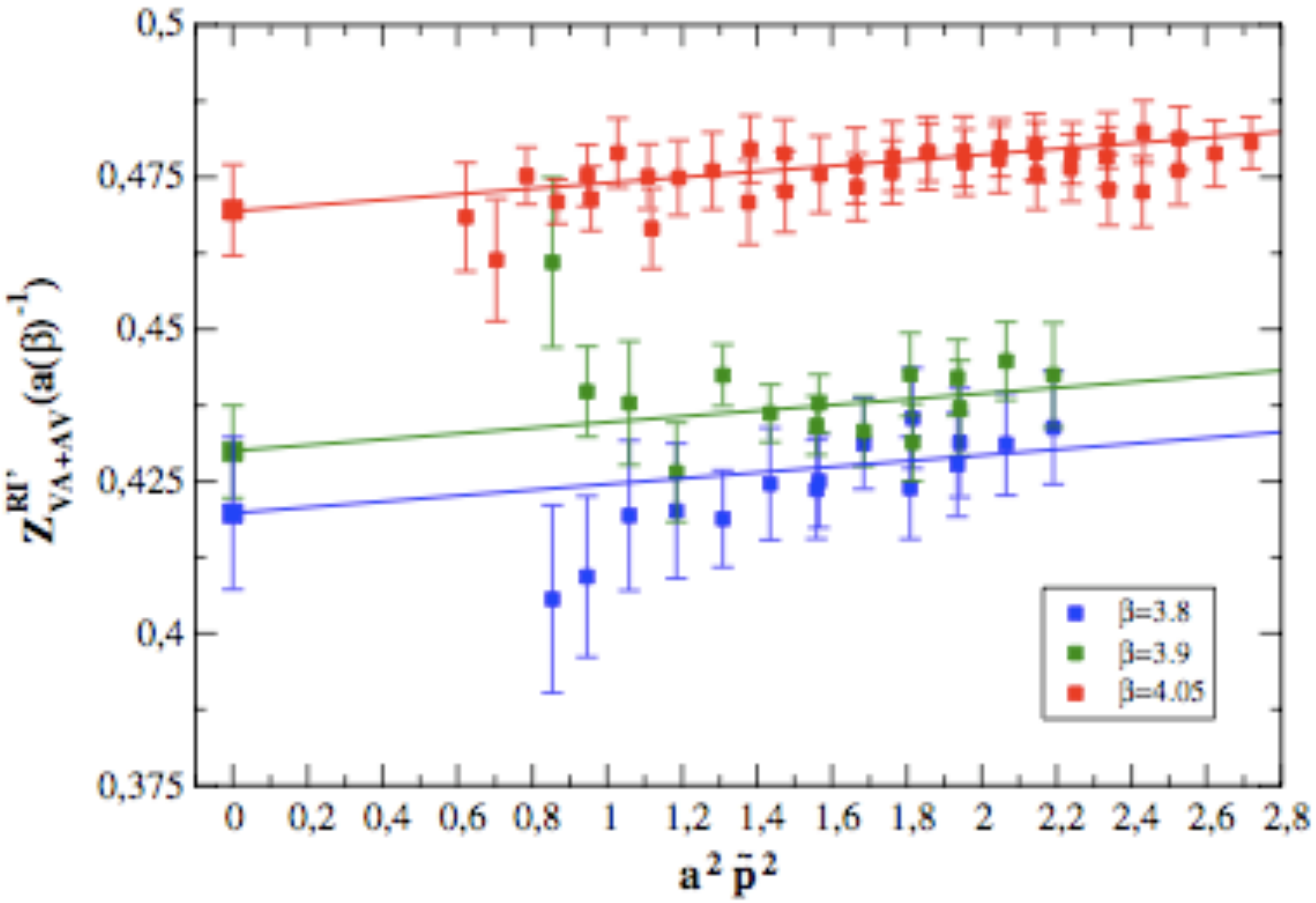
B_K: renormalization (RI/MOM)

- check the "wrong chirality" contributions:



B_K: renormalization (RI/MOM)

- Use NLO PT RG-running from "all scales" to a reference scale $\mu \sim 1/a$ and extrapolate residual $(ap)^2$ dependence as a cutoff effect



B_K: mass extrapolations

- for each β we have a number of bag parameters $B_K(\mu_{l=}\ \mu_{sea},\ \mu_h)$
- The RGI-bag parameter is fit by the SU(2) - χPT Ansatz:

$$B_K^{\rm RGI}(\hat{\mu}_l,\hat{\mu}_s) \;=\; B_\chi^{\rm RGI}(\hat{\mu}_s)\left[1 \;+\; b(\hat{\mu}_s)\,\frac{2\hat{B}_0\hat{\mu}_l}{f_0^2} \;-\; \frac{2\hat{B}_0\hat{\mu}_l}{32\pi^2 f_0^2}\ln\left(\frac{2\hat{B}_0\hat{\mu}_l}{16\pi^2 f_0^2}\right)\right] \;+\; a^2\, f_0^2\, D_B(\hat{\mu}_s)$$

- "hatted" quantities are in $\overline{\rm MS}$ @ 2GeV. **What calibrations are needed?**
- f_0 = 121.0(1) MeV -- the pion decay constant @ chiral limit
- $\hat{B}_\chi$ -- the bag parameter @ chiral limit
- $\hat{B}_0$ = 2.84(11) GeV
- $\hat{\mu}_{u/d}$ = 3.5(1) MeV
- ETMC, M. Constanstinou et al., JHEP08 (2010) 068; ETMC, R. Baron et al., JHEP08 (2010) 097
- need to also know strange quark mass

B_K: mass extrapolations

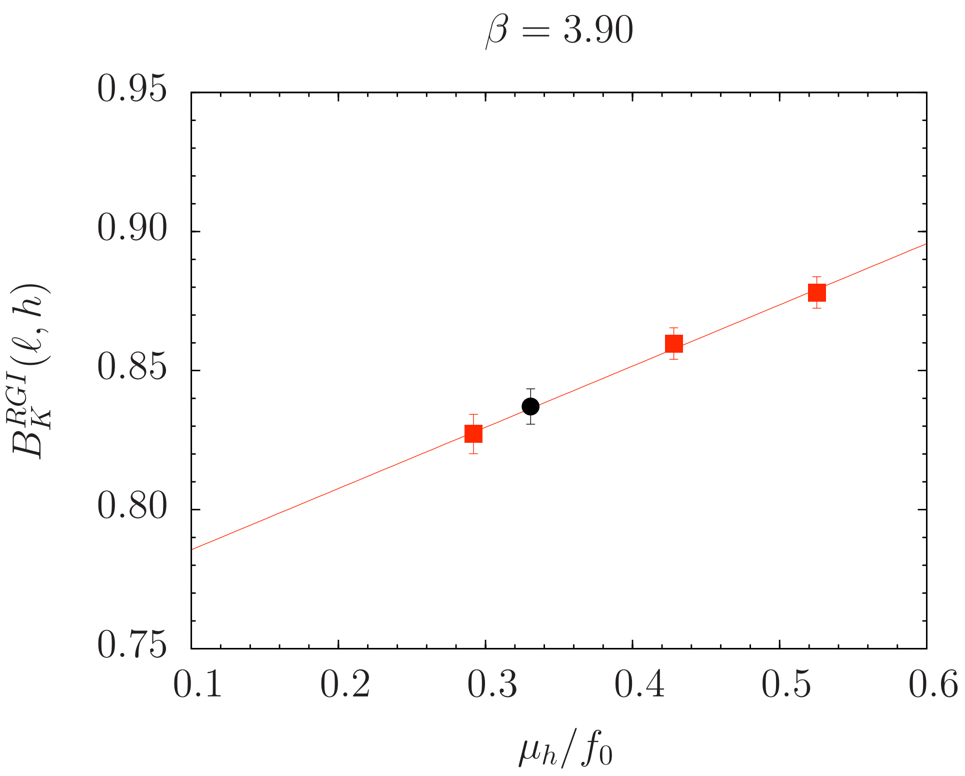
- for each β we have a number of bag parameters $B_K(\mu_{l=}\ \mu_{sea},\ \mu_h)$
- need to also know strange quark mass
- Use the SU(2) - χPT Ansatz for the tmQCD pseudoscalar mass:

$$f_0^2\ M_{34}^2(\hat{\mu}_l,\hat{\mu}_h)\ =\ C_M(\hat{\mu}_h)\left[1\ +\ c(\hat{\mu}_h)\ \frac{2\hat{B}_0\hat{\mu}_l}{f_0^2}\right]\ +\ a^2\ f_0^2\ D_M(\hat{\mu}_h)$$

- for each β and $\hat{\mu}_l$ calculate M_{34} at three reference values 75 MeV < $\hat{\mu}_h^*$ < 105 MeV
- use above Ansatz and known value of $\hat{\mu}_{u/d}$ to compute M_{34} ($\hat{\mu}_{u/d}$, $\hat{\mu}_h^*$) in the continuum
- interpolate $[M_{34}(\hat{\mu}_{u/d}, \hat{\mu}_h^*)]^2$ linearly in $\hat{\mu}_h^*$ to the physical value $M_{34}^2 = (495\ \mathrm{MeV})^2$ and obtain:
- $\hat{\mu}_s$ = 92(5) MeV

B_K: mass extrapolations

- for each β we have a number of bag parameters $B_K(\hat{\mu}_{l} = \hat{\mu}_{sea}, \hat{\mu}_h)$
- now that $\hat{\mu}_s$ is known, we interpolate $B_K(\hat{\mu}_{l} = \hat{\mu}_{sea}, \hat{\mu}_h)$ linearly in $\hat{\mu}_h$, at fixed β and $\hat{\mu}_l$, to get:
- $B_K(\hat{\mu}_l, \hat{\mu}_s)$



B_K: mass extrapolations

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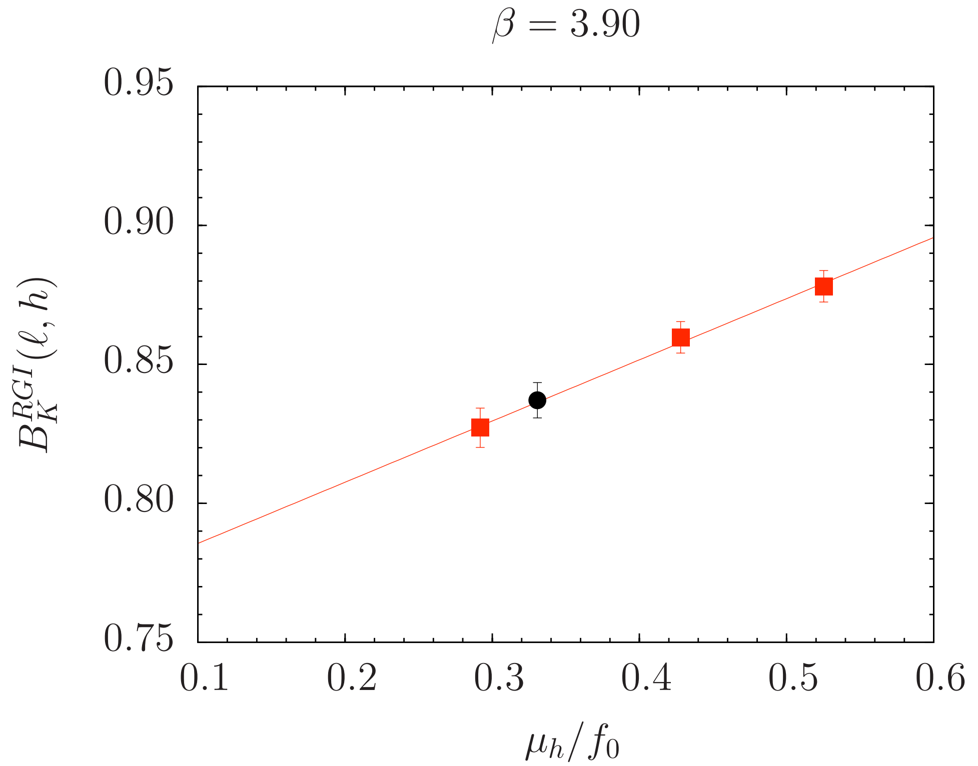
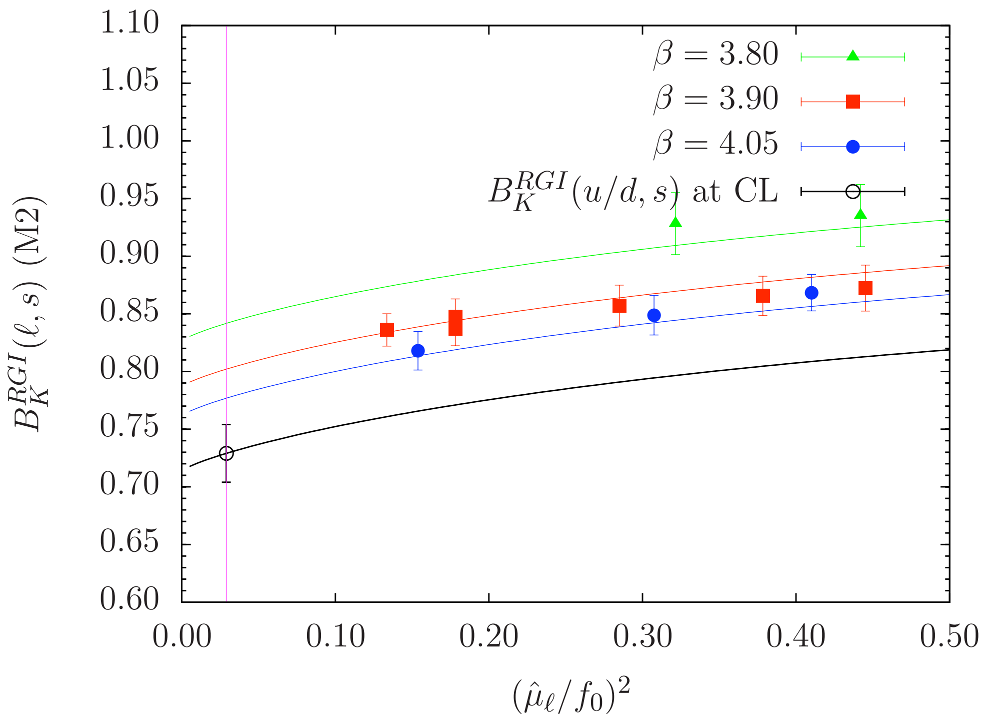
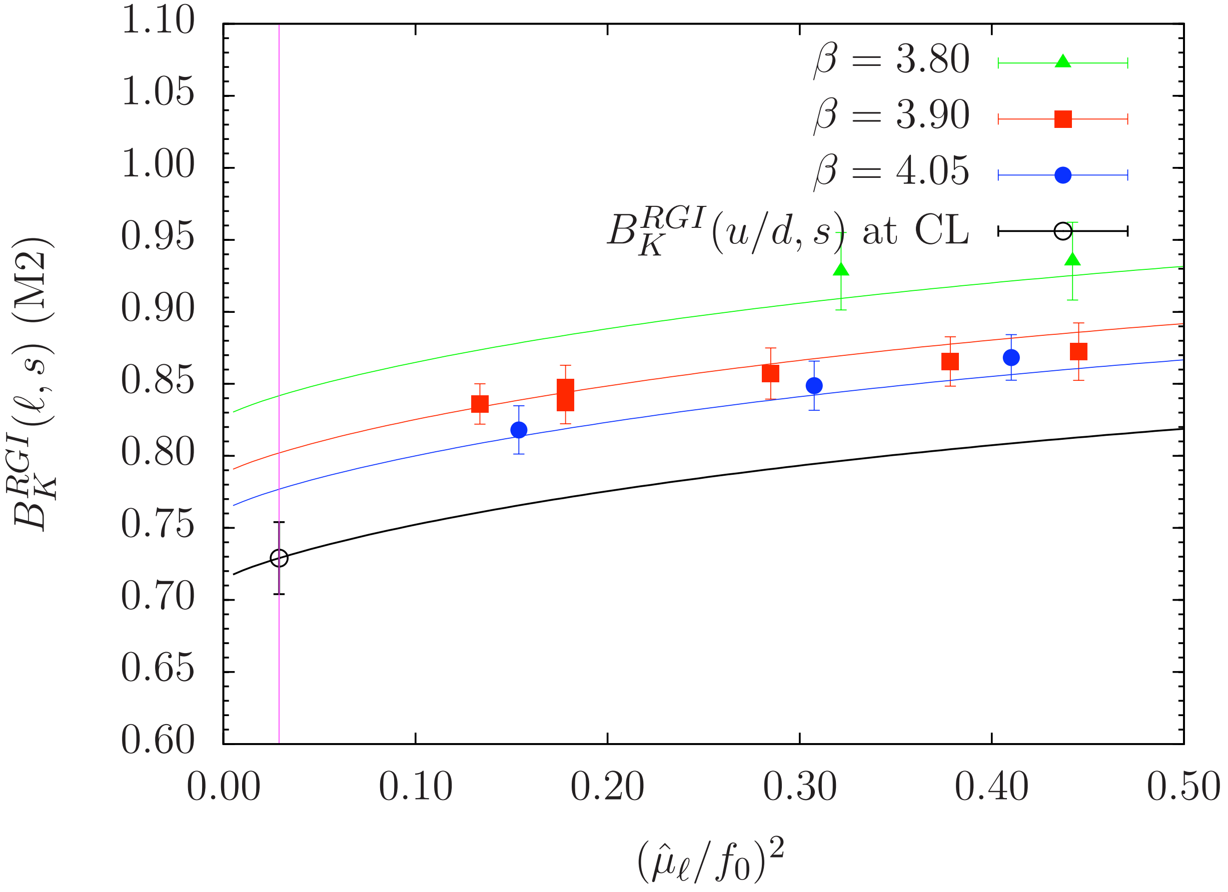
- $B_K(\hat{\mu}_l, \hat{\mu}_s)$
- now use SU(2) - χPT Ansatz (simultaneous chiral and continuum fit), to obtain:
- $B_K(\hat{\mu}_{u/d}, \hat{\mu}_s)$

$$B_K^{\rm RGI}(\hat{\mu}_l, \hat{\mu}_s) \;=\; B_\chi^{\rm RGI}(\hat{\mu}_s)\left[1 \;+\; b(\hat{\mu}_s)\,\frac{2\hat{B}_0\hat{\mu}_l}{f_0^2} \;-\; \frac{2\hat{B}_0\hat{\mu}_l}{32\pi^2 f_0^2}\ln\left(\frac{2\hat{B}_0\hat{\mu}_l}{16\pi^2 f_0^2}\right)\right] \;+\; a^2\; f_0^2\; D_B(\hat{\mu}_s)$$





B_K: mass extrapolations



$$B_K^{\mathrm{RGI}} = 0.729\,(25)\,(17)$$

"statistical error" (bootstrap):

ME fluctuations 1%

Zs determination 2%

CL/chiral extrapolations

strange quark mass interpolations

"systematic error":

lattice calibration from r_0 or f_0

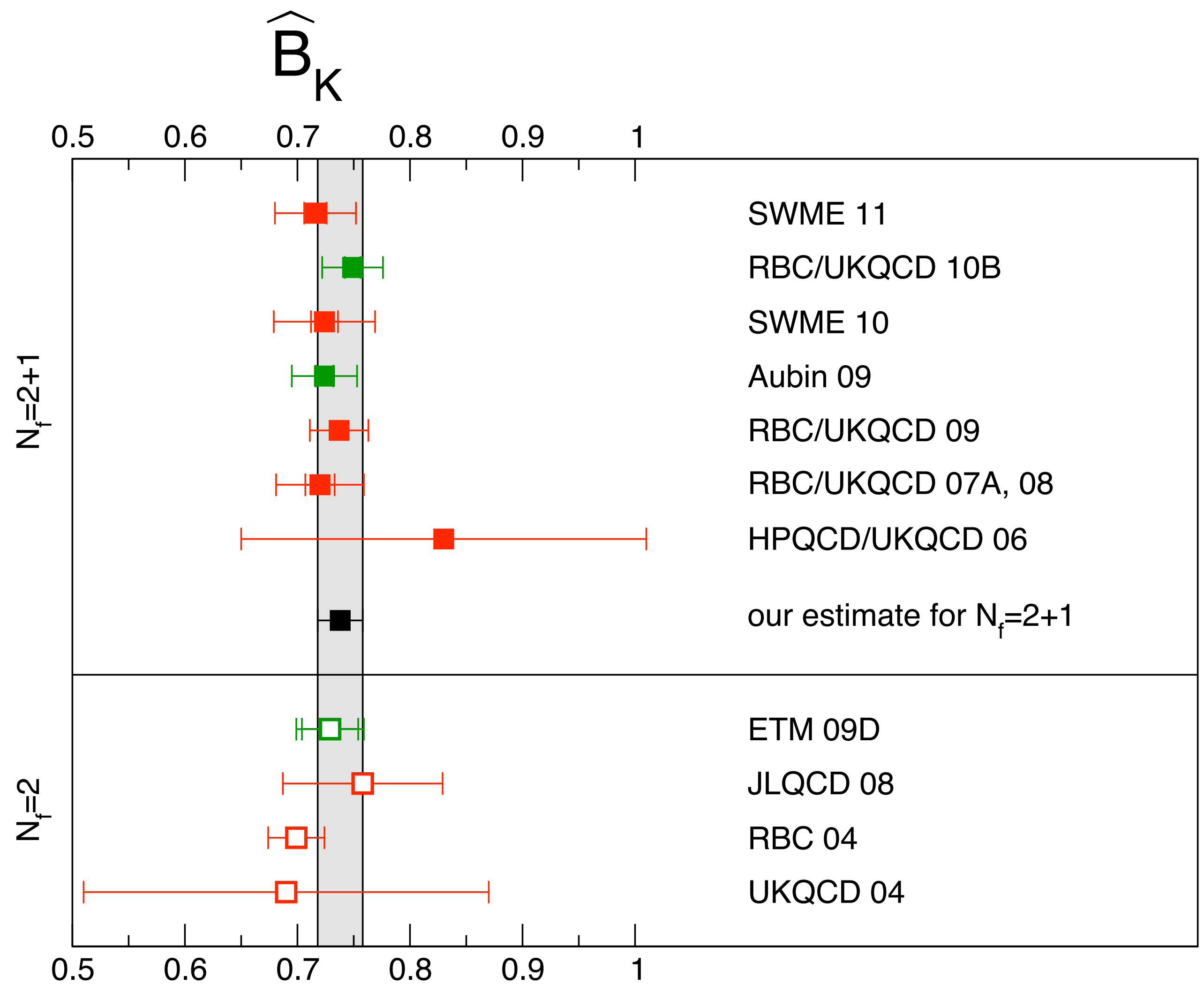
Zs determined with 2 methods

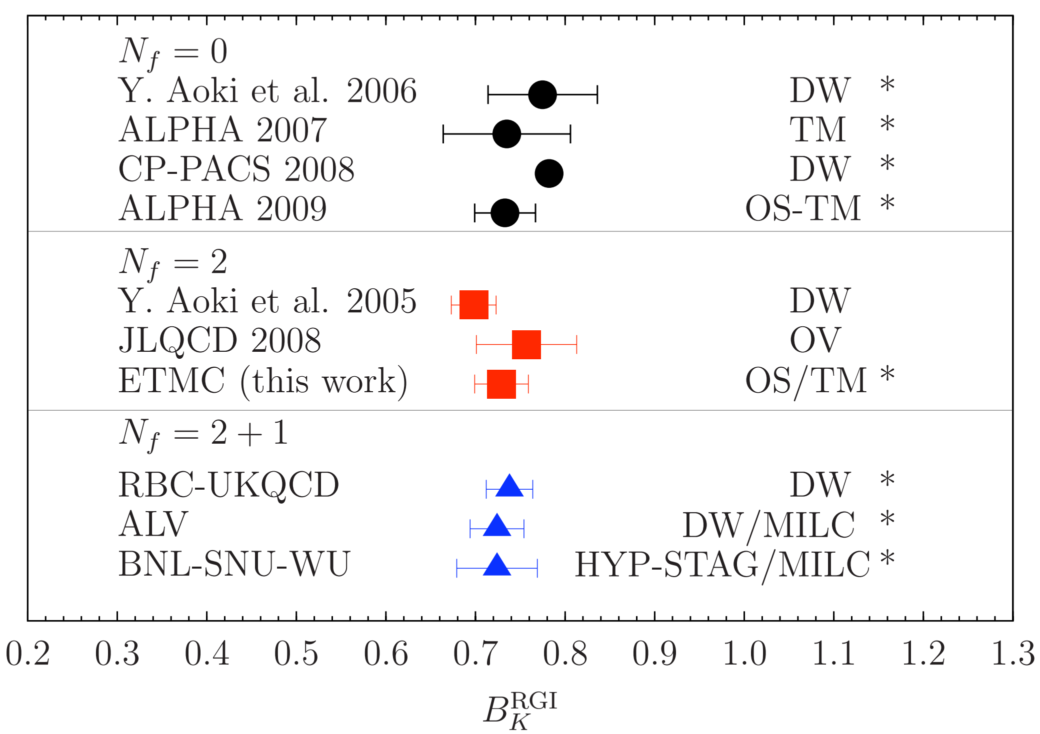
polynomial chiral extrapolations

quark/PS-meson χPT interpolations

B_K comparison with others

FLAG, G. Colangelo et al., arXiv:1011.4408 [hep-lat]





- Hardly any dependence on $N_f \sim 1/a$ is observed

Conclusions

- In 2001 the reliable lattice estimates of B_K were all quenched
- the staggered estimates were clearly the "best", with Ginsparg Wilson computations in their infancy and Wilson computation afflicted by large systematic effects, due to:
  - the complicated renormalization pattern of B_K resulting from loss of chiral symmetry in the bare action
  - the large discretization errors
- thanks to tmQCD, both sources of systematic error are now under control at a (moderate?) price
- quenching is quickly being removed (the last uncontrolled source of systematic error), by either by a partially quenced setup ($N_f = 2$) or an unquenched one ($N_f = 2+1, 2+1+1, ...$)
- B_K is the "simplest" 4-fermion operator, from the lattice point of view (one anomalous dimension, Kaonic mass regime, no finite state interactions, ...)
- controlling its systematics paves the way for more complicated realities, which have been put to stand-by mode for many years (B_K, $K \to \pi\pi$, ...)