Friday, June 17

1 Exercise 5

Compute the $1/a$ result of the sunset diagram of the quark self-energy, for massless Wilson fermions in Feynman gauge.
Hint: The vertex between 2 quarks and 1 gluon is

$$(V_1^a)_\mu^{bc}(p_1, p_2) = -g_0 (T^a)^{bc} \left(\mathrm{i}\gamma_\mu \cos\frac{a(p_1+p_2)_\mu}{2} + r \sin\frac{a(p_1+p_2)_\mu}{2} \right), \tag{1.1}$$

the massless quark propagator is

$$S^{ab}(k, m_0) = \delta^{ab} \cdot a \frac{-\mathrm{i}\sum_\mu \gamma_\mu \sin ak_\mu + 2r\sum_\mu \sin^2\frac{ak_\mu}{2}}{\sum_\mu \sin^2 ak_\mu + \left(2r\sum_\mu \sin^2\frac{ak_\mu}{2}\right)^2}, \tag{1.2}$$

and the gluon propagator in Feynman gauge is

$$G_{\mu\nu}^{ab}(k) = \delta^{ab} \frac{1}{\frac{4}{a^2}\sum_\lambda \sin^2\frac{ak_\lambda}{2}} \delta_{\mu\nu}.$$

Give a momentum p to the external states, and consider the amputated diagram (that is, remove the external states). Rescale the integration variable, $k \to ak$, so that the domain of integration after the rescaling becomes independent of a, and be careful with the various powers of a around.
Notice also that the color factors are the same as in the continuum:

$$\sum_a \sum_b (T^a)_{cb}(T^a)_{bc} = \sum_a (T^a)^2_{cc} = \frac{N_c^2-1}{2N_c} = C_F. \tag{1.3}$$

Advanced: Compute also the result of order zero in a of the sunset diagram (which gives the contribution proportional to $\mathrm{i}\not{p}$, the wave-function renormalization). Do not worry about regularizing the logarithmic divergence.

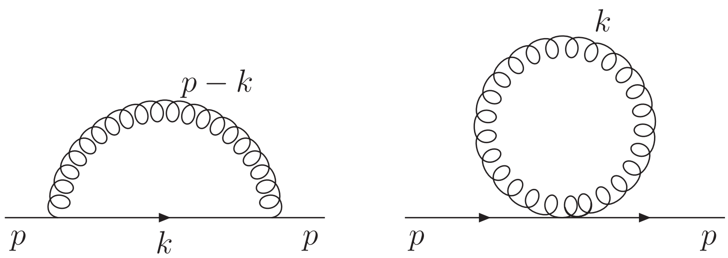

Figure 1: Diagrams for the quark self-energy. On the left the sunset diagram, on the right the tadpole diagram.