Spectroscopy: Glueballs and Exotics

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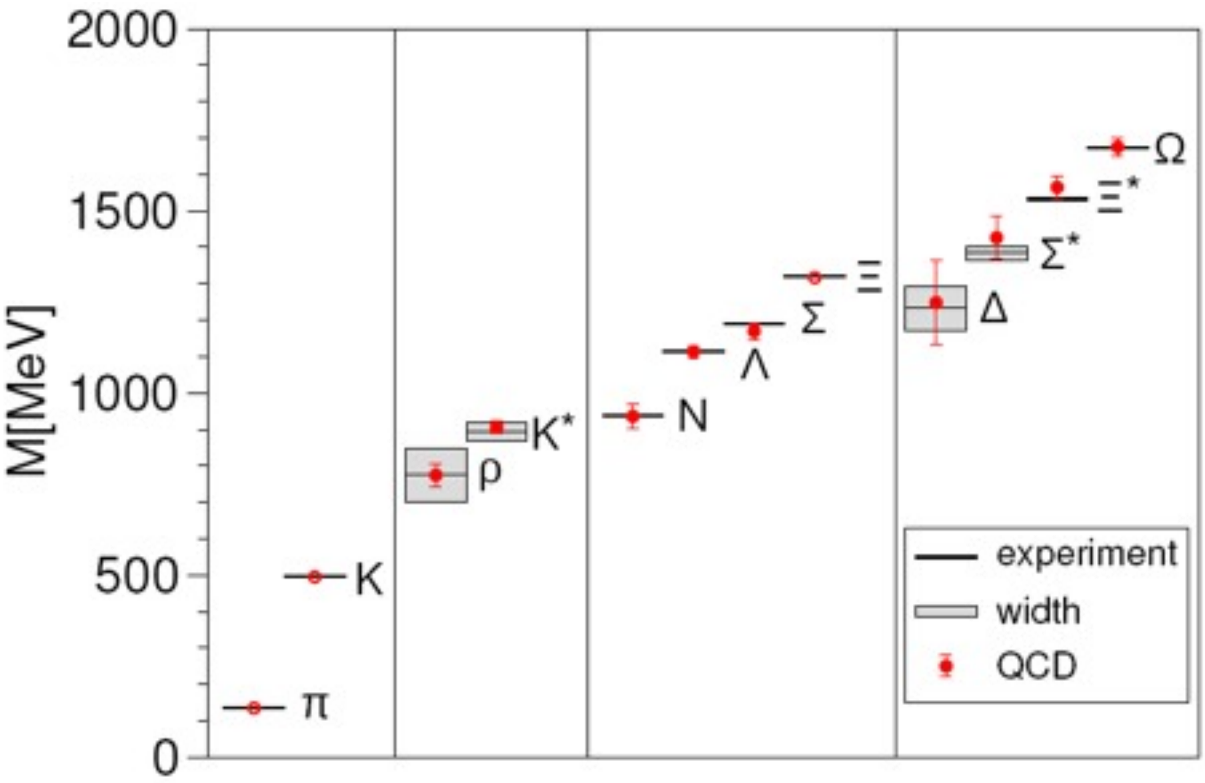
Jefferson Laboratory

StrongNet 2011,
Bielefeld

Plan of Lectures

- Lecture 1
  - What are they and why are they interesting?
  - Experimental searches
  - Review: variational method, distillation
  - Symmetries on the lattice
  - Meson interpolating operators - in the continuum, and on the lattice
  - Identifying spins: *the isovector meson spectrum*
  - Can we learn more - a phenomenology from lattice spectroscopy
  - But they are unstable! *...Back to Christian Lang*
  - What about baryons....
- Lecture 2: Hadron Structure - I
- Lecture 3: Hadron Structure - II
  - ....
  - Structure of excited states: radiative transitions between mesons

Low-lying Hadron Spectrum



Durr et al., BMW Collaboration

Science 2008

Control over:
- ***Quark-mass dependence***
- ***Continuum extrapolation***
- ***finite-volume effects (pions, resonances)***

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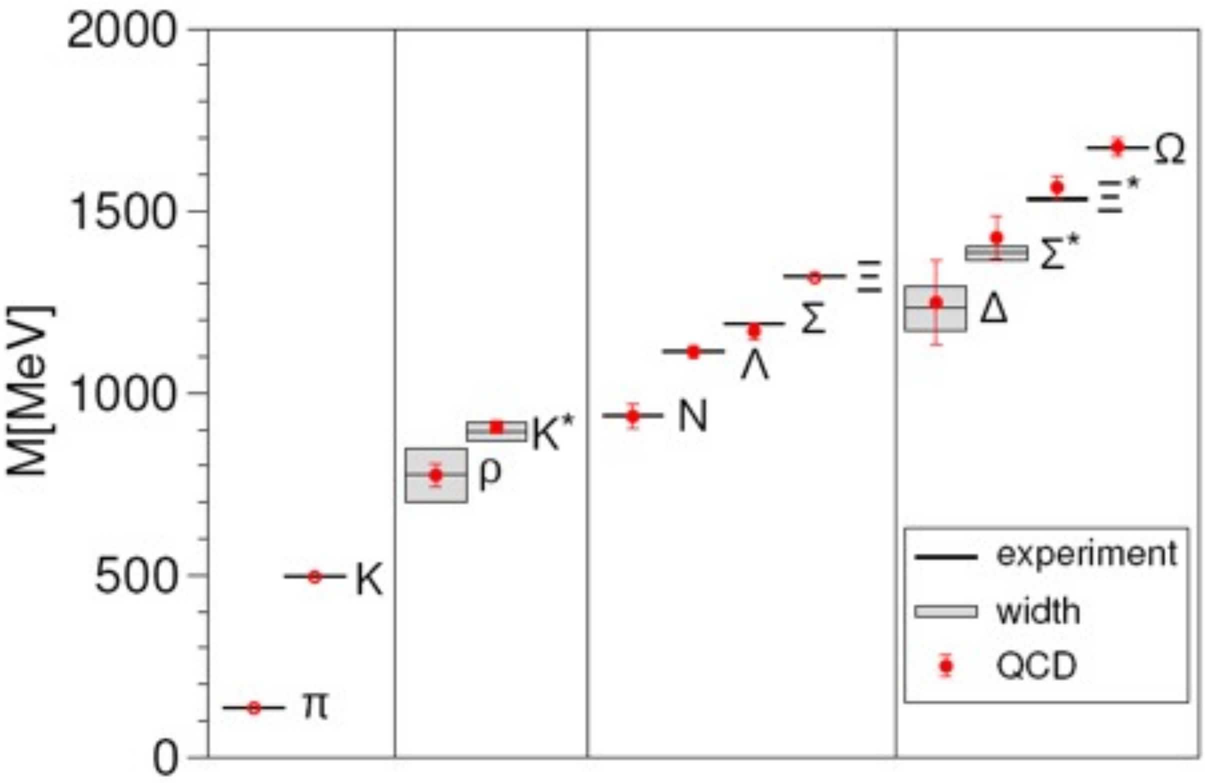

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Control over:
- ***Quark-mass dependence***
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- ***finite-volume effects***

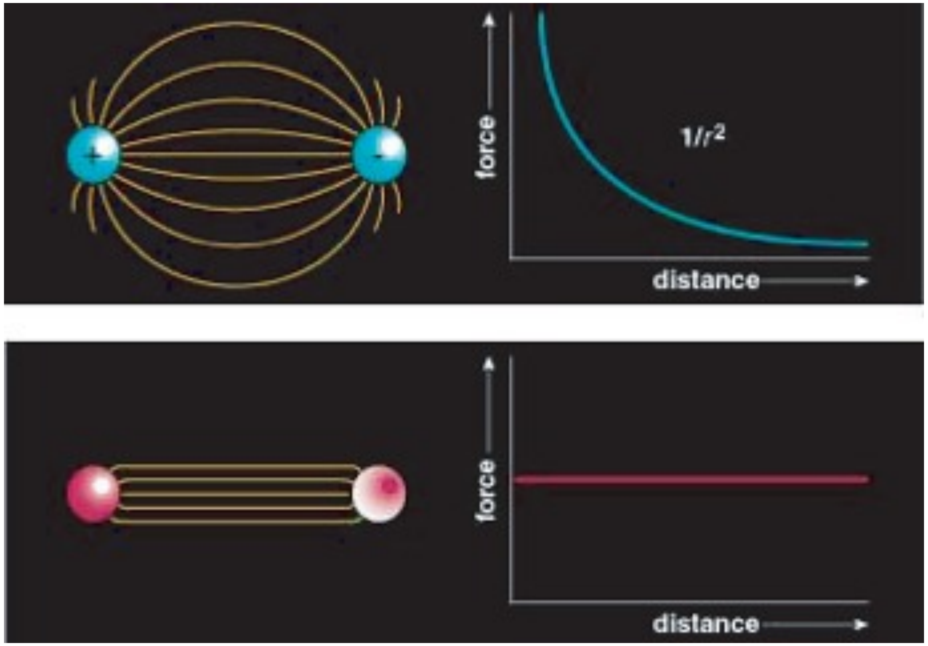
(pions, resonances)

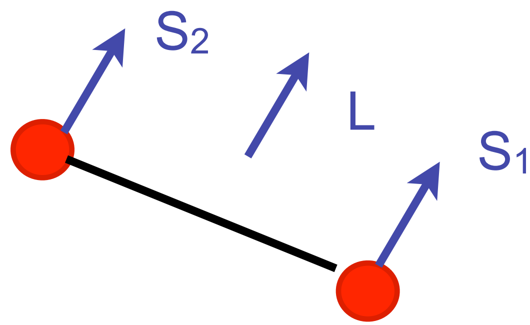
Benchmark calculation of QCD - *enabling us to do something else!*

Goals - I

- *Why is it important?*
  - *What are the key degrees of freedom describing the bound states?*
    - *How do they change as we vary the quark mass?*
  - *What is the origin of confinement, describing 99% of observed matter?*
  - *If QCD is correct and we understand it, expt. data must confront ab initio calculations*
  - *What is the role of the gluon in the spectrum – search for exotics?*

Goals - II



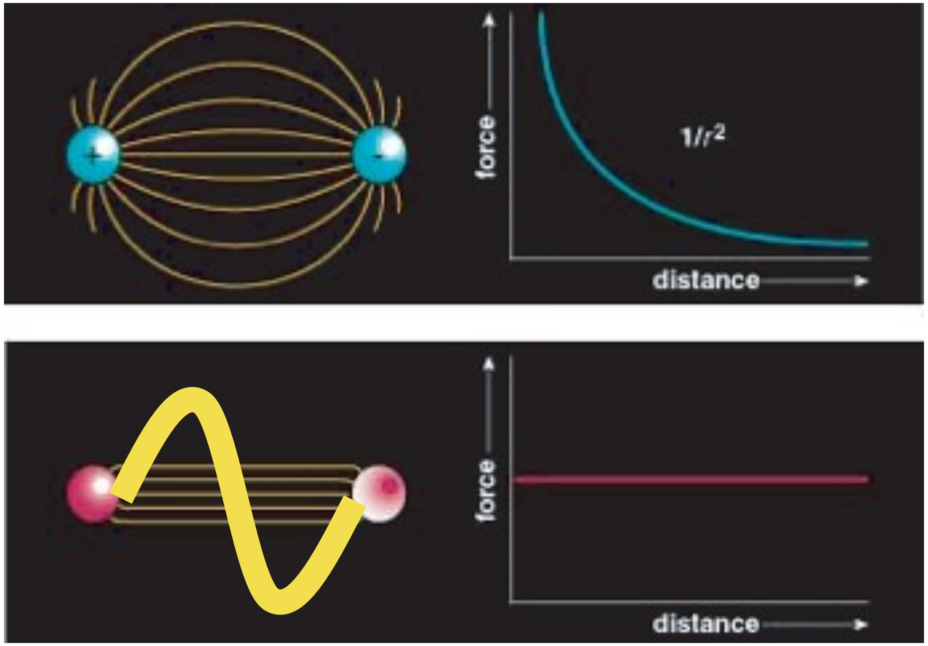


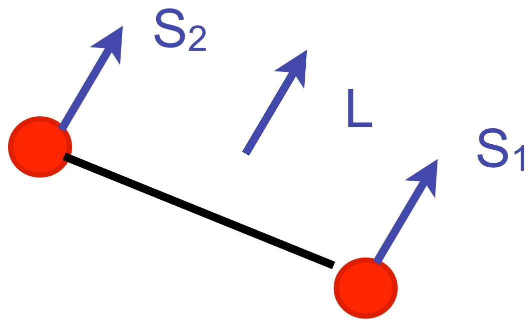
Simple quark model (for neutral mesons) admits only certain values of J^{PC}

$$P = (-1)^{l+1}$$
$$C = (-1)^{l+s}$$

- Exotic Mesons are those whose values of J^{PC} are in accessible to quark model: 0^{+-}, 1^{-+}, 2^{+-}
  - Multi-quark states:
  - Hybrids with *excitations of the flux-tube*
- Study of hybrids: revealing gluonic degrees of freedom of QCD.
- *Glueballs:* purely, or predominantly, gluonic states

Goals - II





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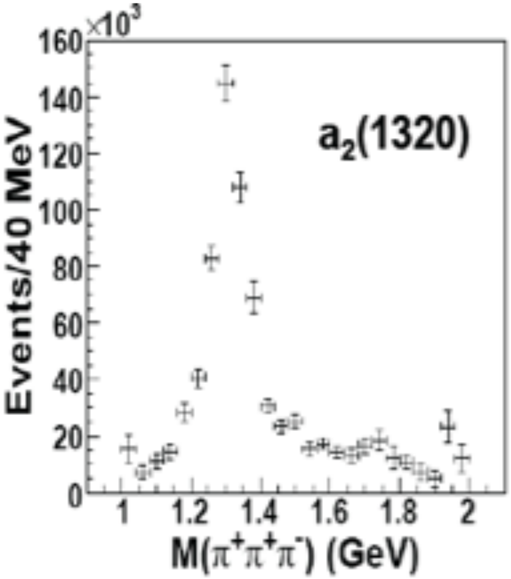
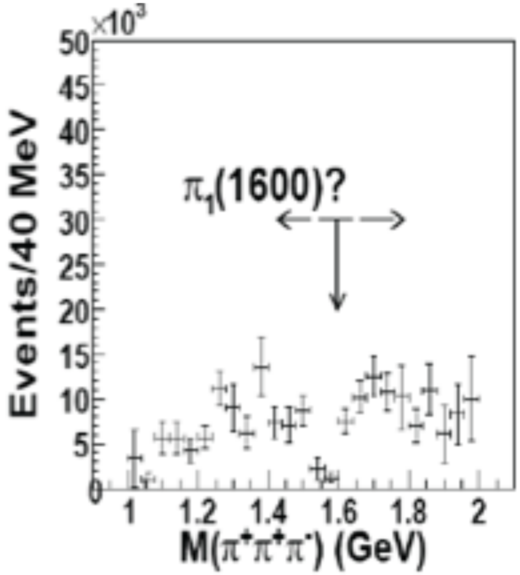
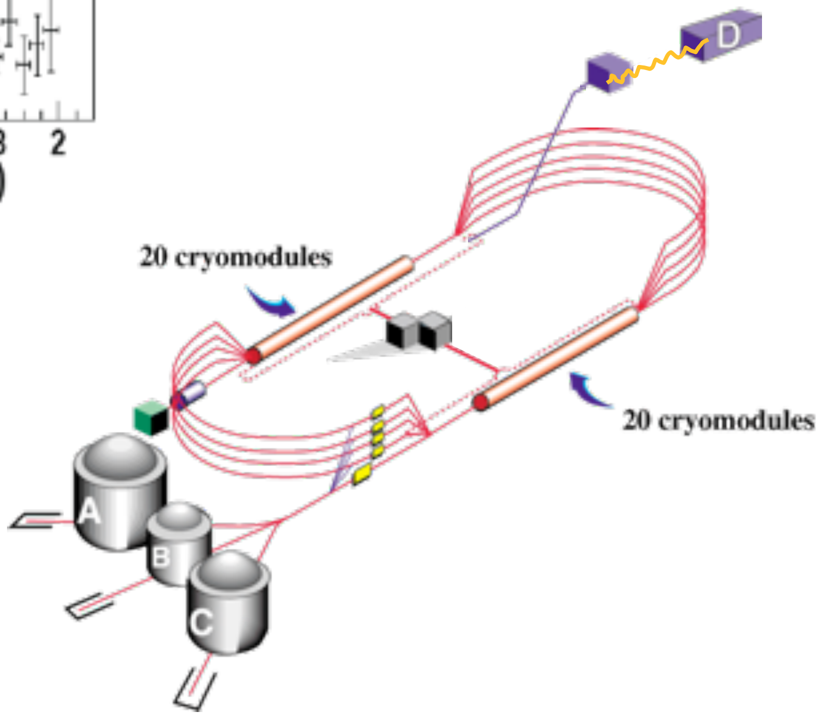
Lattice QCD: Hybrids and GlueX - I





π1(1600) in pion production at BNL

No clear evidence in photoproduction at CLAS



Variational Method

- Construct matrix of correlators

$$C_{\alpha\beta}(t,t_0) = \langle 0 \mid \mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(t_0) \mid 0\rangle \longrightarrow \sum_n Z^n_\alpha Z^{n\dagger}_\beta e^{-M_n(t-t_0)}$$

where $\{\mathcal{O}_\alpha\}$ are basis of operators of definite symmetry: P, C and J?

Delineate contributions using variational method: solve

$$C(t)u(t,t_0) = \lambda(t,t_0)C(t_0)u(t,t_0)$$

$$\lambda_i(t,t_0) \rightarrow e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

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Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

Challenges

- ➡ Resolve energy dependence - *anisotropic lattice*
- ➡ Judicious construction of interpolating operators - *cubic symmetry*

Anisotropic lattices

To appreciate difficulty of extracting excited states, need to understand signal-to-noise ratio in two-point functions. Consider correlation function:

$$C(t) = \langle 0 \mid \mathcal{O}(t)\mathcal{O}(0)^\dagger \mid 0 \rangle \longrightarrow e^{-Et}$$

Then the fluctuations behave as

DeGrand, Hecht, PRD46 (1992)

$$\sigma^2(t) \simeq \left(\langle 0 \mid \mathcal{O}(t)^2 \mathcal{O}(0)^{2^\dagger} \mid 0 \rangle - C(t)^2 \right) \longrightarrow e^{-2m_\pi t}$$

Signal-to-noise ratio degrades with increasing E - Solution: anisotropic lattice with $a_t < a_s$

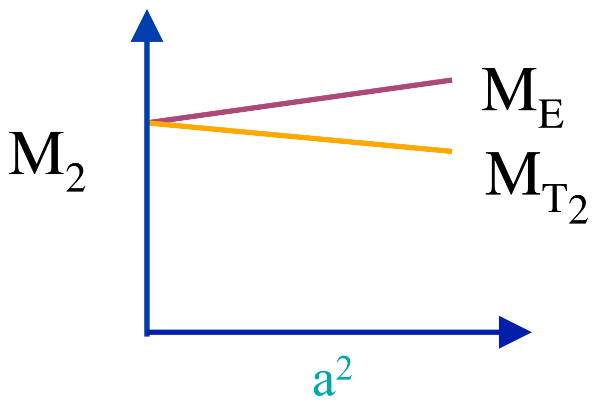
Challenges - II

- States at rest are characterized by their behavior under *rotations - SO(3)*

  Lattice does not possess full symmetry of the continuum - allowed energies characterised by cubic symmetry, or the octahedral point group *O_h*

  - *24 elements*
  - *5 conjugacy classes/5 irreducible representations*
  - *O_h x I_s*: rotations + inversions (parity)

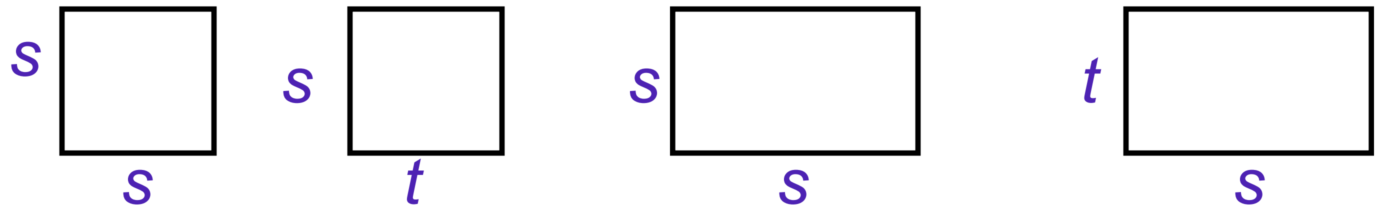
J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$



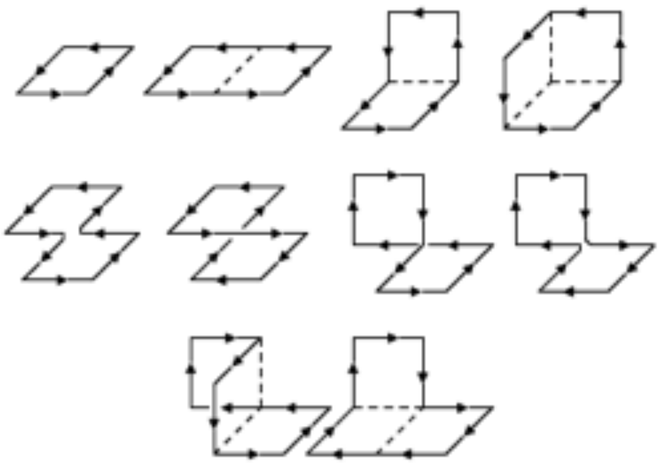
Glueball Spectroscopy - I

Improved anisotropic pure-gauge action Morningstar, Peardon 97,99

$$S[U] = \beta\xi\left\{\frac{5}{3U_s^4}P_{ss'} + \frac{4}{3\xi^2u_s^2u_t^2}P_{st} - \frac{1}{12u_s^6}R_{ss'} - \frac{1}{12\xi^2u_s^4u_t^2}R_{st}\right\}$$



Operators: closed Wilson loops



ξ is bare anisotropy a_s/a_t

Obtain renormalized anisotropy by comparing different Wilson Loops

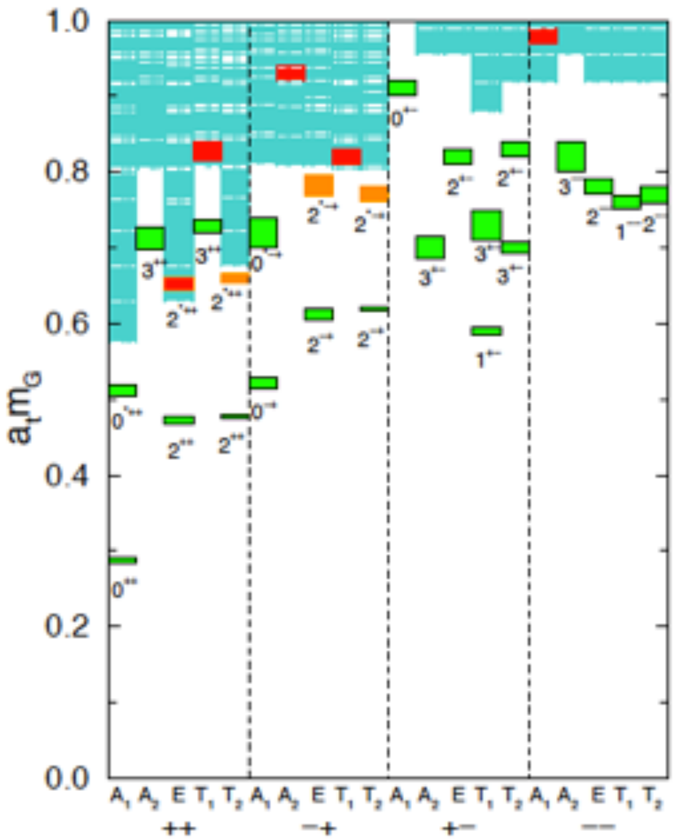
$$W_{xt}(Ia_s, Ja_t) \xrightarrow{J\to\infty} Z_{xt}e^{-Ja_tV(Ia_s,0,0)},$$
$$W_{xy}(Ia_s, Ja_s) \xrightarrow{J\to\infty} Z_{xy}e^{-Ja_s[V(Ia_s,0,0)+V_0]}$$

Ratio at large J gives ξ

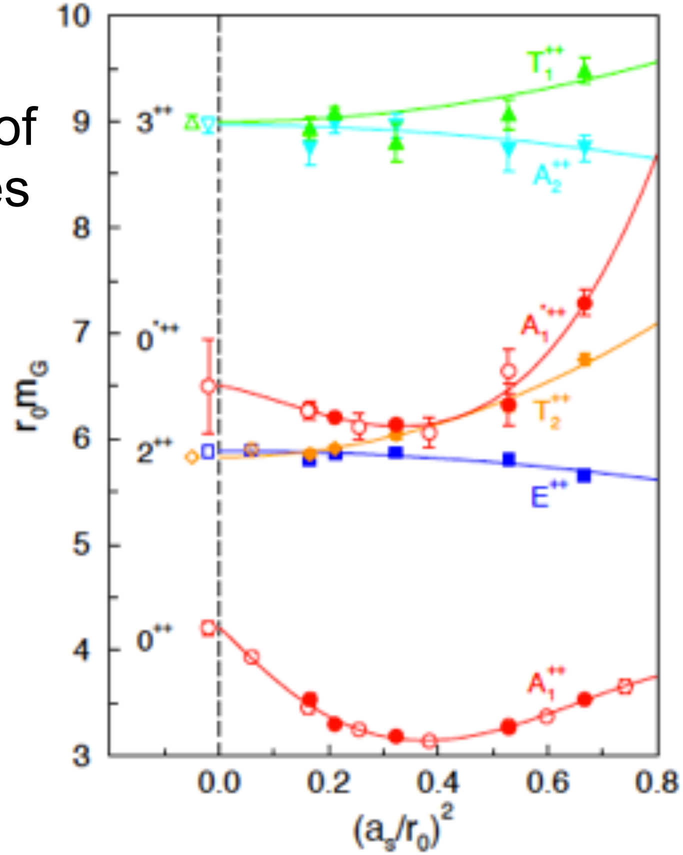
Morningstar, 96

Glueball Spectroscopy - II

$\beta = 2.5 : \ \xi = 5$



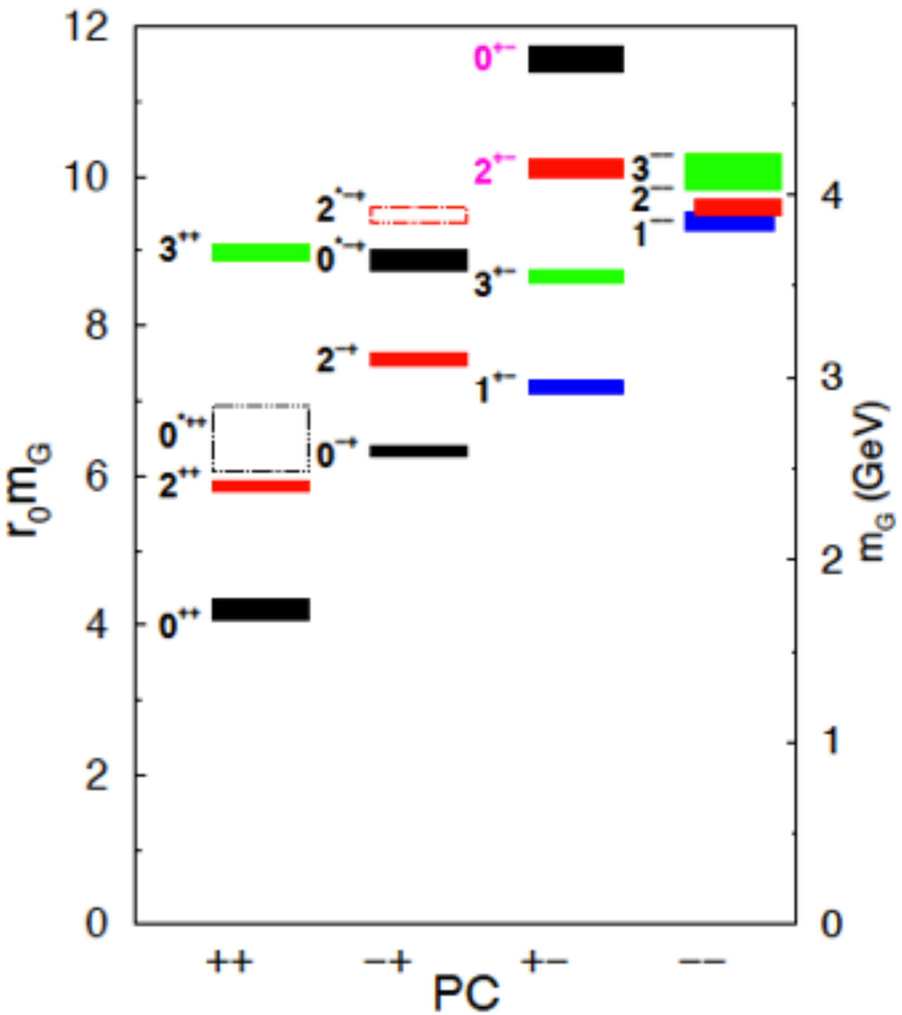
Observe emergence of degeneracies

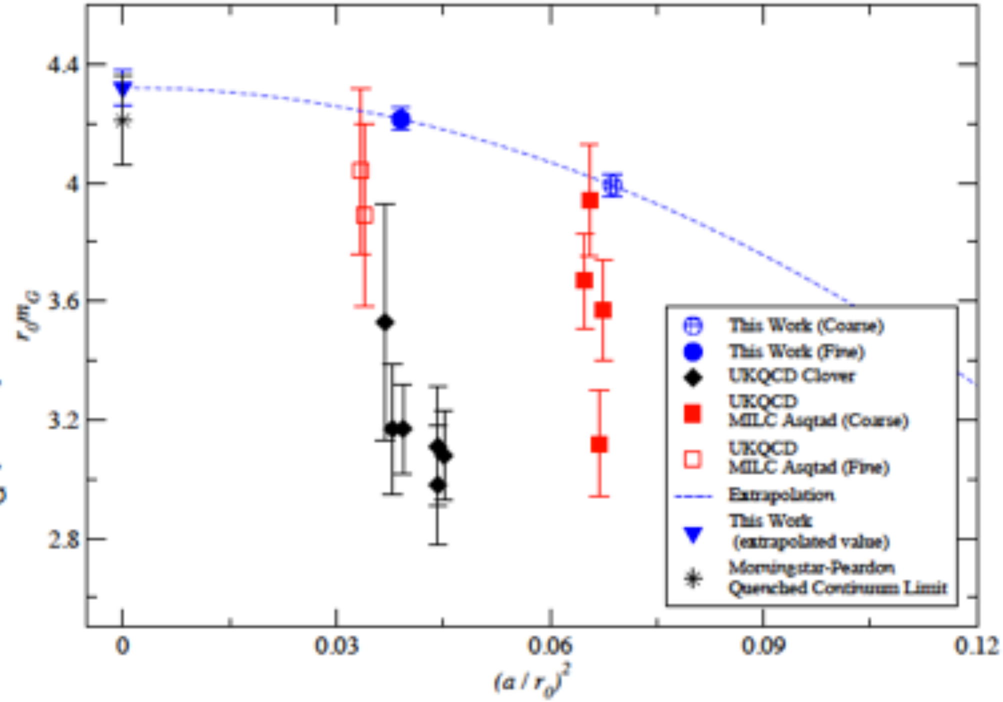


Glueball Spectrum - III

Note that this is the pure Yang-Mills spectrum - not the erroneously named “quenched” glueball spectrum!

UKQCD, C.Richards et al, arXiv:1005.2473





2+1 flavor staggered - can mix with two-pi states!

Meson spectroscopy with Quarks

- Anisotropic lattices - to precisely resolve energies
- Variational method - with sufficient operator basis to delineate states
- Many values of lattice spacing - identification of spin.

Anisotropic fermion action

Edwards, Joo, Lin, PRD78 (2008)

$$S_G^\xi[U] = \frac{\beta}{N_c\gamma_g}\left\{\sum_{x,s>s'}\left[\frac{5}{3u_s^4}\mathcal{P}_{ss'} - \frac{1}{12u_s^6}\mathcal{R}_{ss'}\right] + \sum_{x,s}\left[\frac{4}{3u_s^2u_t^2}\mathcal{P}_{st} - \frac{1}{12u_s^4u_t^2}\mathcal{R}_{st}\right]\right\}$$

$$S_F^\xi[U,\overline{\psi},\psi] = \sum x\overline{\psi}(x)\frac{1}{\tilde{u}_t}\left\{\tilde{u}_t\hat{m}_0 + \hat{W}_t + \frac{1}{\gamma_f}\sum_s \hat{W}_s - \frac{1}{2}\left[\frac{1}{2}\left(\frac{\gamma_g}{\gamma_f} + \frac{1}{\xi}\right)\frac{1}{\tilde{u}_t\tilde{u}_s^2}\sum_s \sigma_{ts}\hat{F}_{ts} + \frac{1}{\gamma_f}\frac{1}{\tilde{u}_s^3}\sum_{s<s'}\sigma_{ss'}\hat{F}_{ss'}\right]\right\}\psi(x).$$

Two anisotropy parameters to tune, in gauge and fermion sectors

$$\xi = 3.5 \qquad \begin{aligned}\gamma_g &= \xi_0 \\ \gamma_f &= \xi_0/\nu\end{aligned}$$

Dispersion Relation

Anisotropic Clover Generation - I

Tuning performed for three-flavor theory

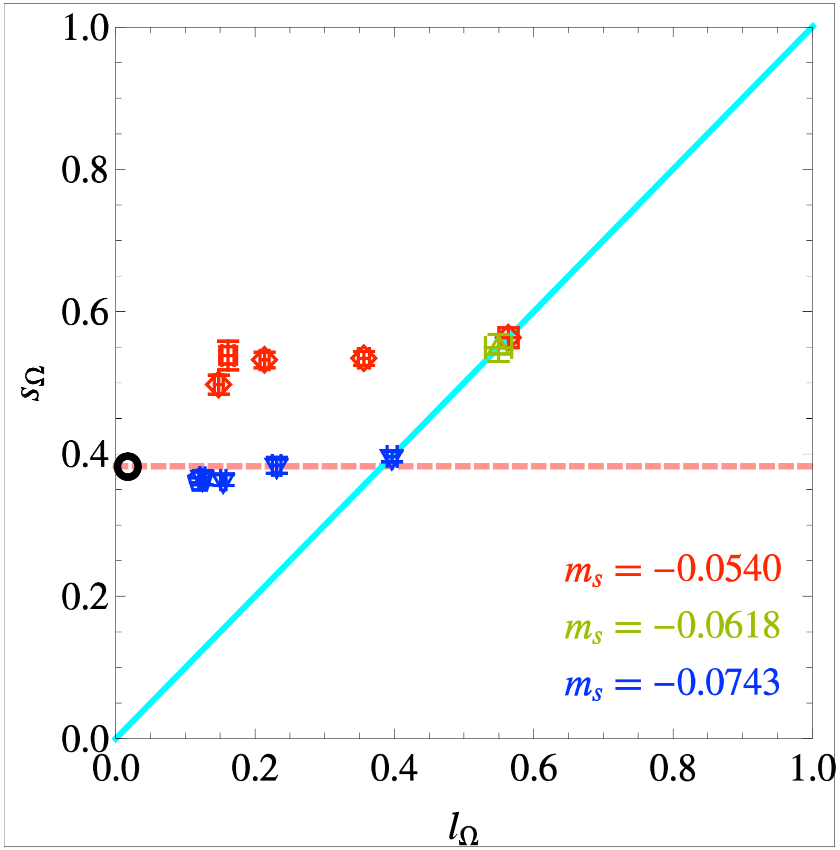
Challenge: setting scale and strange-quark mass

Lattice coupling *fixed*

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega

Express physics in (dimensionless) (l,s) coordinates



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H-W Lin et al (Hadron Spectrum Collaboration), PRD79, 034502 (2009)

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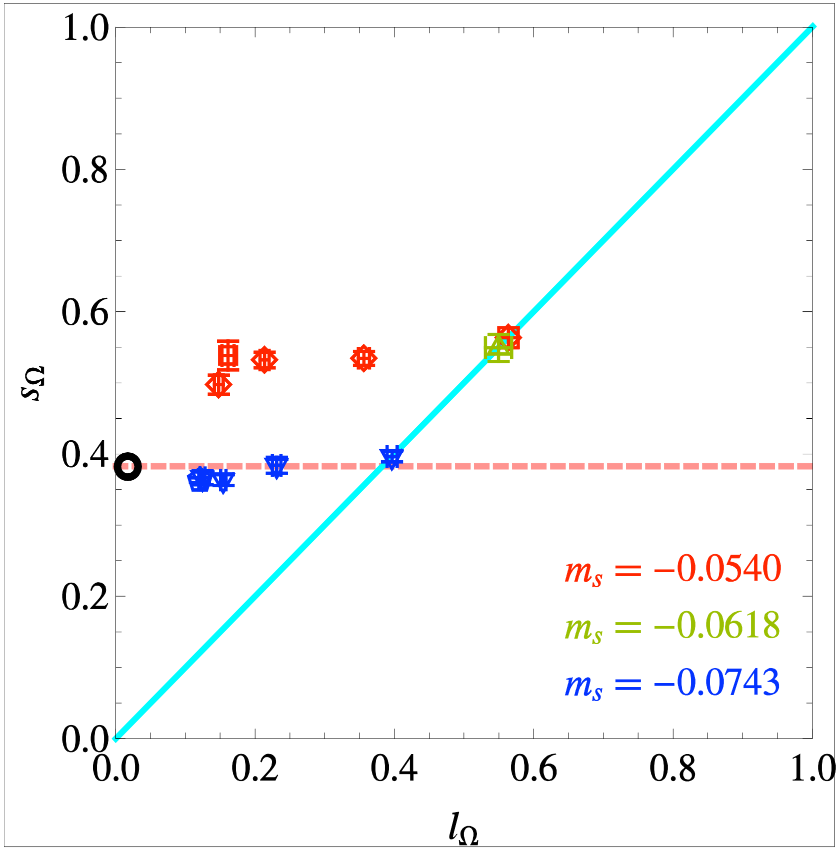

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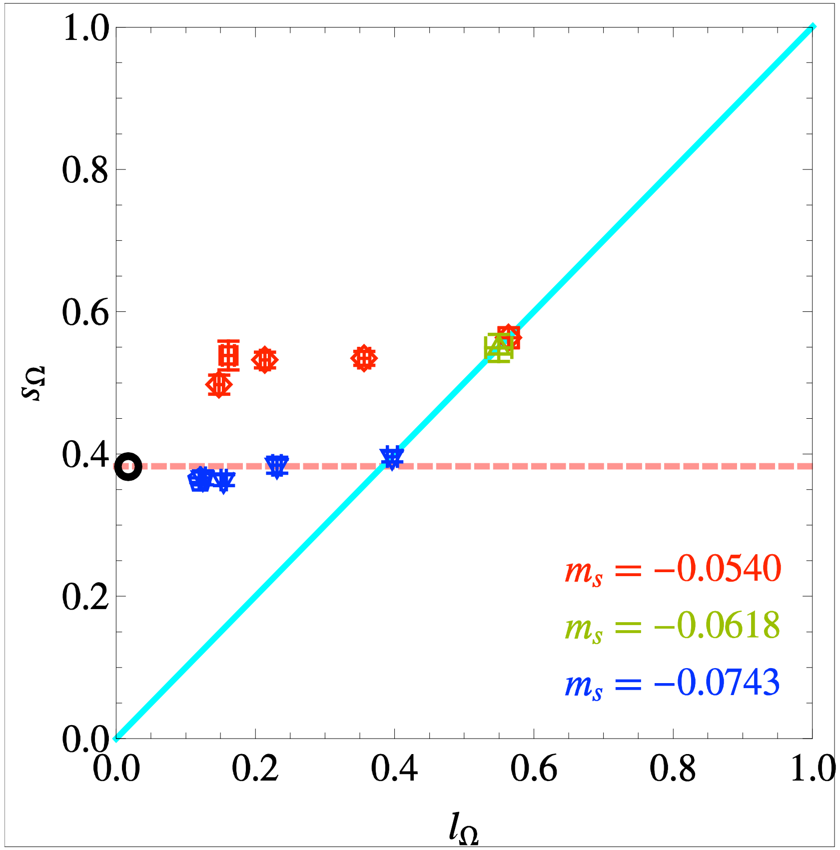

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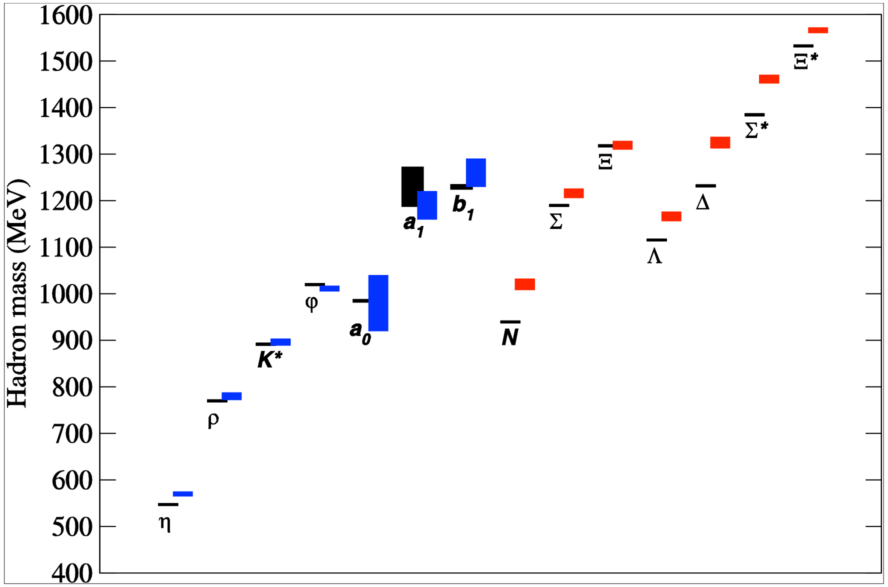
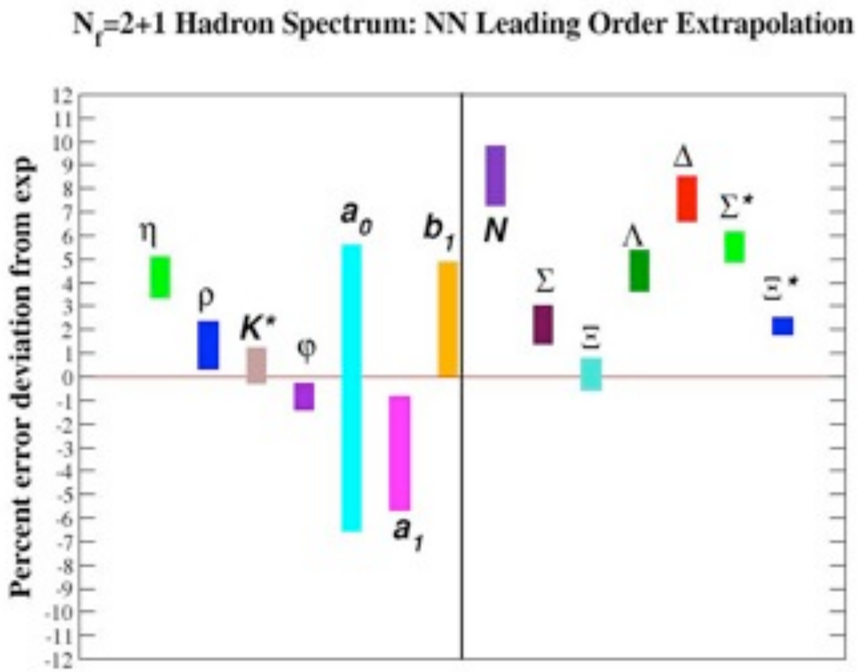
Anisotropic Clover – II



Low-lying spectrum: *agrees with experiment to 10%*



Correlation functions: Distillation

- Use the new "distillation" method.
- Observe

Eigenvectors of Laplacian

$$L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$

- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- Meson correlation function

Includes displacements

$$C_M(t,t') = \langle 0 \mid \bar{d}(t')\Gamma^B(t')u(t')\bar{u}(t)\Gamma^A(t)d(t)|0\rangle$$

- Decompose using "distillation" operator as

M. Peardon *et al.*, PRD80,054506 (2009)

$$C_M(t,t') = \mathrm{Tr}\langle \phi^A(t')\tau(t',t)\Phi^B(t)\tau^\dagger(t',t),\rangle$$

where

Perambulators

$$\begin{aligned}
\Phi^{A,ij}_{\alpha\beta} &= v^{*(i)}(t)[\Gamma^A(t)\gamma_5]_{\alpha\beta} v^{(j)}(t') \\
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Perambulators

Identification of Spin - I

Problem:

- YM glueball requires data at several lattice spacings
- density of states in each irrep large.

Solution: exploit known continuum behavior of overlaps

- **Construct interpolating operators of *definite* (continuum) JM: O^{JM}**

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point

$$\bar{\psi}(\vec{x}, t) \Gamma D_i D_j \ldots \psi(\vec{x}, t)$$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x - i \overleftrightarrow{D}_y \right)$$

$$\overleftrightarrow{D}_{m=0} = i \overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}} \left(\overleftrightarrow{D}_x + i \overleftrightarrow{D}_y \right).$$

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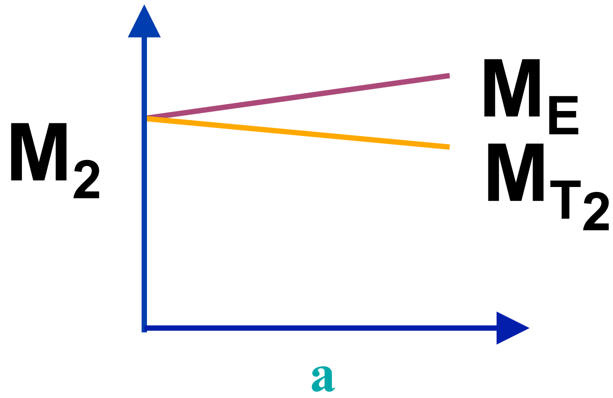

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Identification of spin

Straighforward to project to definite spin: *J = 0, 1, 2*

$$(\Gamma \times D^{[1]}_{J=1})^{J,M} = \sum_{m_1,m_2} \left\langle 1, m_1; 1, m_2 \middle| J, M \right\rangle \bar{\psi}\Gamma_{m_1} \overleftrightarrow{D}_{m_2}\psi.$$

- **Use projection formula to find subduction under irrep. of cubic group - operators are closed under rotation!**

$$O^{[J]}_{\Lambda\lambda}(t,\vec{x}) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R\in O_h^D} D^{(\Lambda)*}_{\lambda\lambda}(R) U_R O^{J,M}(t,\vec{x}) U_R^\dagger$$

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Irrep, Row

Action of R

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Irrep, Row **Irrep of R in Λ** **Action of R**

Exercise: check LHS transforms irreducibly

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} \equiv (\Gamma \times D^{[n_D]}_{\dots})^J_{\Lambda,\lambda} =$$

$$\sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \, (\Gamma \times D^{[n_D]}_{\dots})^{J,M} \equiv \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \mathcal{O}^{J,M}$$

Identification of Spin - II

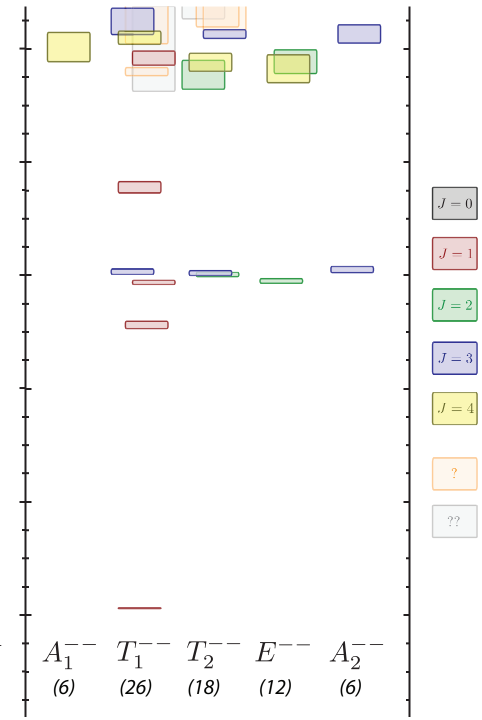
Hadspec collab. (dudek et al), 0909.0200, PRL

Overlap of state onto subduced operators

$$\langle 0 \,|\, O^{J,M} \,|\, J', M' > \;=\; Z_J \delta_{J,J'} \delta_{M,M'}$$

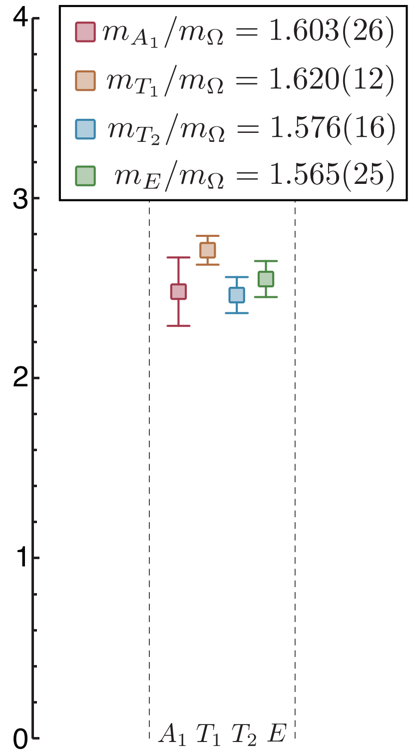
$$\langle 0 \,|\, O^{J}_{\Lambda,\lambda} \,|\, J', M' \rangle \;=\; S^{J,M'}_{\Lambda,\lambda} Z_J \delta_{J,J'}$$

Common across irreps.



J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

$$N_f = 3$$



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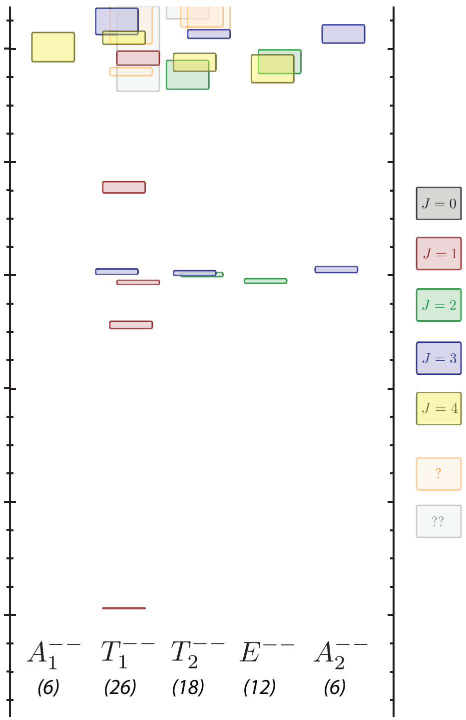

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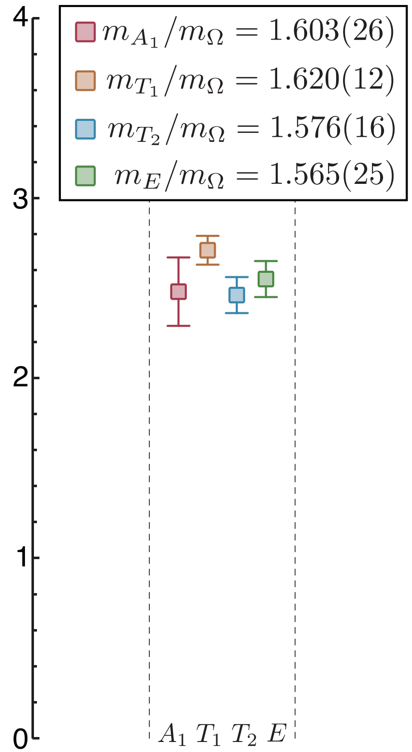

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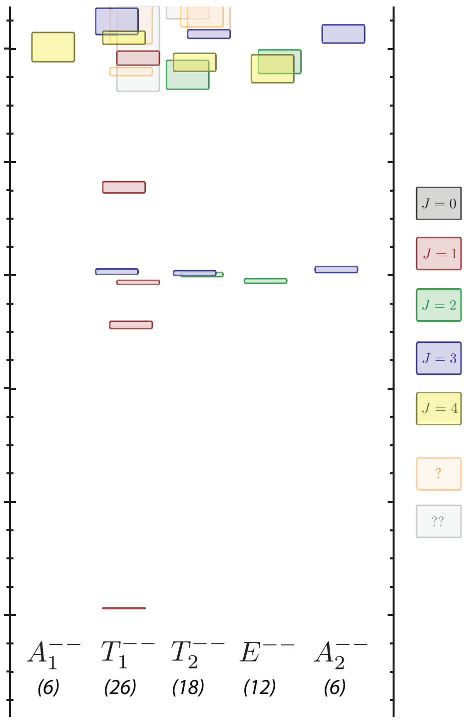

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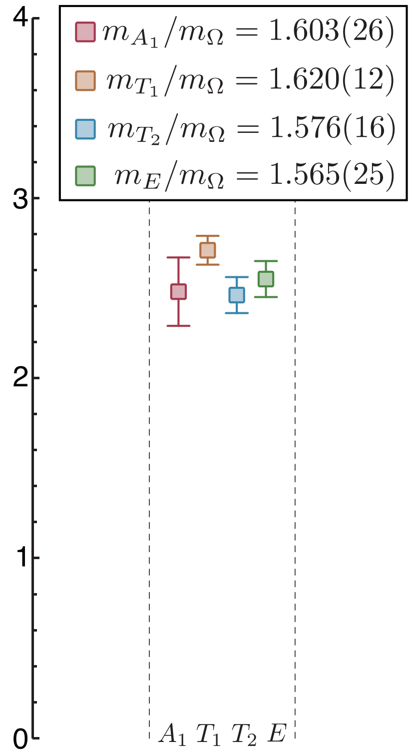
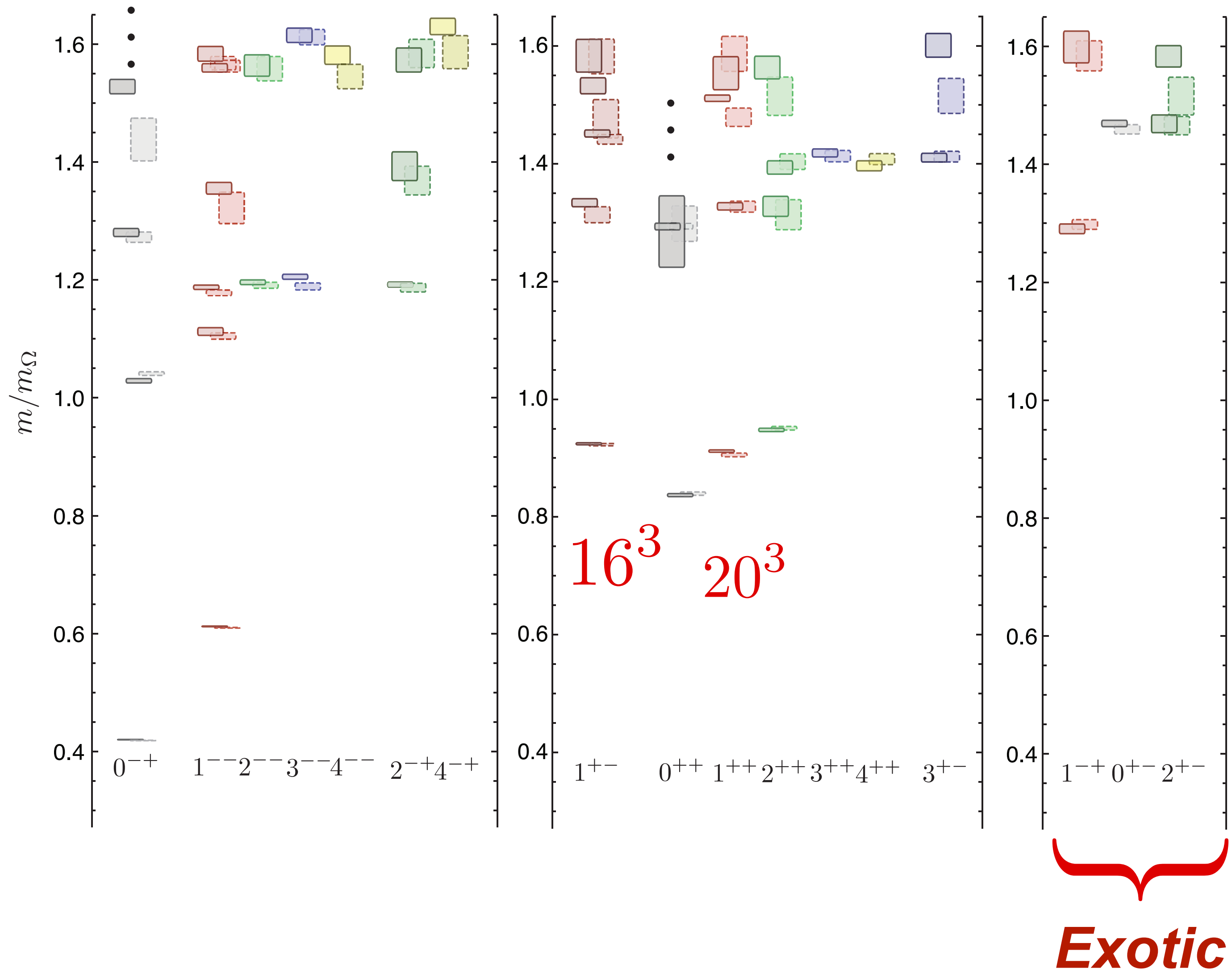


Lattice ops. retain memory of their continuum ancestors

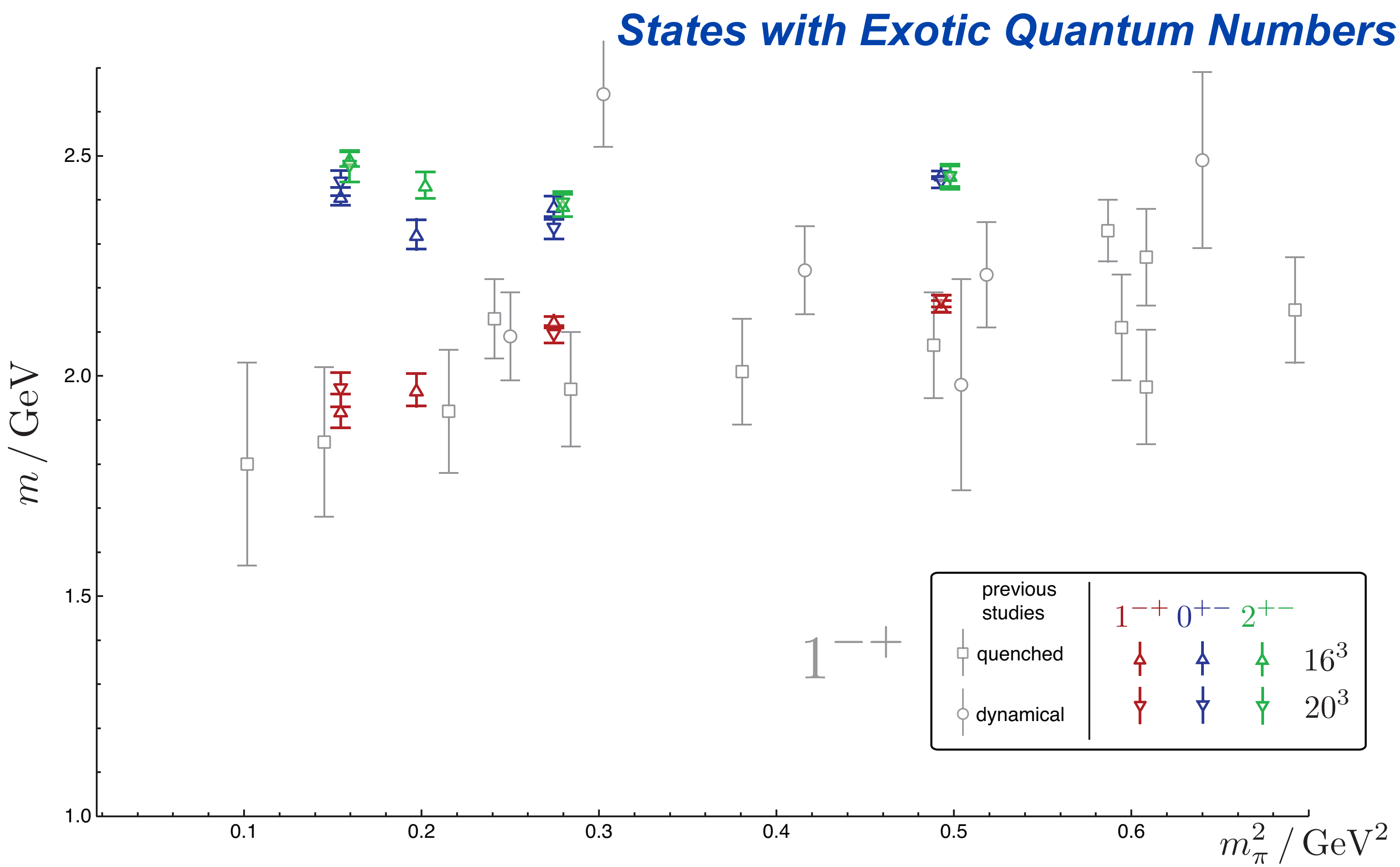
Isovector Meson Spectrum - I



PRL
103:262001,2009

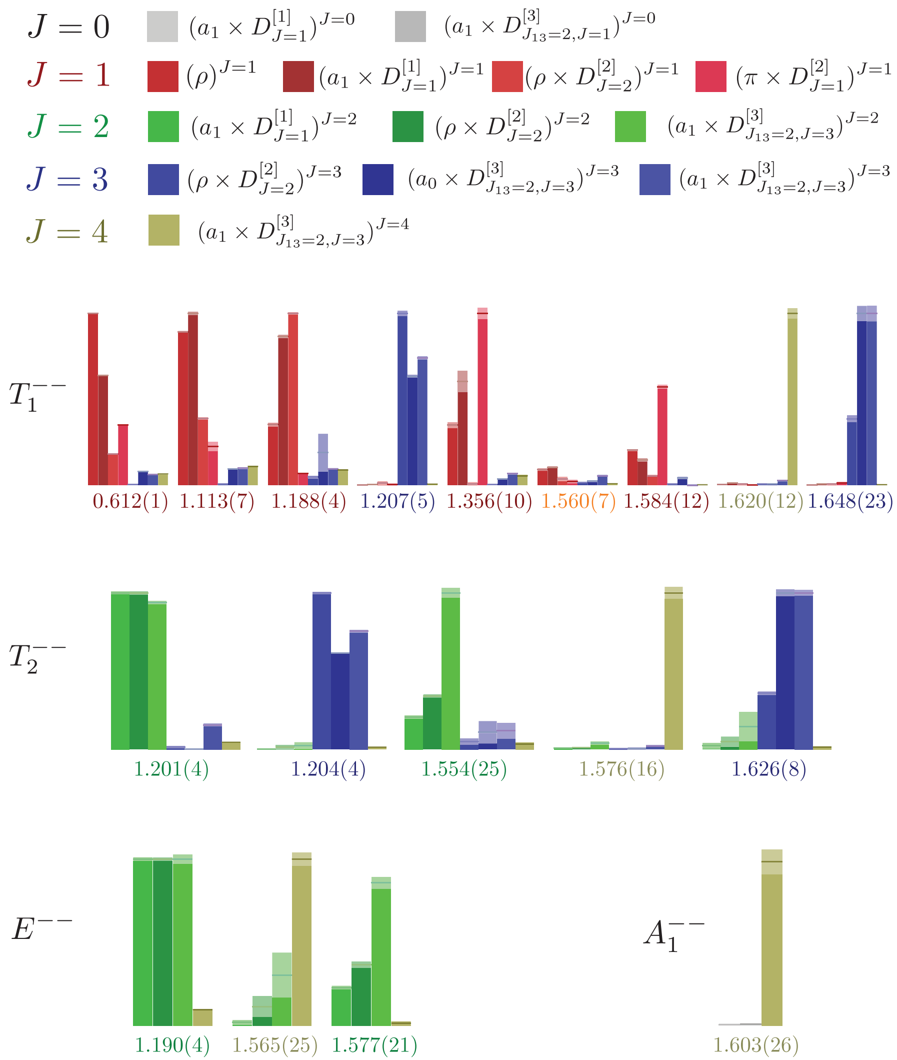
Isovector spectrum with quantum numbers reliably identified

Isovector Meson Spectrum - II



Dudek, Edwards, DGR, Thomas, arXiv:1004.4930

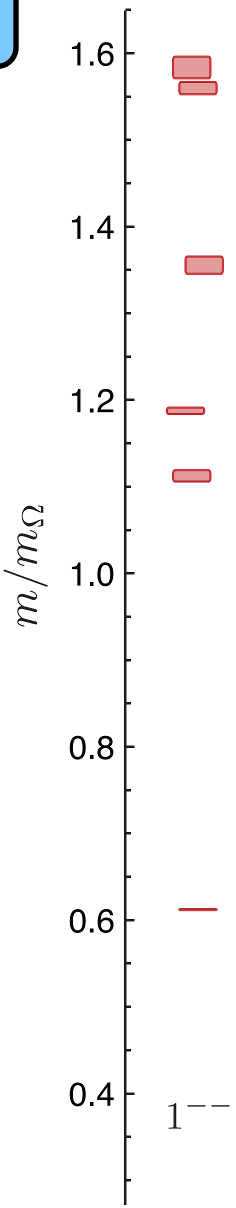
Interpretation of Meson Spectrum



In each Lattice Irrep, state dominated by operators of particular J

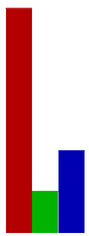
1^{--}

look at the 'overlaps' $Z_n^{\mathbf{\Gamma}} = \langle n | \bar{\psi} \mathbf{\Gamma} \psi | 0 \rangle$



ρ	$^3S_1$
$\left(\rho \times D^{[2]}_{J=2}\right)^{J=1}$	$^3D_1$
$\left(\pi \times D^{[2]}_{J=1}\right)^{J=1}$	hybrid?

Anti-commutator of covariant derivative: vanishes for unit gauge!



ground state is dominantly $^3S_1$

1^{--}

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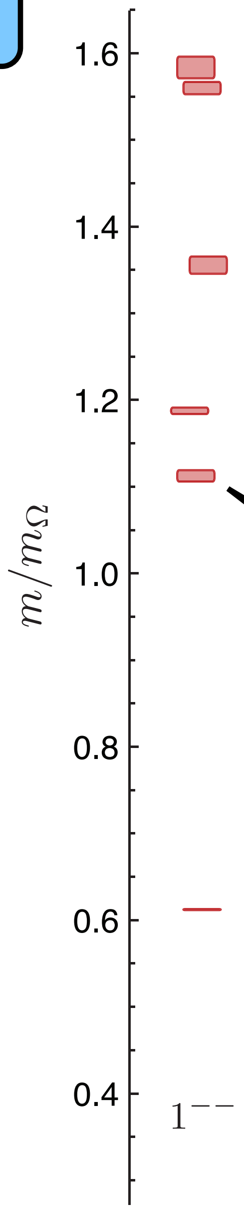
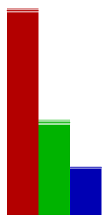
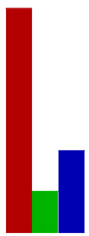




1^{st} excited state is dominantly
$^3S_1$
with some
$^3D_1$



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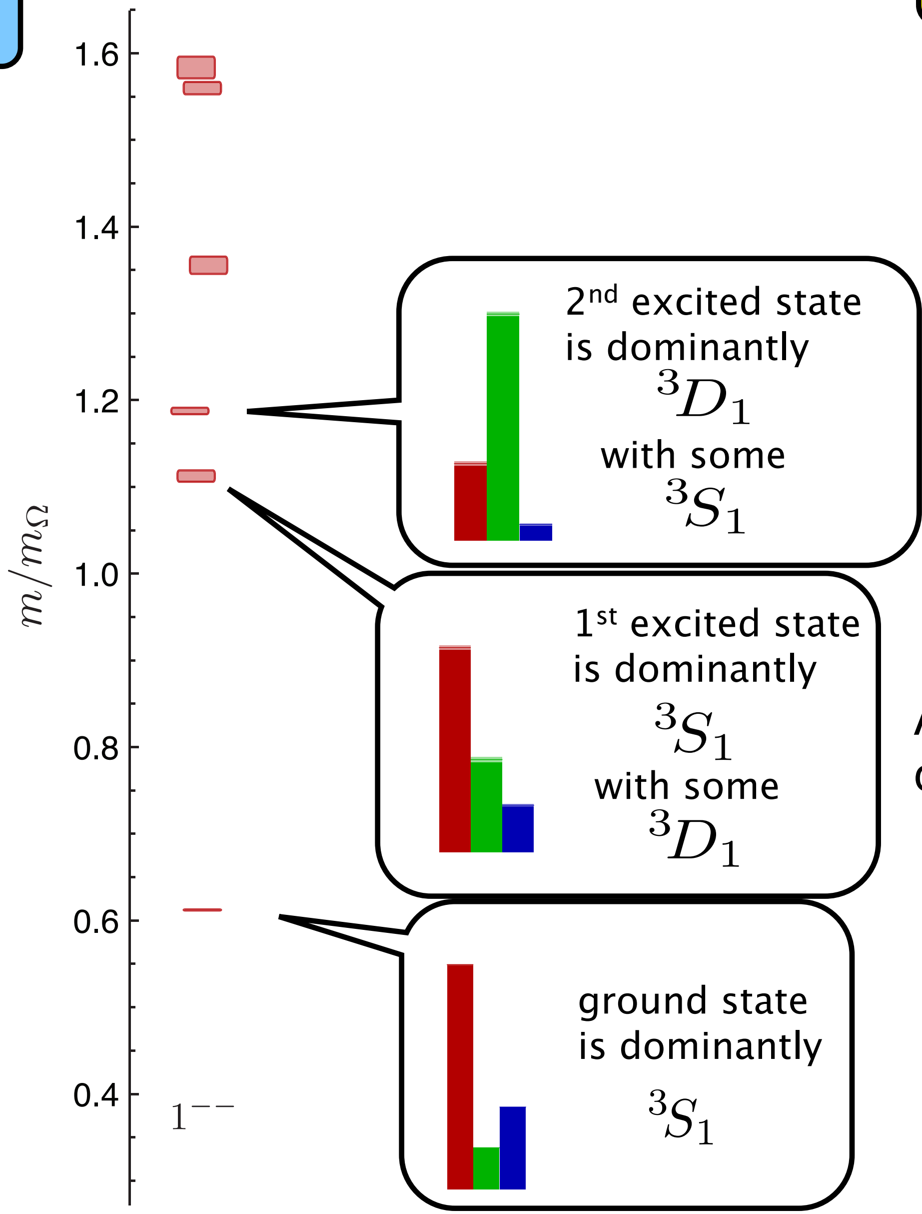

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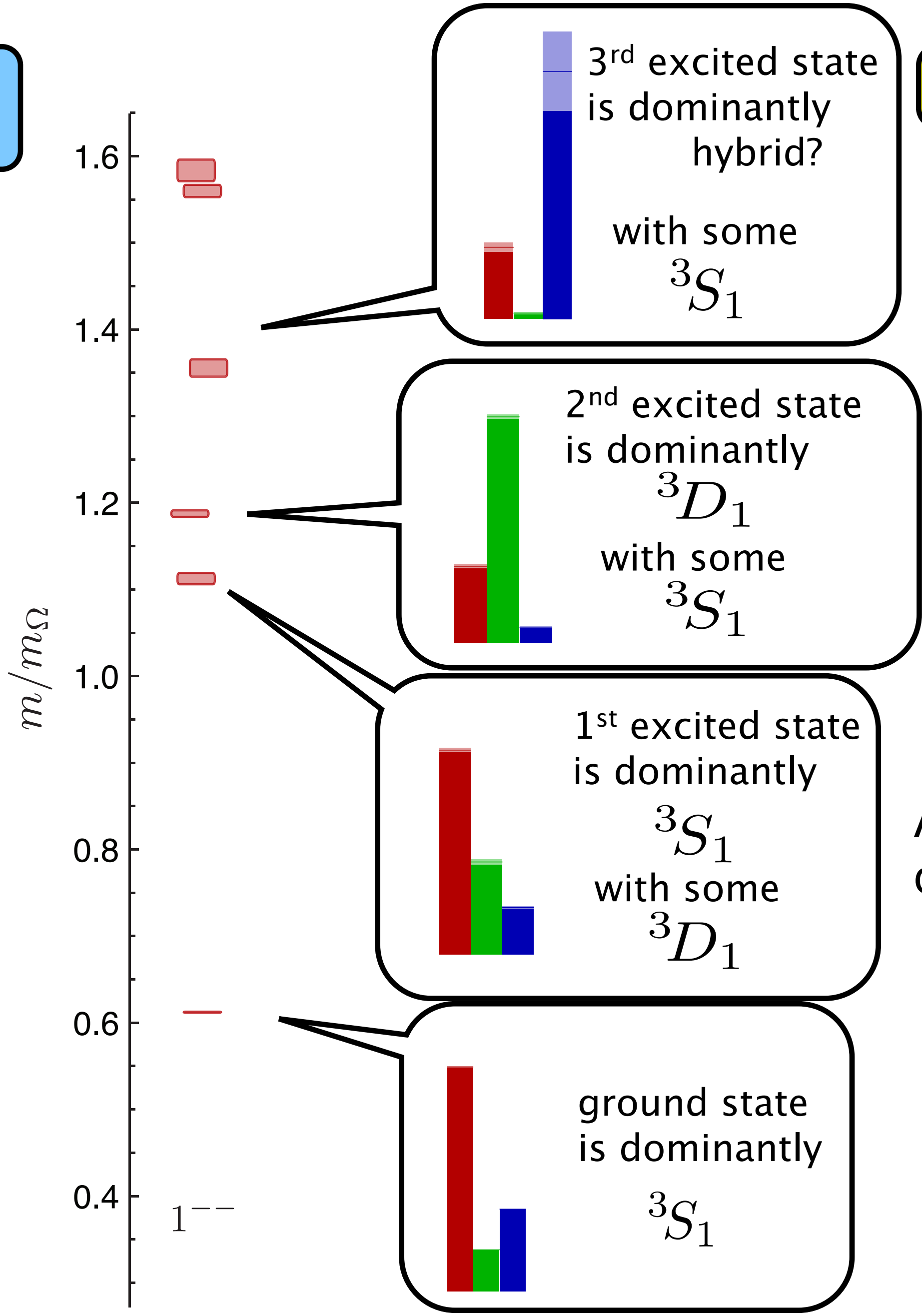

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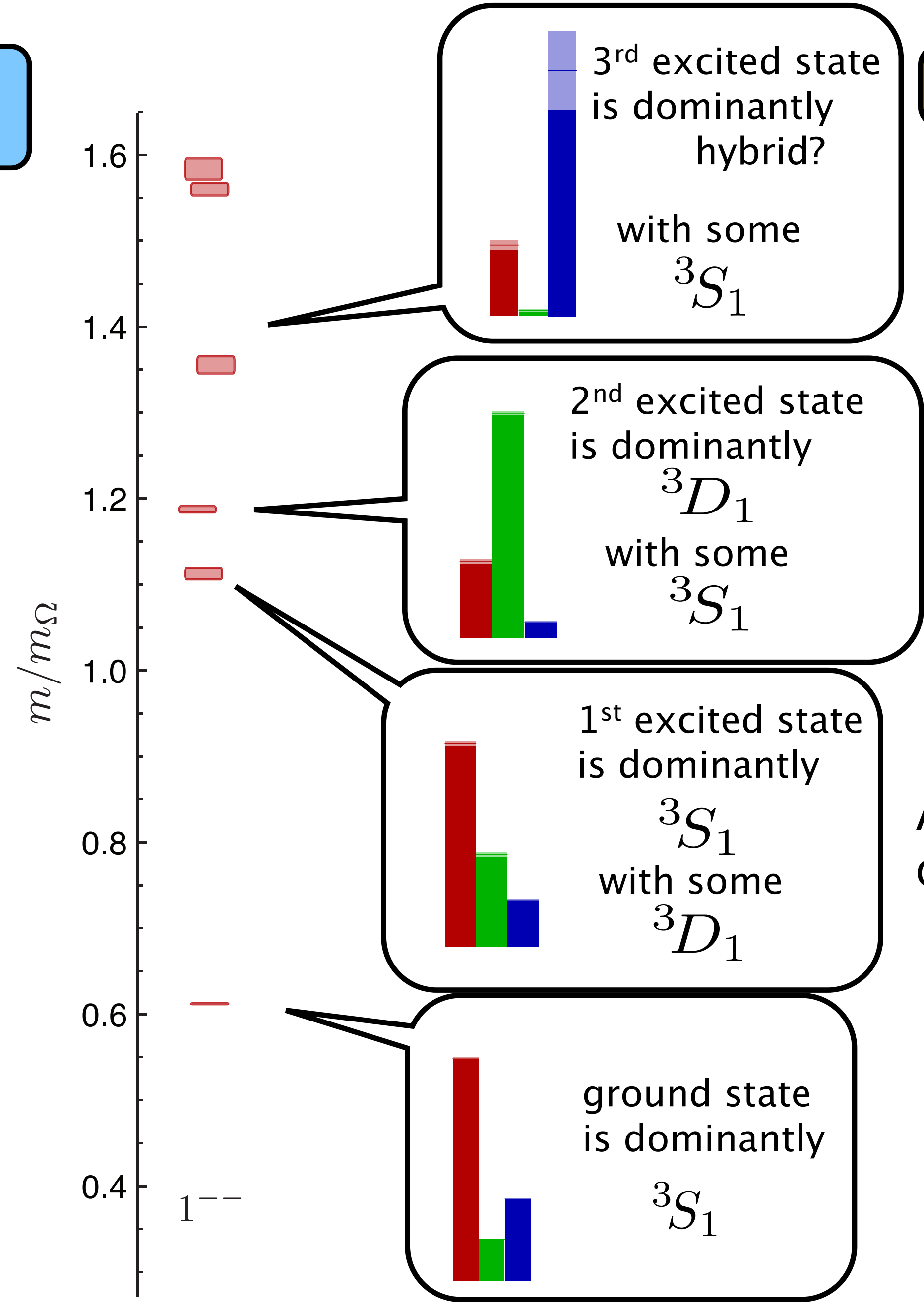

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Anti-commutator of covariant derivative: vanishes for unit gauge!

build a **bound state model** phenomenology comparable to the quark model using non-perturbative **QCD** calculations

Isoscalar Meson Spectrum

Isoscalar requires disconnected contributions

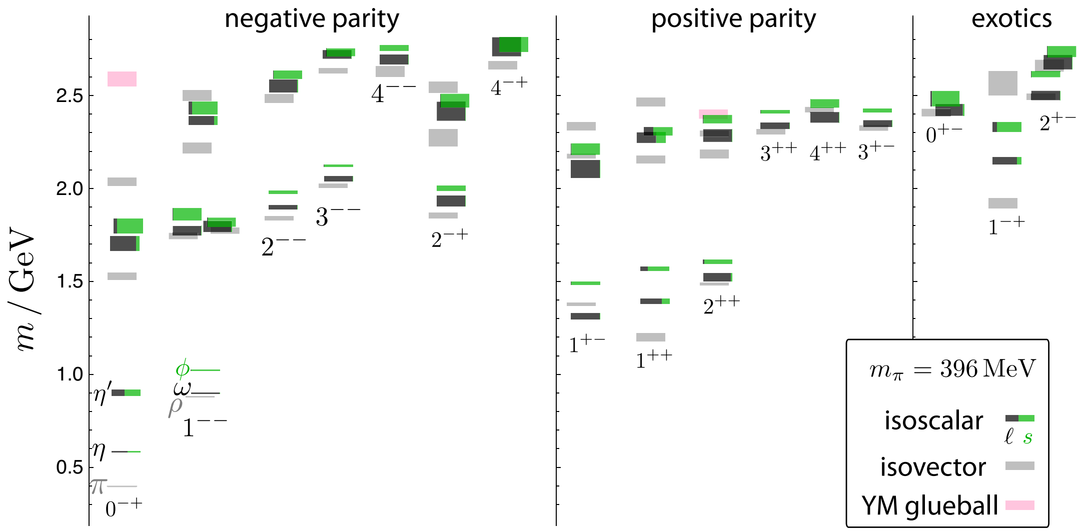
Require perambulators at each timeslice

$$\tau_q(t_1,t_2) = V^\dagger_{t_1} M_q^{-1}(t_1,t_2) V_{t_2}$$

t' t

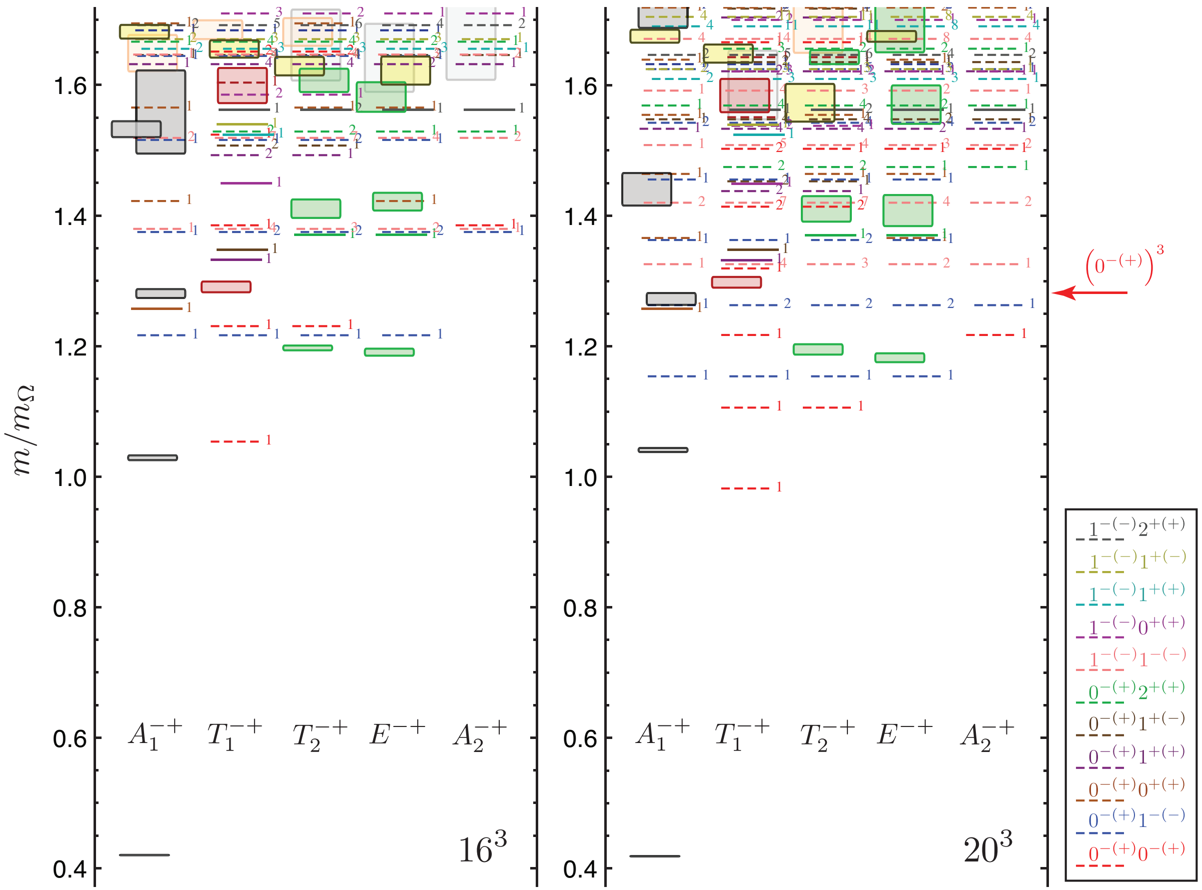
Dominated by quark-propagator inversions - ENABLED BY GPU

J. Dudek *et al.*, arXiv:1102.4299



- **Spin-identified single-particle spectrum: states of spin as high as four**
- **Hidden flavor mixing angles extracted - except 0^{-+}, 1^{++} near ideal mixing**
- ***First determination of exotic isoscalar states: comparable in mass to isovector***

Where are the multi-hadrons?



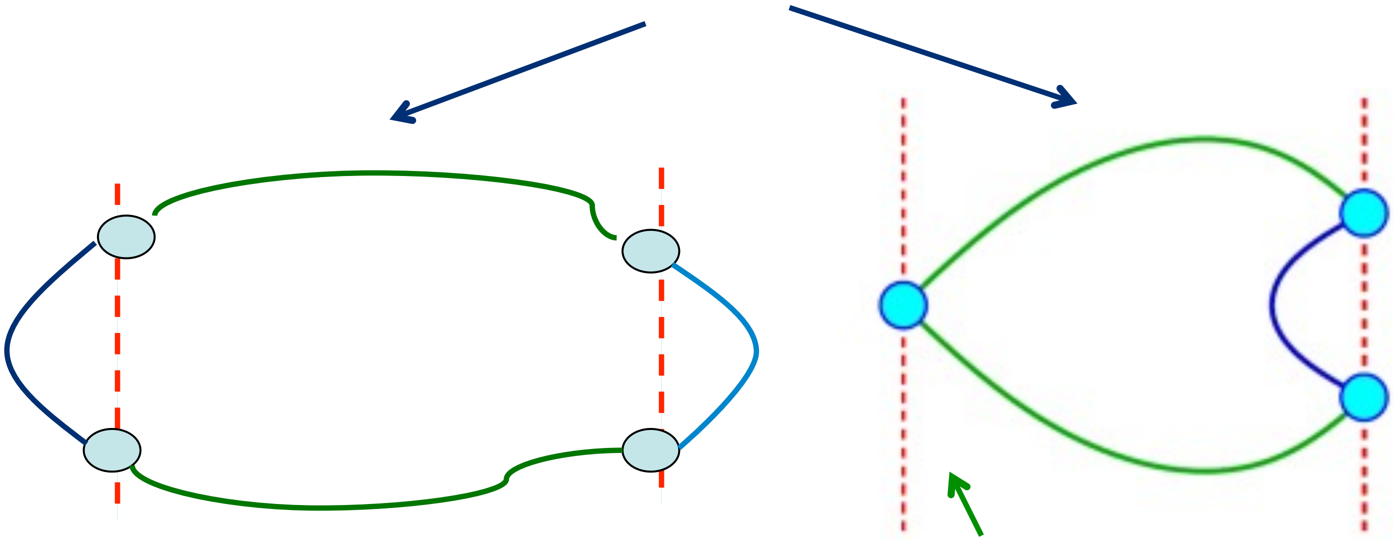
Meson spectrum on two volumes: dashed lines denote expected (non-interacting) multi-particle energies.

Allowed two-particle contributions governed by cubic symmetry of volume

Calculation is incomplete.

Multi-hadron Operators

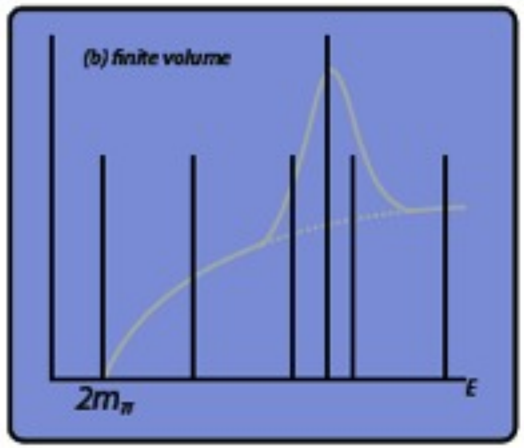
Need "all-to-all"



Usual methods give "point-to-all"

Strong Decays

- In QCD, even ρ is unstable under strong interactions – *resonance in π-π scattering (quenched QCD not a theory – won't discuss).*
- Spectral function continuous; finite volume yields discrete set of energy eigenvalues



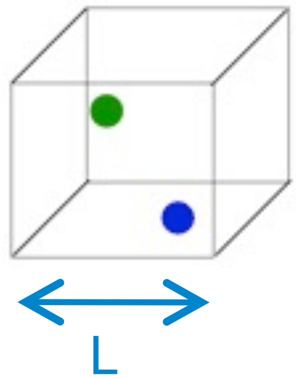
Momenta quantised: known set of free-energy eigenvalues

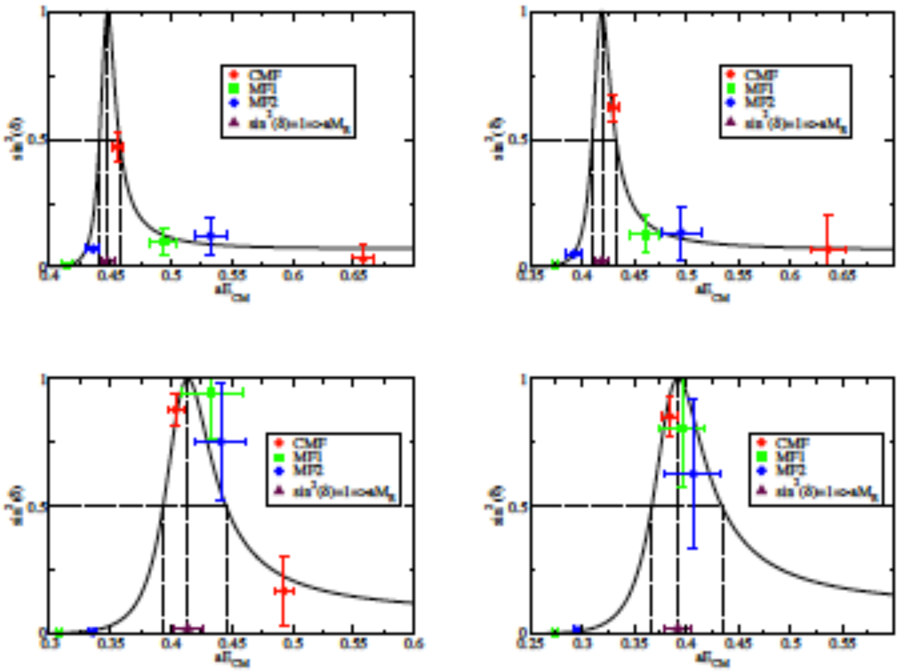
$$E_n = 2\sqrt{m_\pi^2 + (\frac{2n\pi}{L})^2}$$

Strong Decays - II

- For interacting particles, energies are shifted from their free-particle values, by an amount that depends on the energy.
- Luscher: relates shift in the free-particle energy levels to the phase shift at the corresponding E.

$$\delta E(L) \leftrightarrow \delta(E)$$





$$\tan\delta_1 = \frac{{g_{\rho\pi\pi}}^2}{6\pi}\frac{p^3}{E_{\mathrm{CM}}({m_\rho}^2 - E_{\mathrm{CM}}^2)}$$

$$p = \sqrt{E_{\mathrm{CM}}^2/4 - m_\pi^2}$$

Feng, Jansen, Renner, 2010

Lang, these lectures

Momentum-dependent I = 2 ππ Phase Shift

Dudek *et al.*, Phys Rev D83, 071504 (2011)

Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

$$\det\left[e^{2i\boldsymbol{\delta}(k)} - \mathbf{U}_{\Gamma}\left(k\frac{L}{2\pi}\right)\right] = 0$$

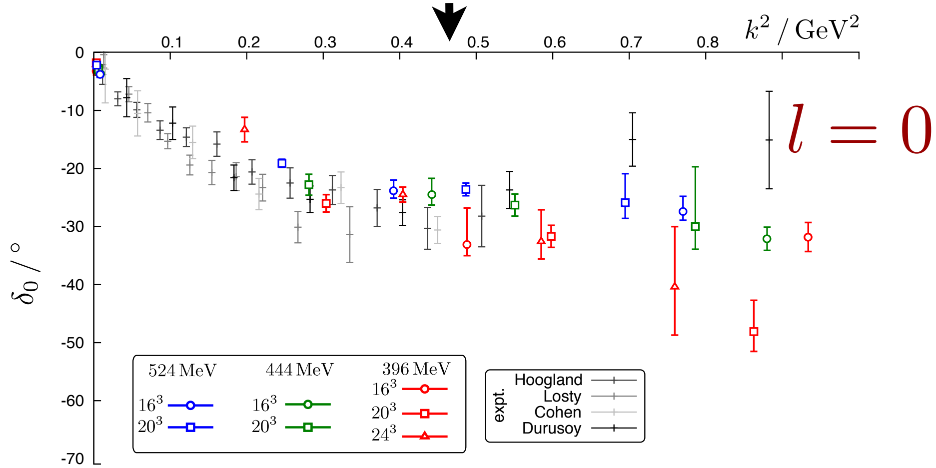
Matrix in l

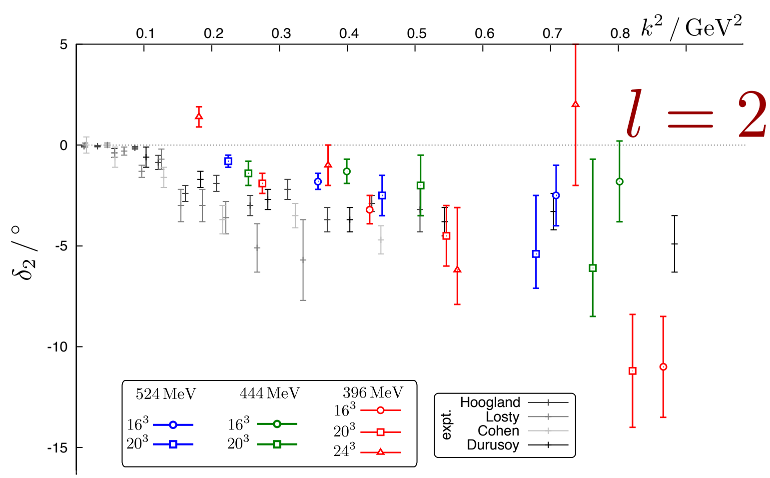
lattice irrep

Operator basis $\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_{m} \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^{m}_{\ell}(\hat{p})\, \mathcal{O}_{\pi}(\vec{p})\mathcal{O}_{\pi}(-\vec{p})$

Total momentum zero - pion momentum ±*p*

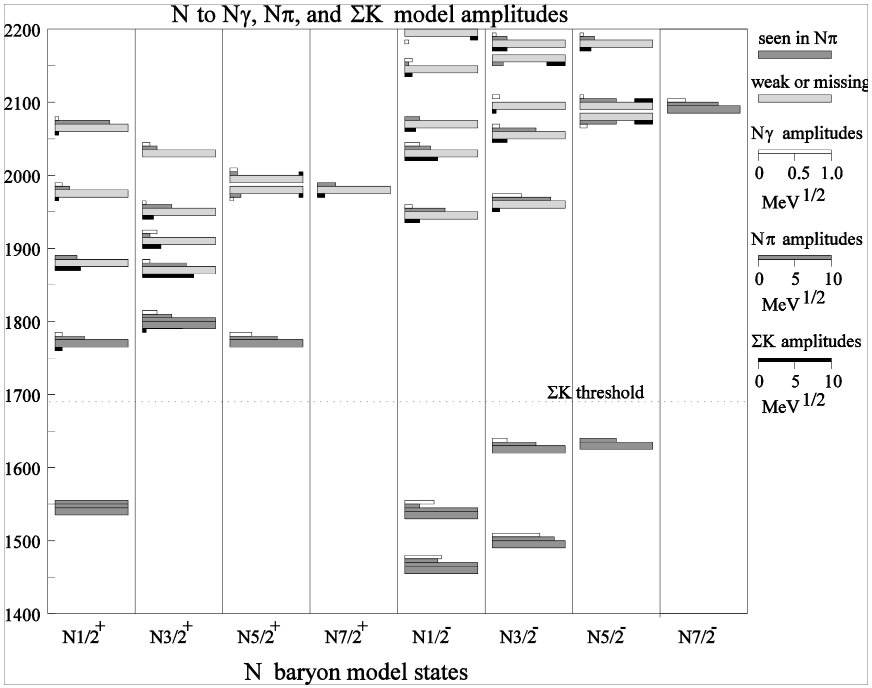
4π at $m_\pi = 396$ MeV





Excited Baryon Spectrum

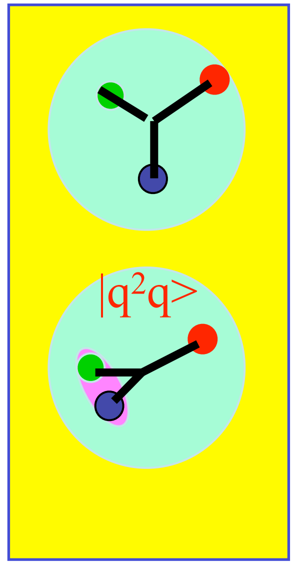
- No baryon "exotics", ie quantum numbers not accessible with simple quark model; but may be hybrids!
- Nucleon Spectroscopy: Quark model masses and amplitudes – states classified by isospin, parity and spin.



Capstick and Roberts, PRD58 (1998) 074011

- Missing, because our pictures do not capture correct degrees of freedom?
- Do they just not couple to probes?

$|q^3\rangle$



CLAS at JLab

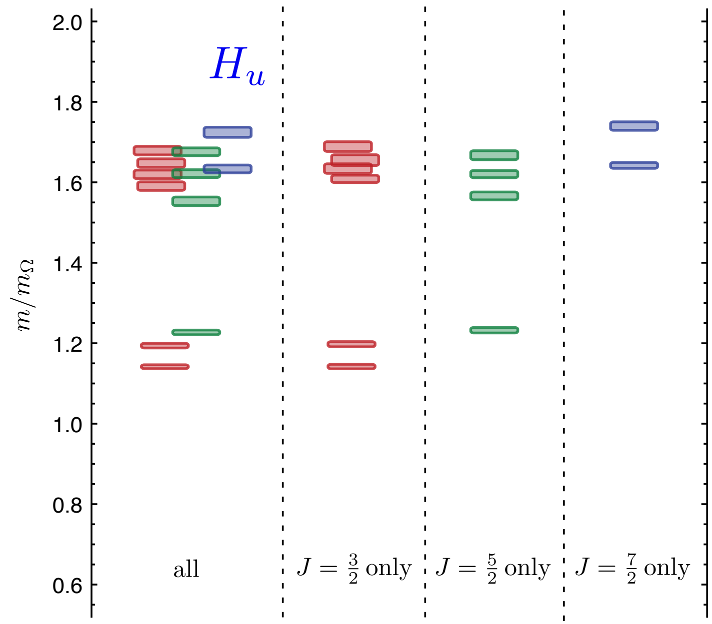
Excited Baryon Spectrum - I

- **Construct basis of 3-quark interpolating operators in the continuum:**

$$\left(N_{\mathsf{M}} \otimes \left(\tfrac{3}{2}^-\right)^{1}_{\mathsf{M}} \otimes D^{[2]}_{L=2,\mathsf{S}}\right)^{J=\frac{7}{2}}$$

- **Subduce to lattice irreps:**

$$\mathcal{O}^{[J]}_{n\Lambda,r} = \sum_{M} \mathcal{S}^{J,M}_{n\Lambda,r}\, \mathcal{O}^{[J,M]} : \Lambda = G_{1g/u}, H_{g/u}, G_{2g/u}$$



Continuum antecedents

R.G.Edwards et al., arXiv:1104.5152

$16^3 \times 128$ lattices $m_\pi = 524, 444$ and 396 MeV

Observe remarkable realization of rotational symmetry at hadronic scale: *reliably determine spins up to 7/2, for the first time in a lattice calculation*

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"Flavor" x Spin x Orbital

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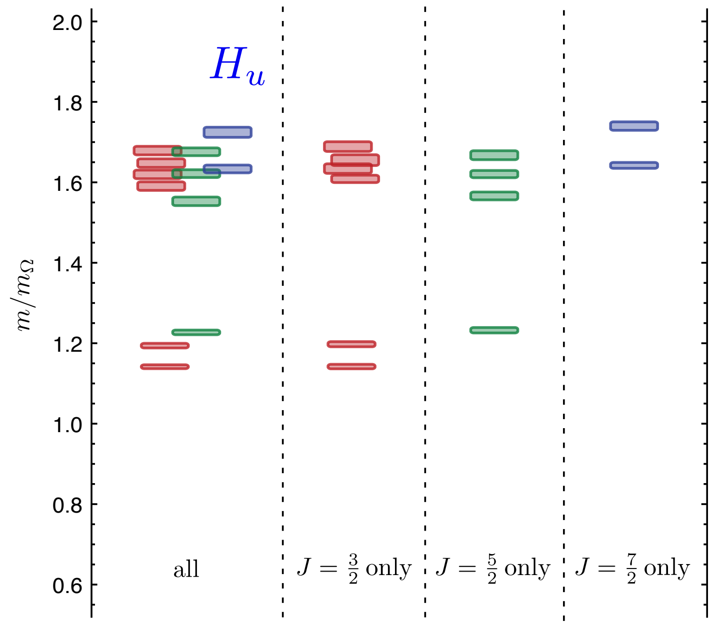

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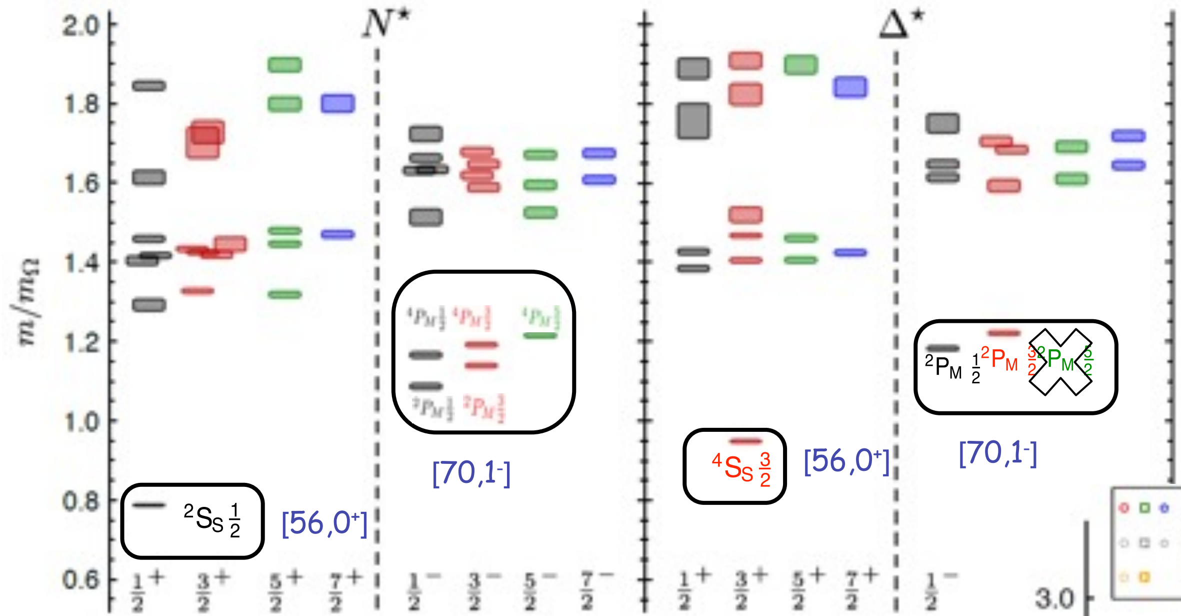
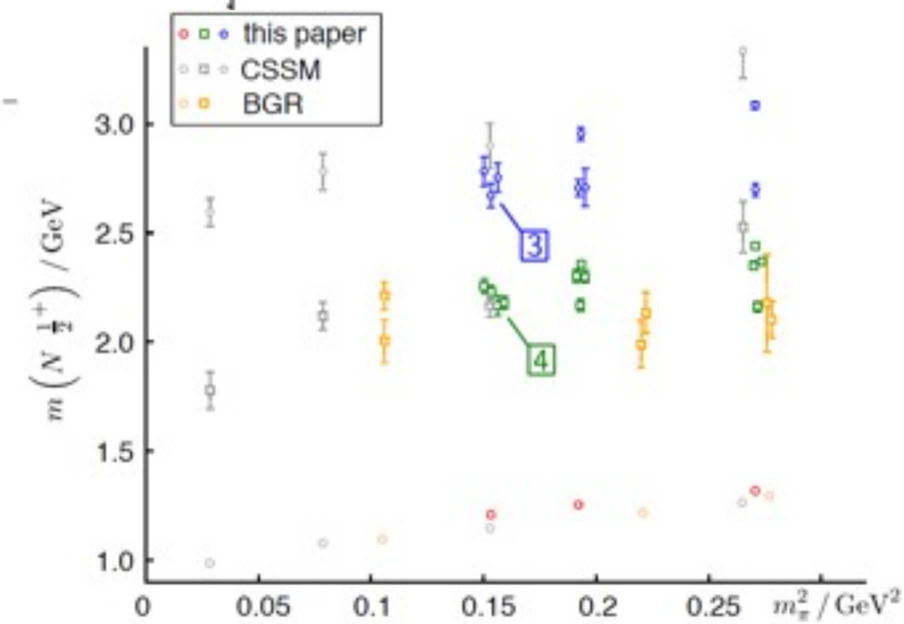
Excited Baryon Spectrum - II



- **Broad features of SU(6)xO(3) symmetry**
- **Counting of states consistent with NR quark model**
- **Inconsistent with quark-diquark picture or parity doubling**

N $^{1/2+}$ sector: need for complete basis to faithfully extract states



Summary

- Spectroscopy of excited states affords an excellent theatre in which to study QCD in low-energy regime.
- Major progress at reliable determinations of the *single-particle* spectrum, *with quantum numbers identified*
- Lattice calculations used to construct new “phenomenology” of QCD
- Next step for lattice QCD:
  - Complete the calculation: where are the multi-hadrons?
  - Determine the *phase shifts* - model dependent extraction of resonance parameters
- Final Lecture: Structure and Electromagnetic Properties of excited states