Tuesday, June 14

1 Exercise 1

Derive the quark-quark-gluon vertex from the fermionic part of the Wilson action:

$$\begin{aligned} S_W^f &= a^4 \sum_x \Big[-\frac{1}{2a} \sum_\mu \big[\overline{\psi}(x)(r-\gamma_\mu)U_\mu(x)\psi(x+a\hat{\mu}) \\ &\quad + \overline{\psi}(x+a\hat{\mu})(r+\gamma_\mu)U_\mu^\dagger(x)\psi(x) \big] + \overline{\psi}(x)\left(m_0 + \frac{4r}{a}\right)\psi(x) \Big], \end{aligned}$$

knowing that

$$U_\mu(x) = \mathrm{e}^{\mathrm{i} g_0 a T^a A_\mu^a(x)} \qquad (a = 1, \ldots, N_c^2 - 1),$$

and that the action enters in the functional integral as e^{-S}.

Hint: The Fourier transforms on the lattice are

$$\begin{aligned} \psi(x) &= \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4p}{(2\pi)^4}\, \mathrm{e}^{\mathrm{i}xp}\, \psi(p), \\ \overline{\psi}(x) &= \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4p}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}xp}\, \overline{\psi}(p), \\ A_\mu(x) &= \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4}\, \mathrm{e}^{\mathrm{i}(x+a\hat{\mu}/2)k}\, A_\mu(k), \end{aligned}$$

and the lattice δ-function is momentum space is

$$\delta^{(4)}(p) = \frac{a^4}{(2\pi)^4} \sum_x \mathrm{e}^{-\mathrm{i}xp}.$$

Optional exercise: Derive also the quark propagator (by inverting the quadratic form in the spinor fields, in momentum space).

2 Exercise 2

Reduce the following expressions:

(a)

$$\sum_\rho \gamma_\rho \gamma_\mu \gamma_\rho$$

(b)

$$\sum_\rho \gamma_\rho \gamma_\mu \gamma_\rho \cos^2 k_\rho$$

(c)

$$\sum_\rho \gamma_\rho \gamma_\mu \gamma_\rho \cos^2 k_\mu$$

(d)

$$\sum_\rho \gamma_\rho \gamma_\mu \gamma_\rho \cos k_\mu \cos k_\rho$$