Wednesday, June 15

1 Exercise 3

Compute the tadpole diagram of the quark self-energy, for Wilson fermions in Feynman gauge (i.e., $\alpha = 1$, where the form of the gluon propagator is simpler). Explain the meaning of the two terms in the result.
Hint: The vertex between 2 quarks and 2 gluons is

$$(V_2^{ab})^{cd}_{\mu_1\mu_2}(p_1,p_2) = -\frac{1}{2} a g_0^2\, \delta_{\mu_1\mu_2}\, \{T^a, T^b\}^{cd} \left(-\mathrm{i}\gamma_\mu \sin\frac{a(p_1+p_2)_\mu}{2} + r\cos\frac{a(p_1+p_2)_\mu}{2} \right)$$

and the gluon propagator in Feynman gauge is

$$G^{ab}_{\mu\nu}(k) = \delta^{ab} \frac{1}{\frac{4}{a^2}\sum_\lambda \sin^2\frac{ak_\lambda}{2}}\, \delta_{\mu\nu}.$$

Give a (small) momentum p to the external states, and consider the amputated diagram (that is, remove the external states). Rescale the integration variable, $k \to ak$, so that the domain of integration after the rescaling becomes independent of a, and be careful with the various powers of a around.

In particular, extract also the $1/a$ contribution, and take the limit of small external momentum ap in the rest of the expressions.

Remember also that $\sum_a (T^a)^2_{bb} = (N_c^2-1)/(2N_c) = C_F$.

2 Exercise 4

Reduce the following expressions:

(e)

$$\sum_{\rho,\alpha,\beta} \gamma_\rho\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\rho \sin k_\alpha \sin k_\beta$$

(f)

$$\sum_{\rho,\alpha,\beta} \gamma_\rho\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\rho \sin k_\alpha \sin k_\beta \cos k_\mu \cos k_\rho$$

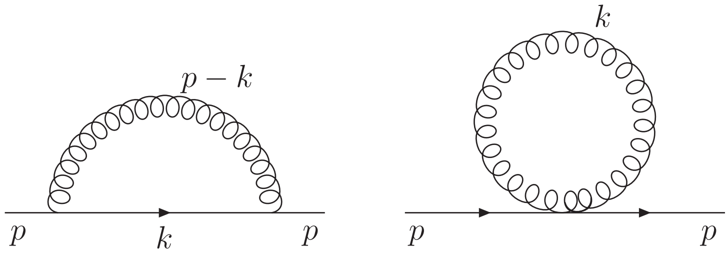

Figure 1: Diagrams for the quark self-energy. On the left the sunset diagram, on the right the tadpole diagram.