Hadron Structure

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StrongNet 2011, Bielefeld

Plan of Lectures

- Lecture 2 - Hadron Structure I
  - What are we studying, and how do we encapsulate it?
  - Paradigm: electromagnetic form factor of pion
  - Nucleon EM form factors
  - Polarized and unpolarized structure functions
  - Three-dimensional imaging of hadrons: Generalized Parton Distributions
- Lecture 3: Hadron Structure - II
  - Recent advances: Transverse-Momentum-Dependent distributions
  - Flavor-singlet contributions: role of sea quarks and gluons
  - Structure of excited states: radiative transitions between mesons

Hadron Structure

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- momentum
- spin and angular momentum

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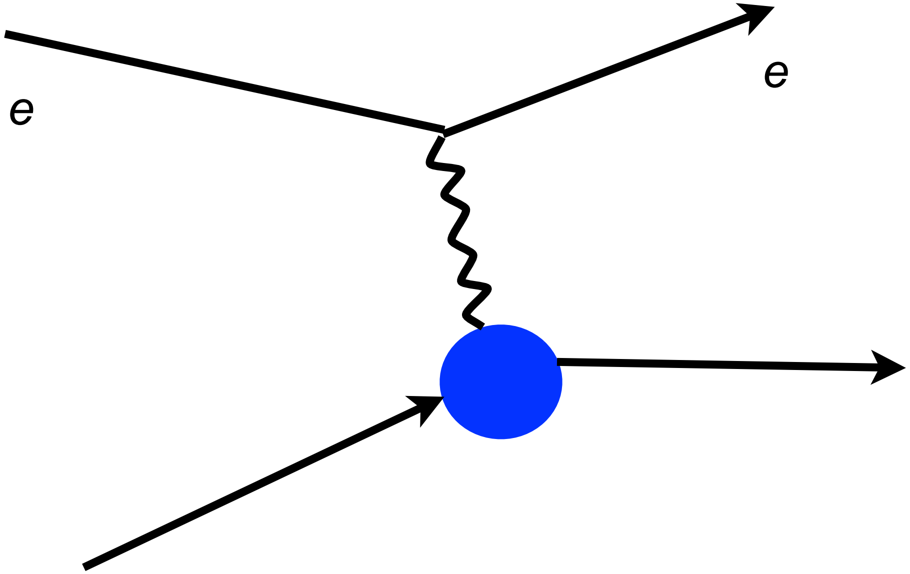
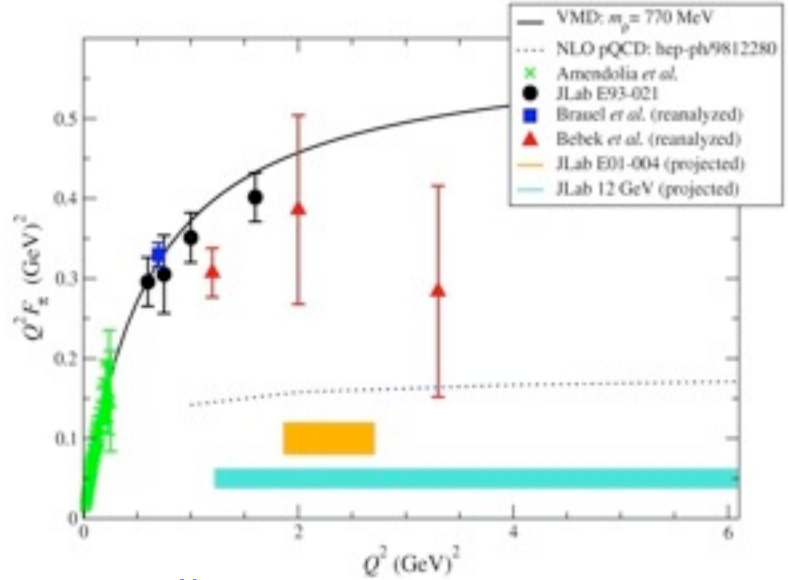
Technique: calculation of hadronic matrix elements - analogous to WME.

To paraphrase Nathan Isgur: “Tassos Vladikas wants to eliminate QCD, I want to understand it!”

Paradigm: Pion EM form factor





$$\langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2)$$

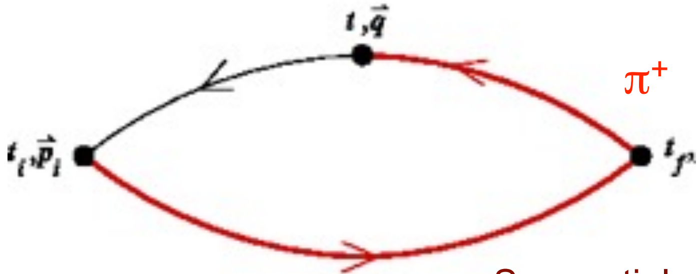
where

$$\begin{aligned} V_\mu &= \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d \\ -Q^2 &= [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}_i)^2 \end{aligned}$$

Anatomy of a Matrix Element Calculation - I

Pion Interpolating Operator

$$\begin{aligned} \phi(x) &= \bar{d}(x)\gamma_5 u(x) \\ \phi^\dagger(x) &= -\bar{u}(x)\gamma_5 d(x) \\ V_\mu(x) &= e_u \bar{u}(x)\gamma_\mu u(x) + e_d \bar{d}(x)\gamma_\mu d(x). \end{aligned}$$



Sequential-source propagator

$$\Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x}, t_f) V_\mu(\vec{y}, t)\phi^\dagger(\vec{0}, 0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

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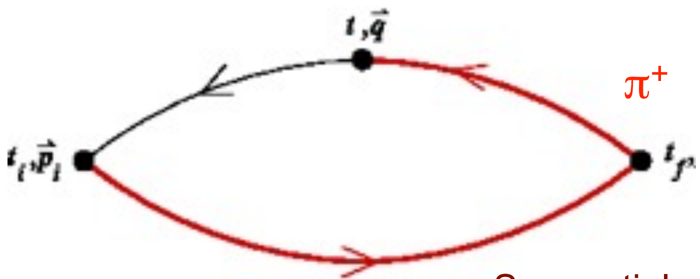

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$$V_\mu^{\rm cont} = Z_V V_\mu^{\rm lattice}; Z_V = 1 \quad \text{for conserved current}$$

Anatomy of a Matrix Element Calculation - II

Construction of three-point function

Introduce quark propagators

$$U^{ij}_{\alpha\beta}(x,y) = \langle u^i_\alpha(x)\bar{u}^j_\beta(y)\rangle$$
$$D^{ij}_{\alpha\beta}(x,y) = \langle d^i_\alpha(x)\bar{d}^j_\beta(y)\rangle,$$

Then U-contribution to three-point function given by

$$\Gamma^U_{\pi^+\mu\pi^+} = e_u \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\cdot\vec{x}-i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{\gamma_5 U(x,y)\gamma_\mu U(y,0)\gamma_5 D(0,x)\right\}$$

Quark propagator: $G^{ij}_{\alpha\beta}(x,y) = \langle q^i_\alpha(x)\bar{q}^j_\beta(y)\rangle$ satisfies

$$M^{ik}_{\alpha\gamma}(x,z)G^{kj}_{\gamma\beta}(z,y) = \delta_{ij}\delta_{\alpha\beta}\delta_{xy}; \quad G(y,x) = \gamma_5 G(x,y)^\dagger\gamma_5$$

Introduce Sequential Quark Propagator $H^u(y,0;t_f,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} U(y,x)\gamma_5 D(x,0)\gamma_5$

Satisfies: $M(z,y)H^u(y,0;t_f,\vec{p}) = \delta_{t_z,t_f} e^{i\vec{p}\cdot\vec{z}}\gamma_5 D(z,0)\gamma_5$

Finally: $\Gamma^U_{\pi^+\mu\pi^+} = e_u \sum_{\vec{y}} e^{-i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{H^u(y,0;t_f,\vec{p})^\dagger\gamma_5\gamma_\mu U(y,0)\gamma_5\right\}$

Anatomy of a Matrix Element Calculation - II

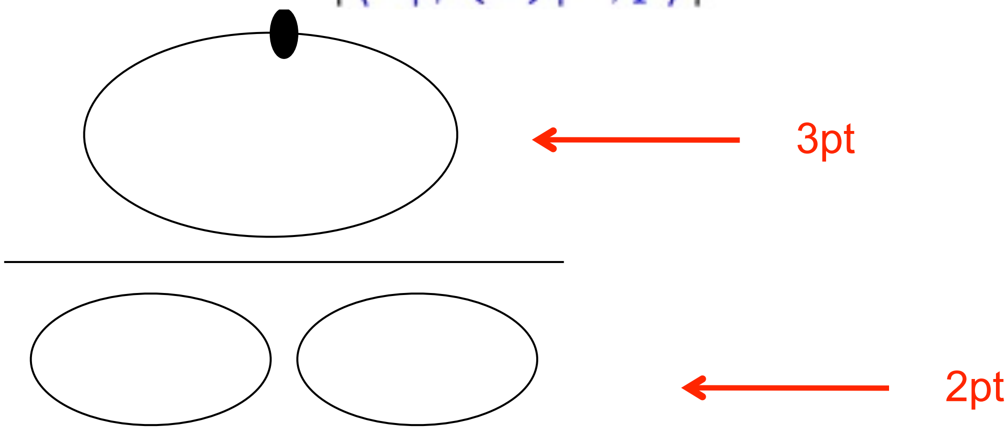
$$\Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x}, t_f) V_\mu(\vec{y}, t) \phi^\dagger(\vec{0}, 0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

Resolution of unity – insert states

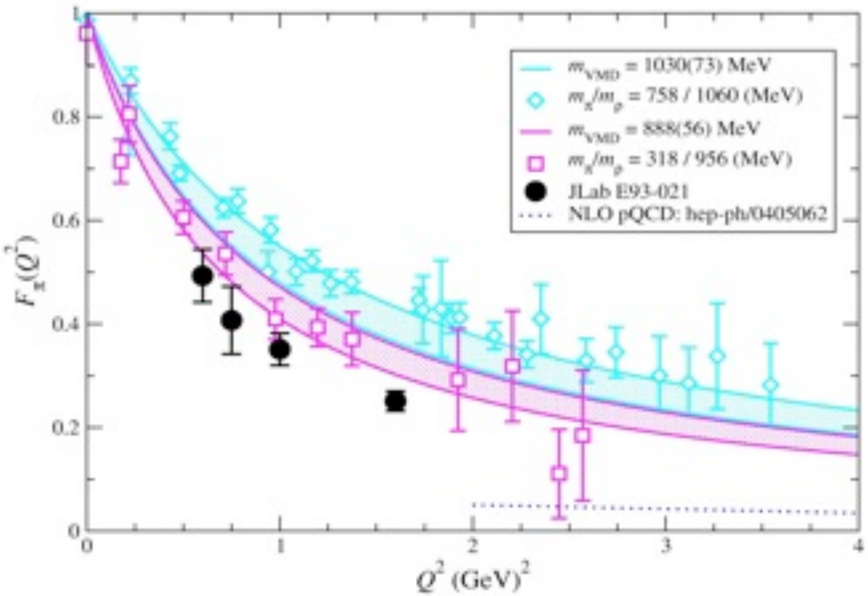
$$\langle 0 \mid \phi(0) \mid \pi, \vec{p}+\vec{q}\rangle\langle\pi, \vec{p}+\vec{q} \mid V_\mu(0) \mid \pi, \vec{p}\rangle\langle\pi, \vec{p} \mid \phi^\dagger \mid 0\rangle e^{-E(\vec{p}(t-t_i)} e^{-E(\vec{p}+\vec{q})(t_f-t)}$$

$$\Gamma_{\pi^+\pi^+}(t, 0; \vec{p}) = \sum_{\vec{x}} \langle 0 \mid \phi(\vec{x}, t_f) \phi^\dagger(0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}}$$

$$\propto |\langle 0|\phi(0)|\pi, \vec{p}\rangle|^2 e^{-E(\vec{p})t}$$



Pion Form Factor



LHPC, Bonnet et al,
Phys.Rev. D72 (2005) 054506

$$F(Q^2) = \frac{1}{1 + Q^2/{M_{\text{VMD}}}^2}$$

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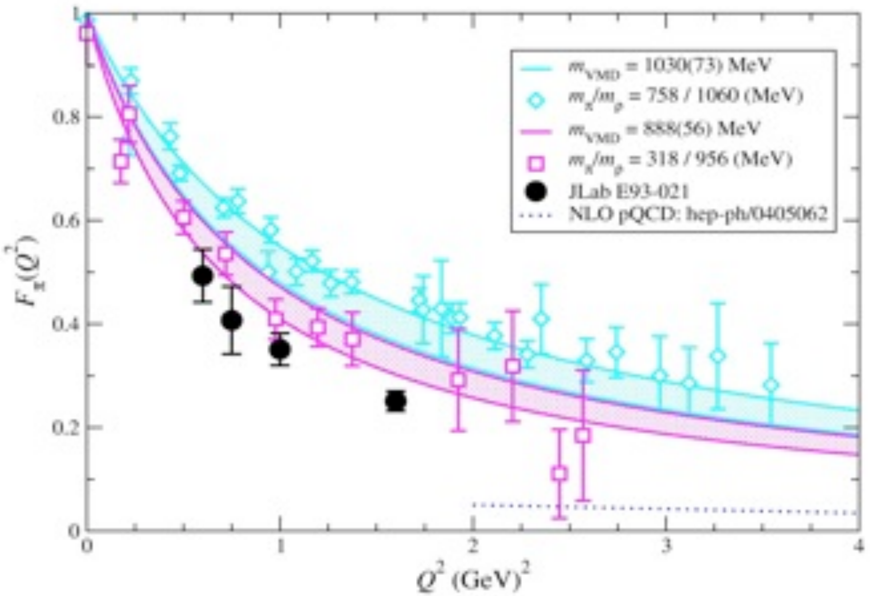

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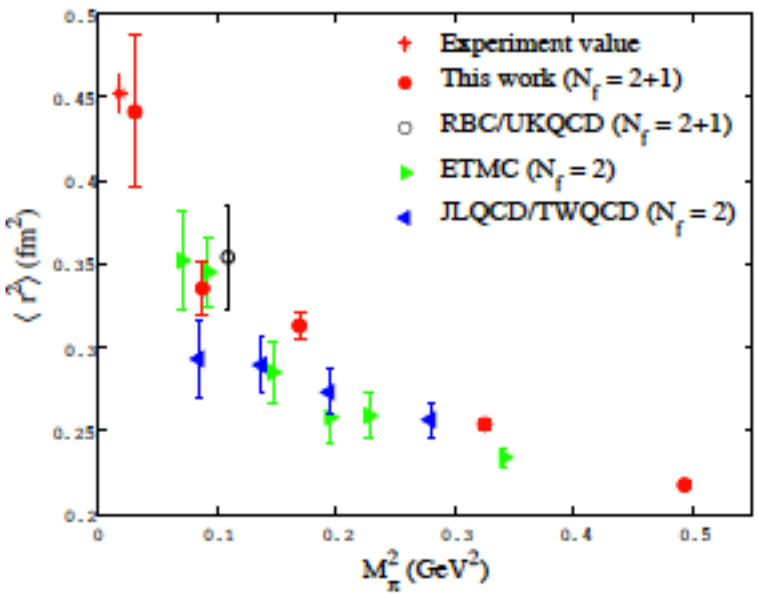
$$F(Q^2) = \frac{1}{1 + Q^2/{M_{\mathrm{VMD}}}^2}$$

Charge radius Nguyen et al, 1102.3652

$$\langle r^2 \rangle = 6 \left. \frac{dF(q^2)}{dq^2} \right|_{q^2=0}$$



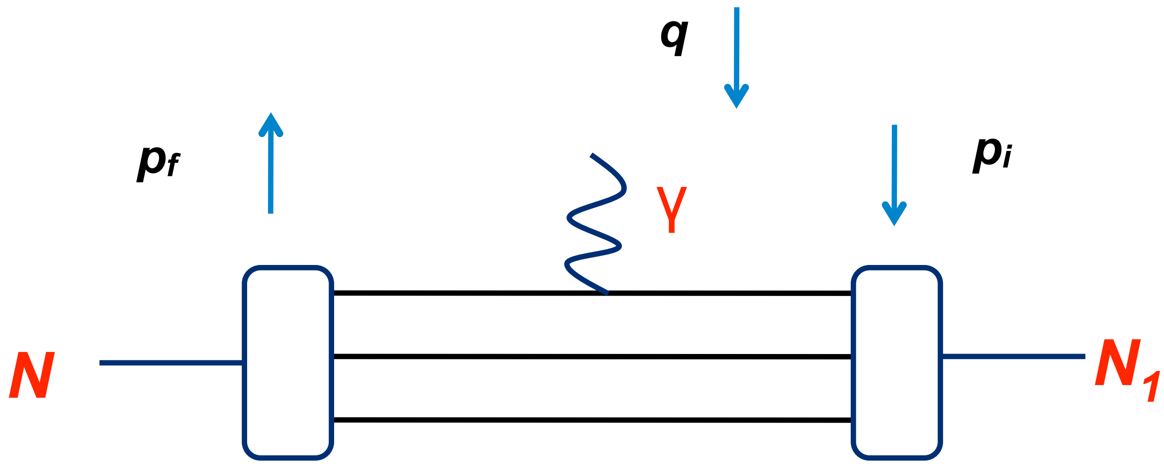
Nucleon EM Form Factors

Two form factors

$$\langle p_f \mid V_\mu \mid p_i \rangle = \bar{u}(p_f)\left[\gamma_\mu \underbrace{F_1(q^2)}_{\text{Dirac}} + i q_\nu \frac{\sigma_{\mu\nu}}{2m_N} \underbrace{F_2(q^2)}_{\text{Pauli}}\right] u(p_i)$$

Related to familiar Sach's electromagnetic form factors through

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2m_N)^2} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$



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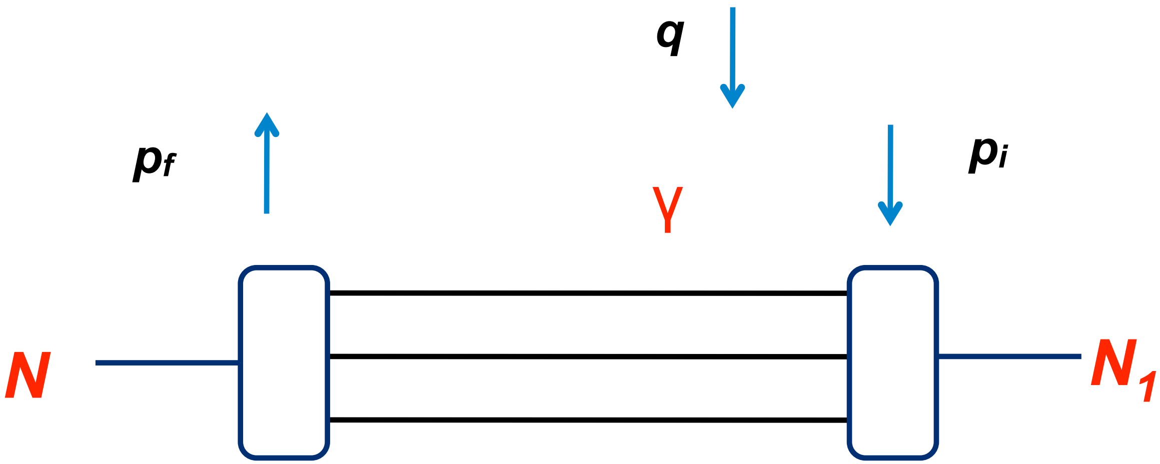

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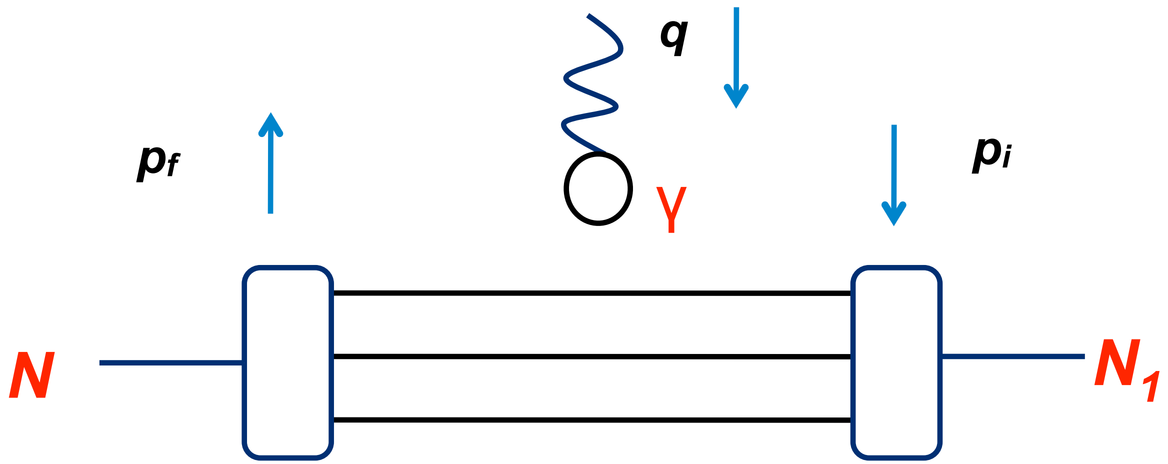

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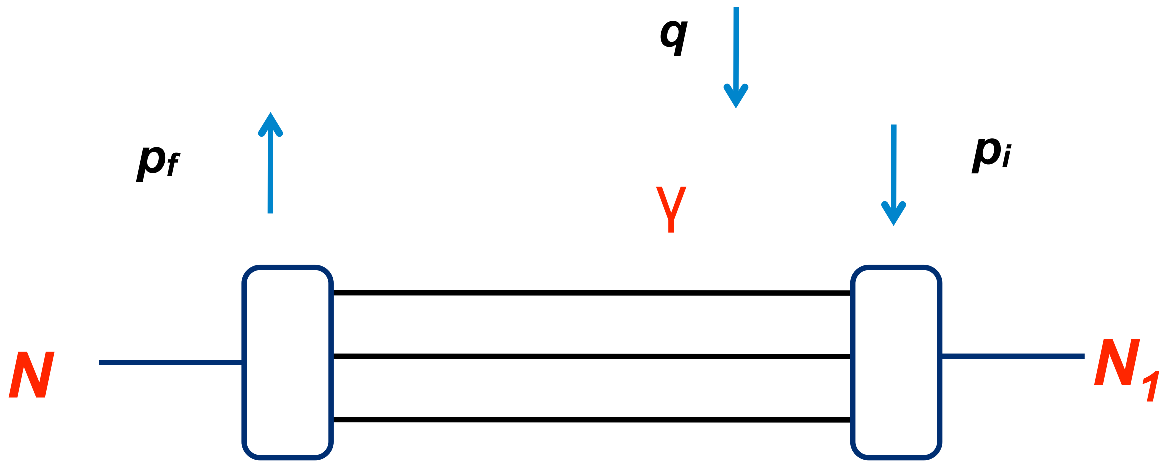

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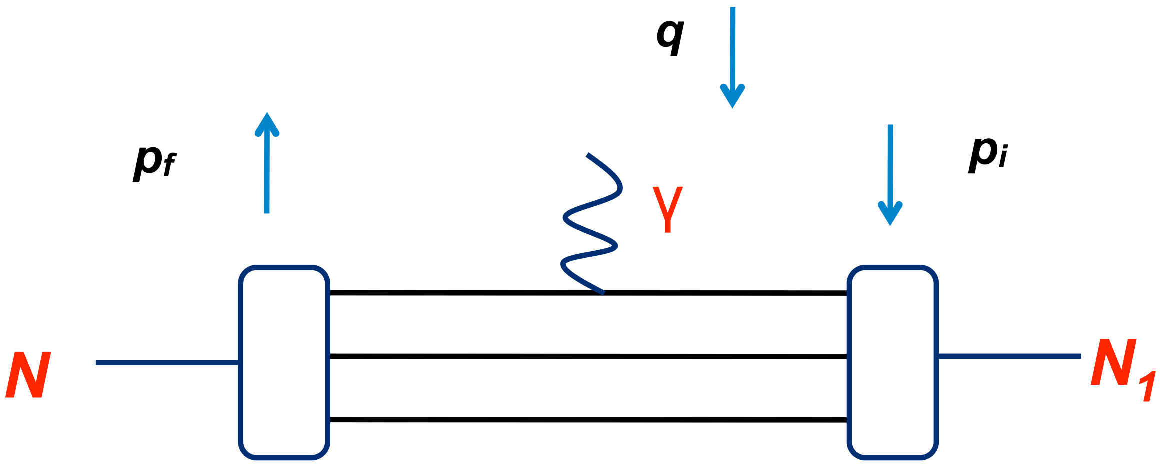

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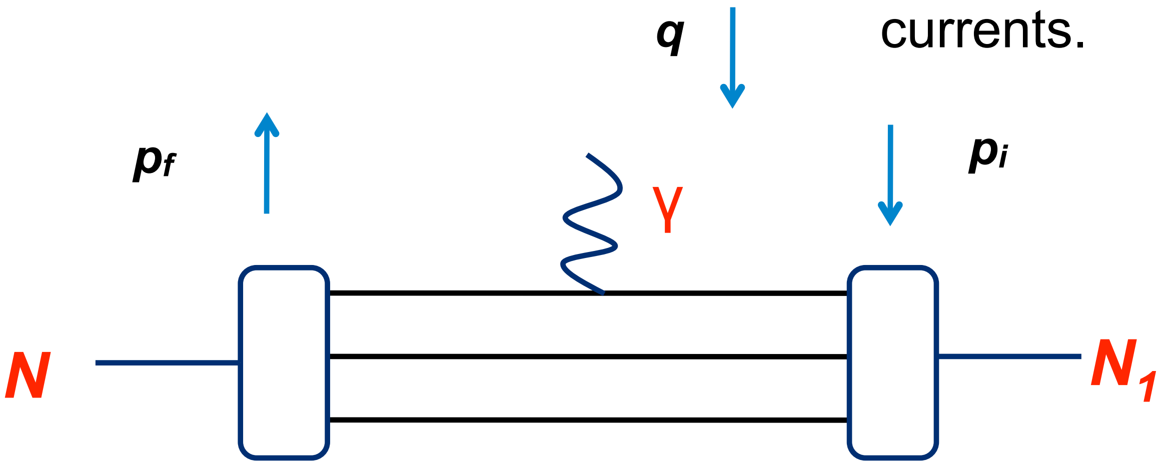
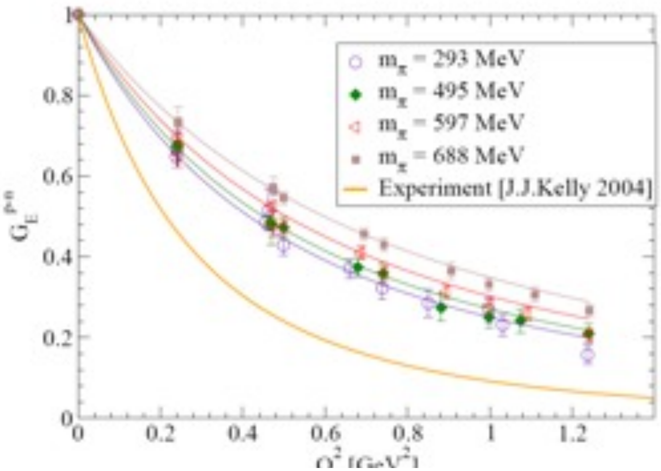
Isovector: difference between p and n or difference between u and d currents.



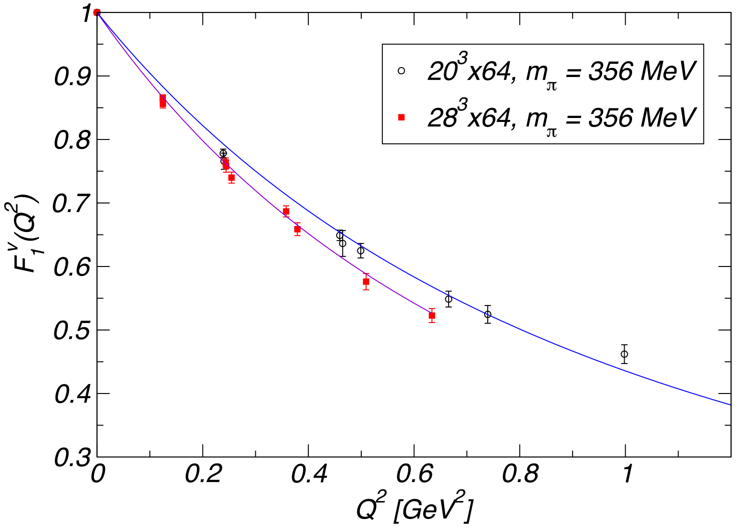
Isovector Form Factor

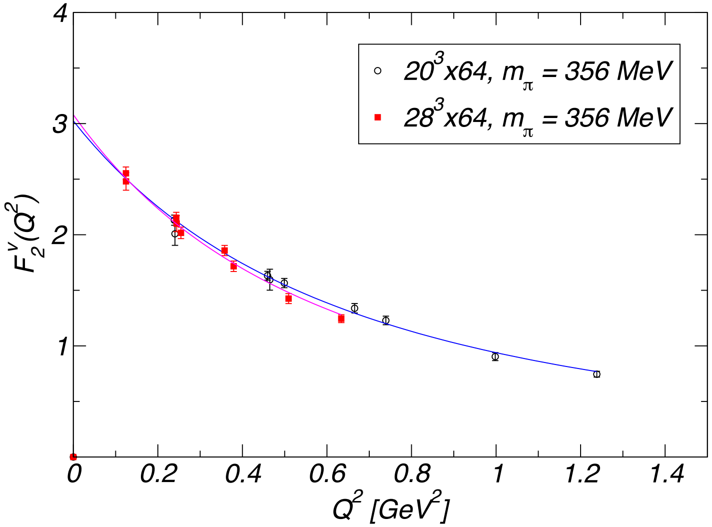


DWF valence/Asqtad sea

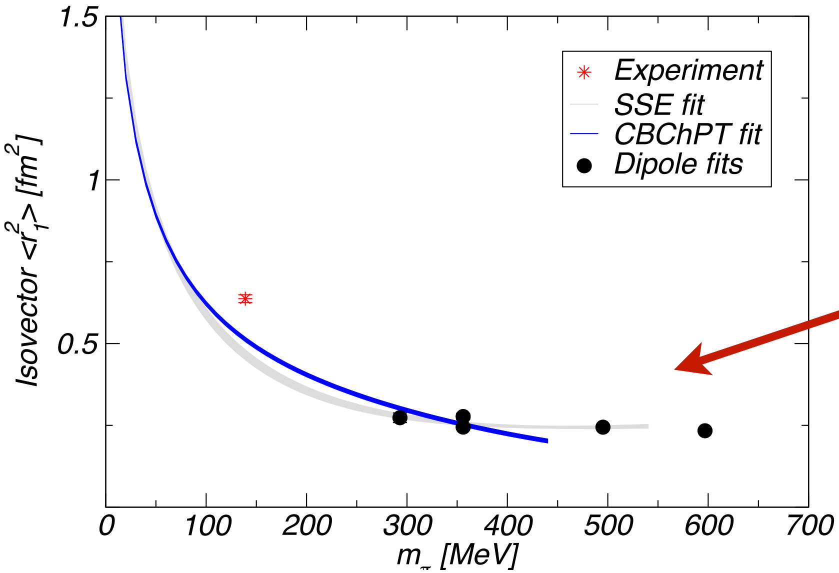
J.D.Bratt et al (LHPC), arXiv:0810.1933

Data well described by dipole form - but example of notable finite-volume effect:





Nucleon Form Factors - III

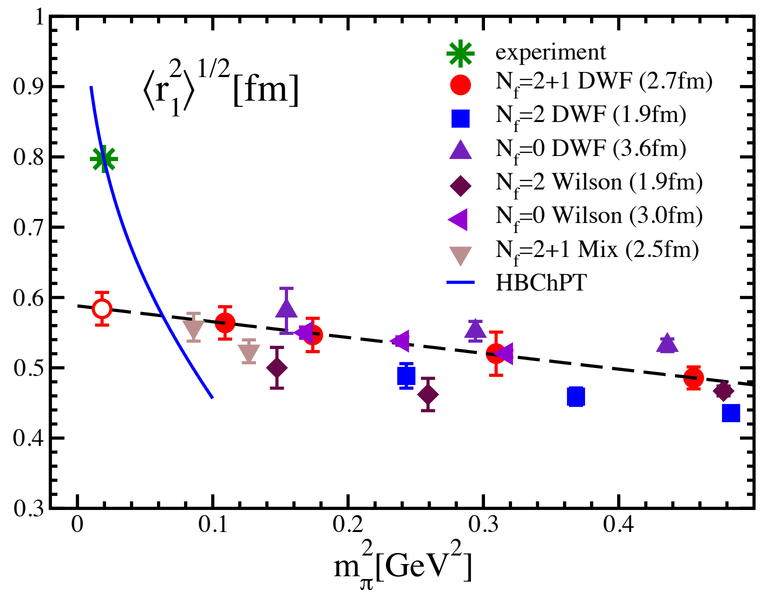


LHPC, arXiv:1001.3620

Dipole fits at each pion mass

RBC/UKQCD, arXiv:0904.2039

Large extrapolation to Chiral limit

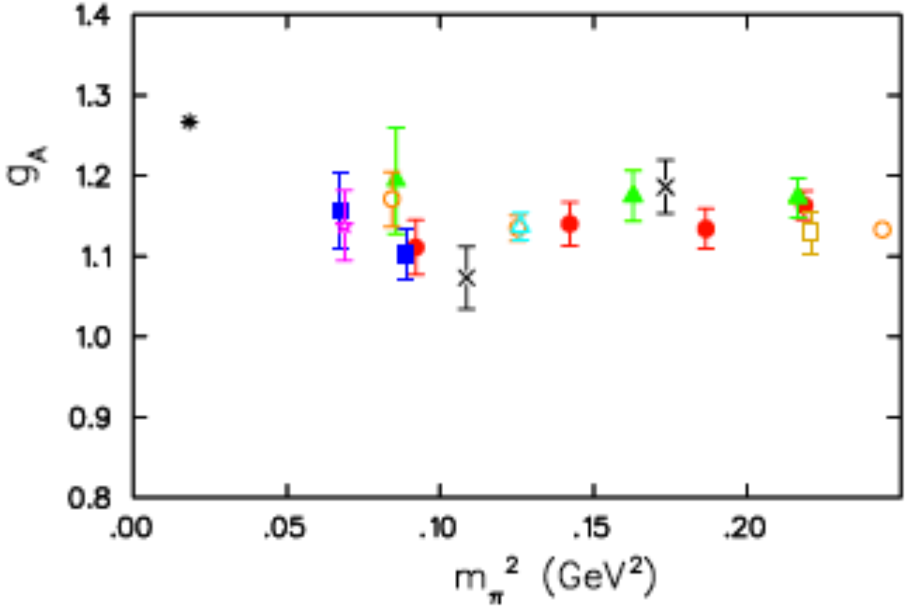


Nucleon Axial-Vector Charge - I

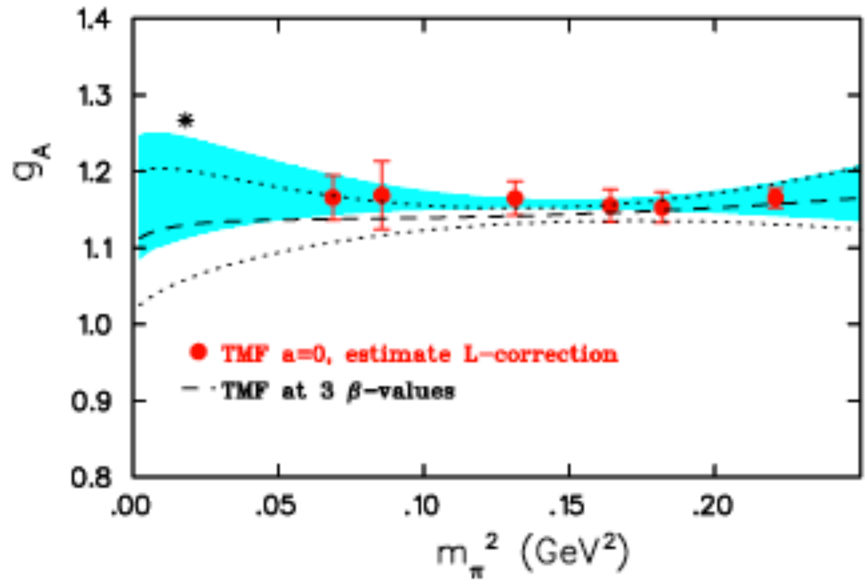
Nucleon's axial-vector charge g_A: benchmark of lattice QCD

Precisely measured in neutron β decay

$$\langle N(p,S) \mid \bar{\psi}\gamma_\mu\gamma_5\psi \mid N(p,S)\rangle$$



D. Alexandrou, Lattice 2010

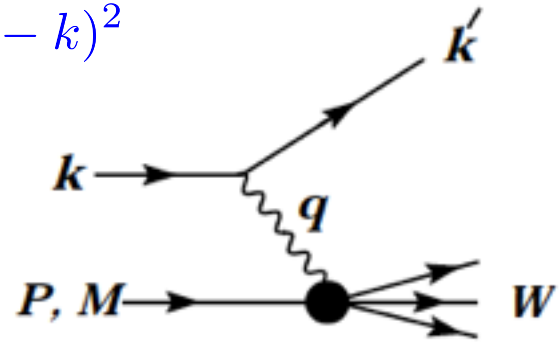


Twisted mass QCD: volume corrected

Different actions, volumes

Structure Functions - I

$$Q^2 = -q^2 = (k' - k)^2$$
$$\nu = q \cdot P/M$$
$$x = \frac{Q^2}{2M\nu}$$



Bjorken limit:
$$Q^2 \longrightarrow \infty, \nu \longrightarrow \infty, x \text{ fixed}$$

The structure functions are defined in terms of the hadronic tensor:

$$W_{\mu\nu} = \frac{1}{4\pi} \int dz e^{iq\cdot z} \langle N(p,S) \mid J_\mu(z) J_\mu(0) \mid N(p,S) \rangle$$

Yields two unpolarized structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$, and two polarized structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$

Leading twist structure functions: product of currents at light-like $z^2 \to 0$

In Euclidean lattice QCD, use OPE to write in terms of local operators whose matrix elements we can compute in Euclidean space

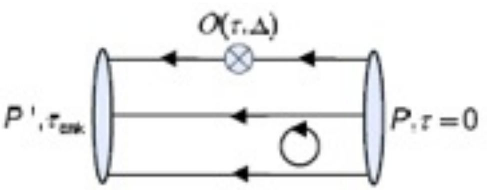
Structure Functions - II

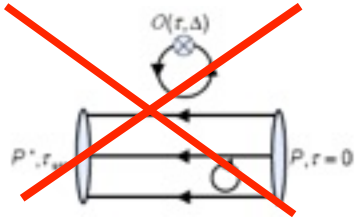
Operators

polarized

Capitani, this school

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q\gamma_5\gamma^{\{\mu_1}iD^{\mu_2}\ldots D^{\mu_n\}}\psi_q$$





Matrix elements related to moments of structure functions

Wilson coeffs

Operator renormalization

$$\int_0^1 dx\, x^{n-1}F_2(x,Q^2) = \sum_{q=u,d} C_n(\mu^2/Q^2), g(\mu))\langle x^n\rangle(\mu)$$

where

$$\langle N(p) \mid O_q^{\mu_1\cdots\mu_{n+1}} \mid N(p)\rangle = \langle x^n\rangle(\mu)[p_{\mu_1}\ldots p_{\mu_{n+1}}]$$

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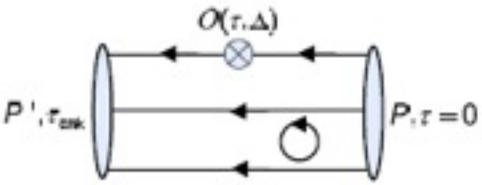

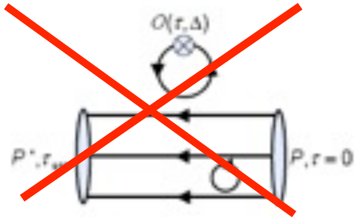

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$$\mathcal{O}^{\rm cont} = Z\mathcal{O}^{\rm latt}$$

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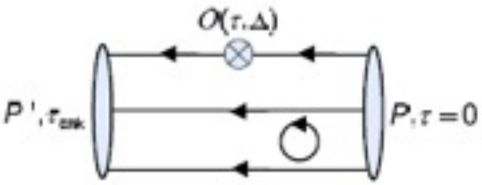

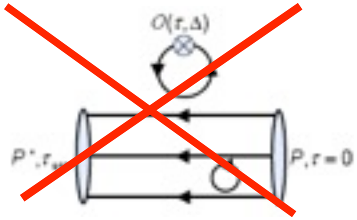

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Perturbation theory

$$\mathcal{O}^{\rm cont} = Z\mathcal{O}^{\rm latt}$$

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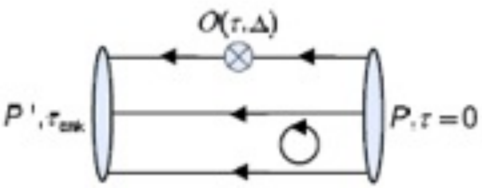

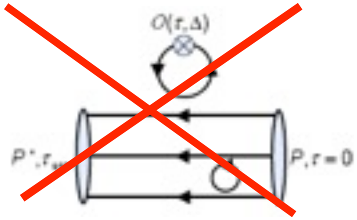

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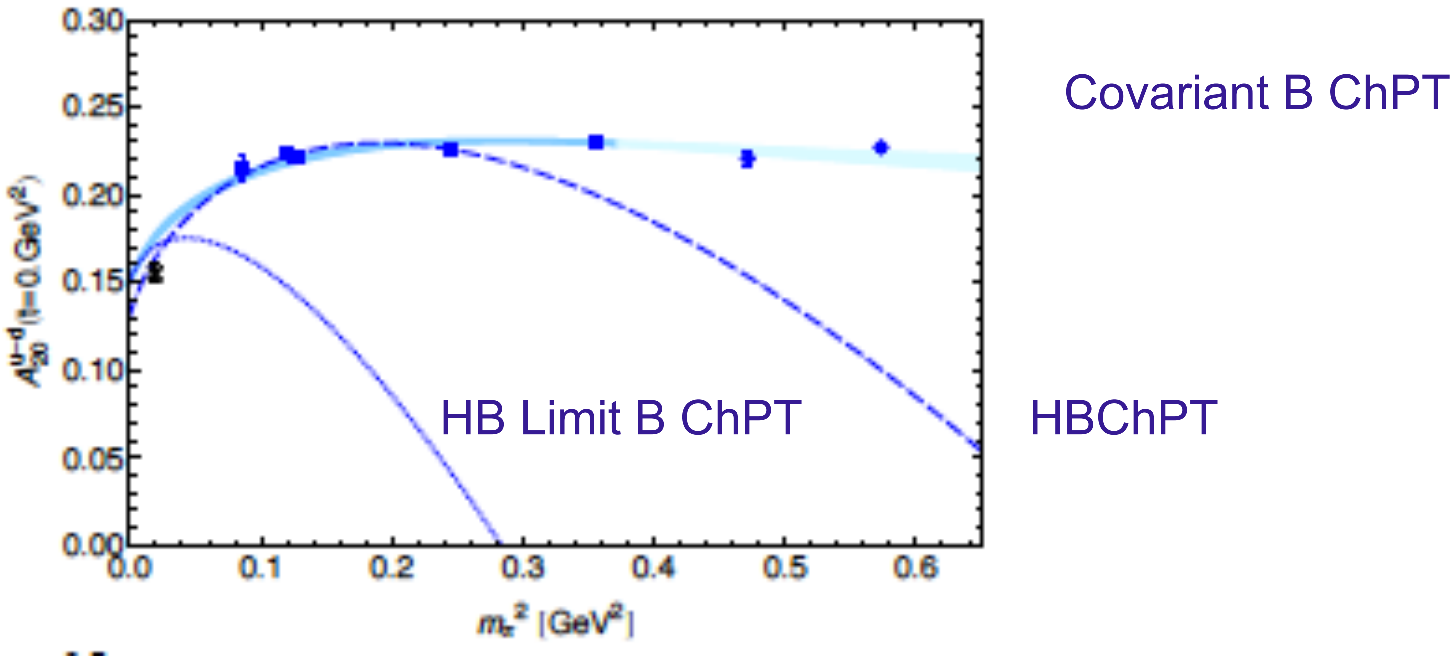
Perturbation theory

Non-perturbatively

Quark Momentum Fraction - I

LHPC, 2010: DWF valence, Asqtad sea



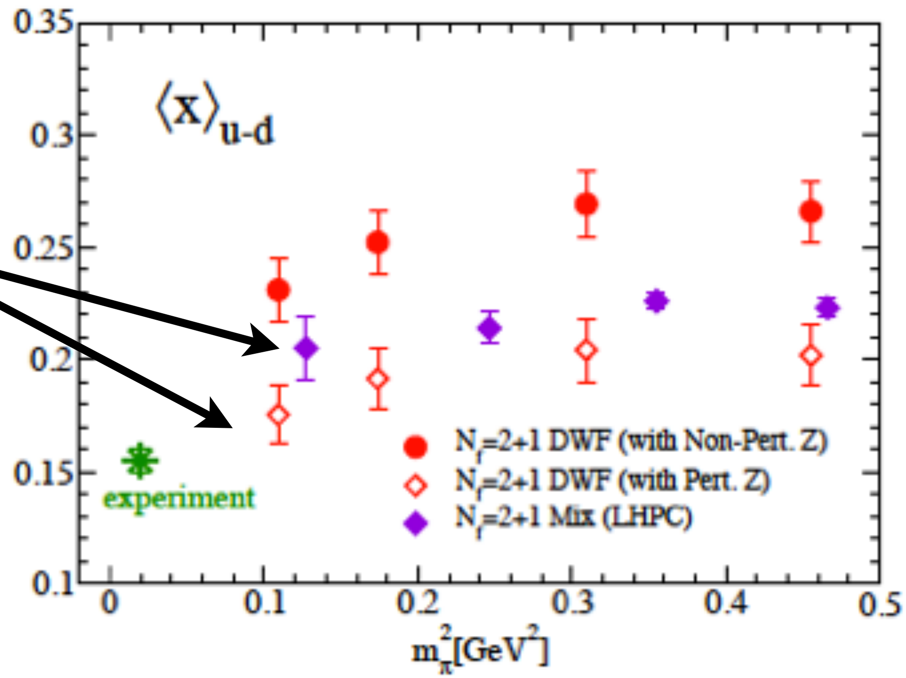
Heavy-Baryon Ch PT (HBChPT)

$$\langle x \rangle_{u-d} = C\left[1 - \frac{3g_A^2+1}{(4\pi F_\pi)^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{\pi^2}\right)\right] + e(\mu^2)\frac{m_\pi^2}{(4\pi F_\pi)^2}$$

Quark Momentum Fraction - II

RBC/UKQCD 2010: DWF

Similar renormalization prescription



• Need to go to approach physical light-quark masses: chiral behavior

Quark Momentum Helicities

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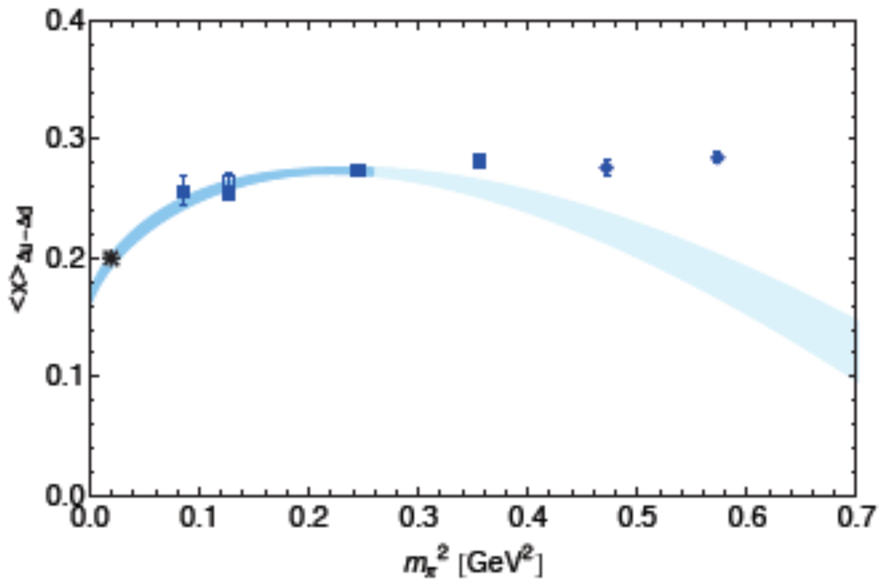
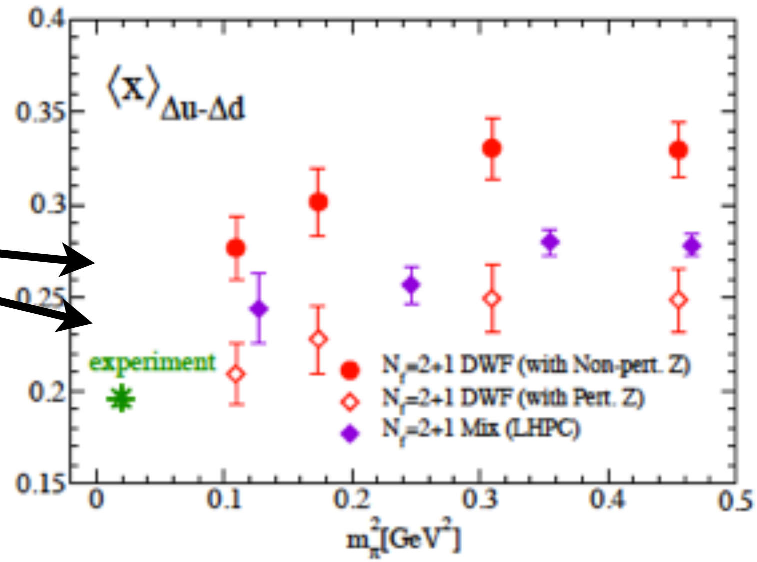


HBChPT

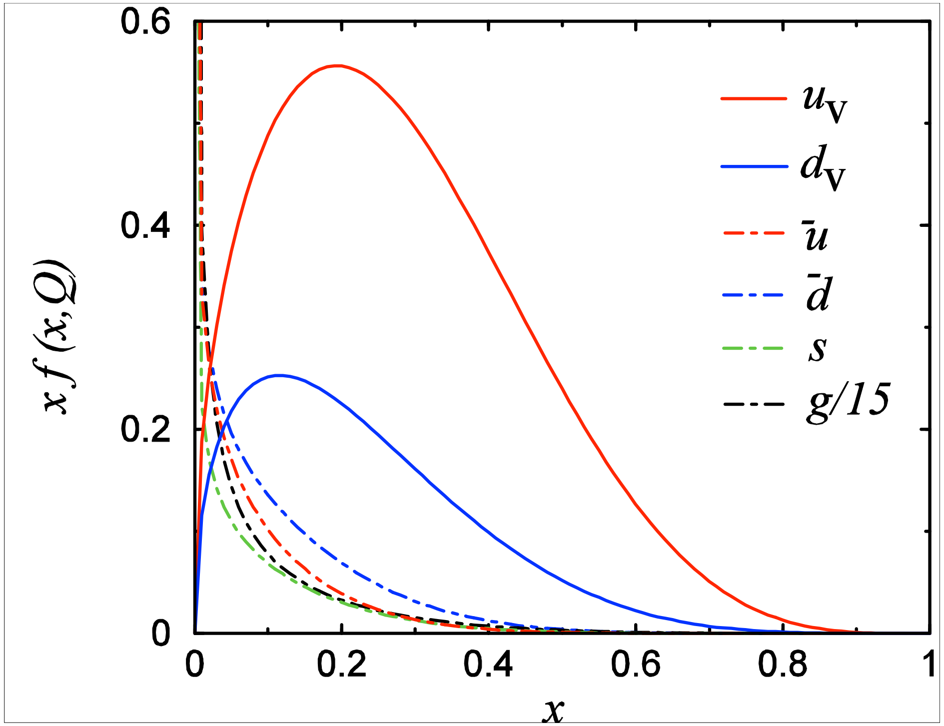
RBC/UKQCD 2010: DWF



Similar renormalization prescription

- Need to go to approach physical light-quark masses: chiral behavior

Moments of Parton Distributions



We are computing moments

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q\gamma_5\gamma^{\{\mu_1}iD^{\mu_2}\dots D^{\mu_n\}}\psi_q$$

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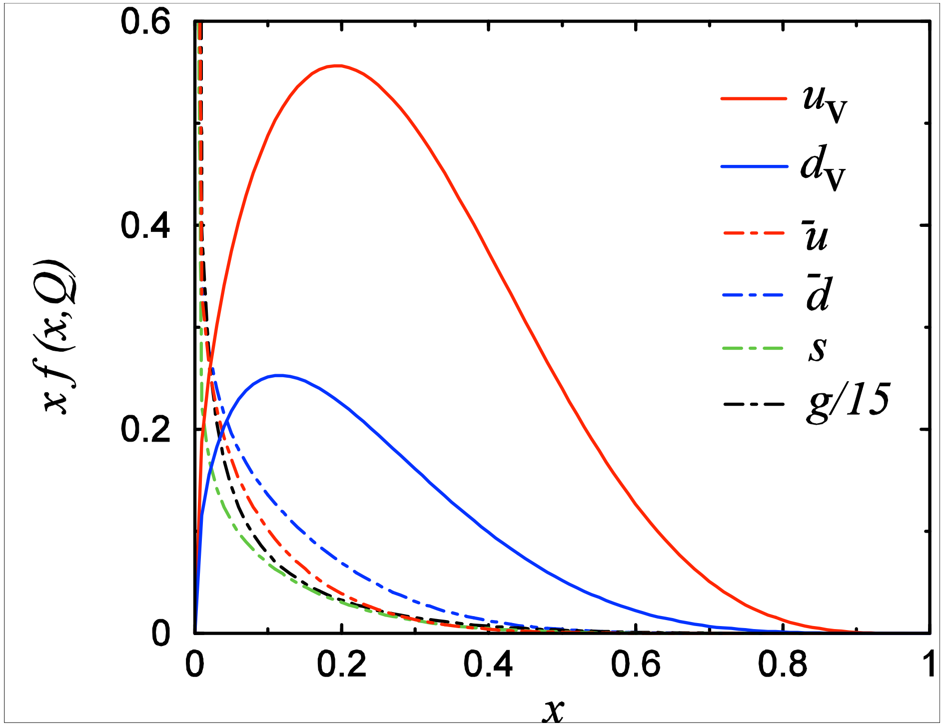

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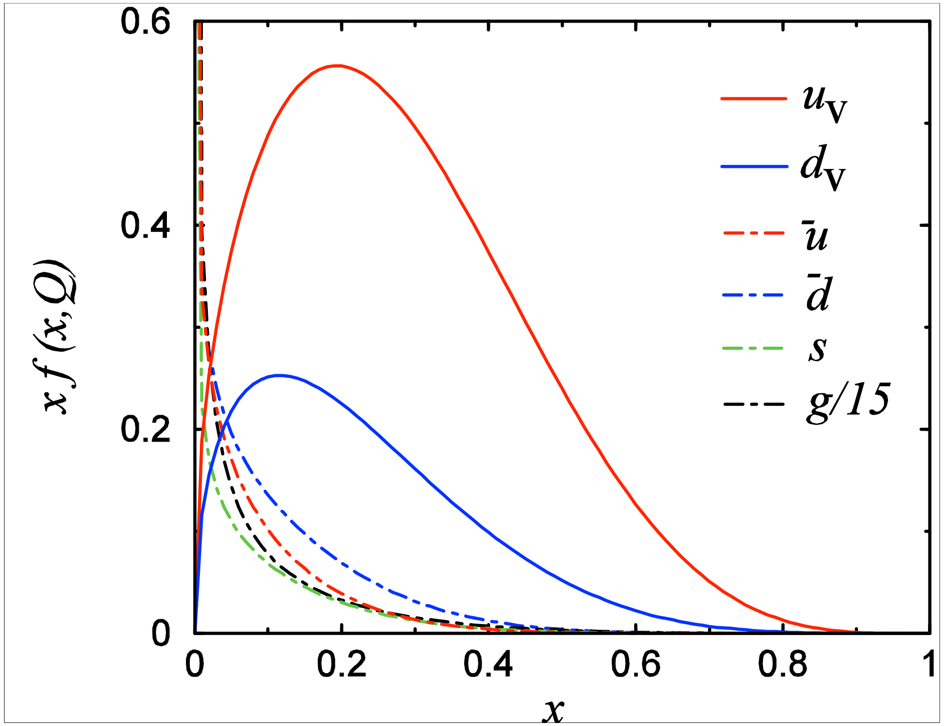

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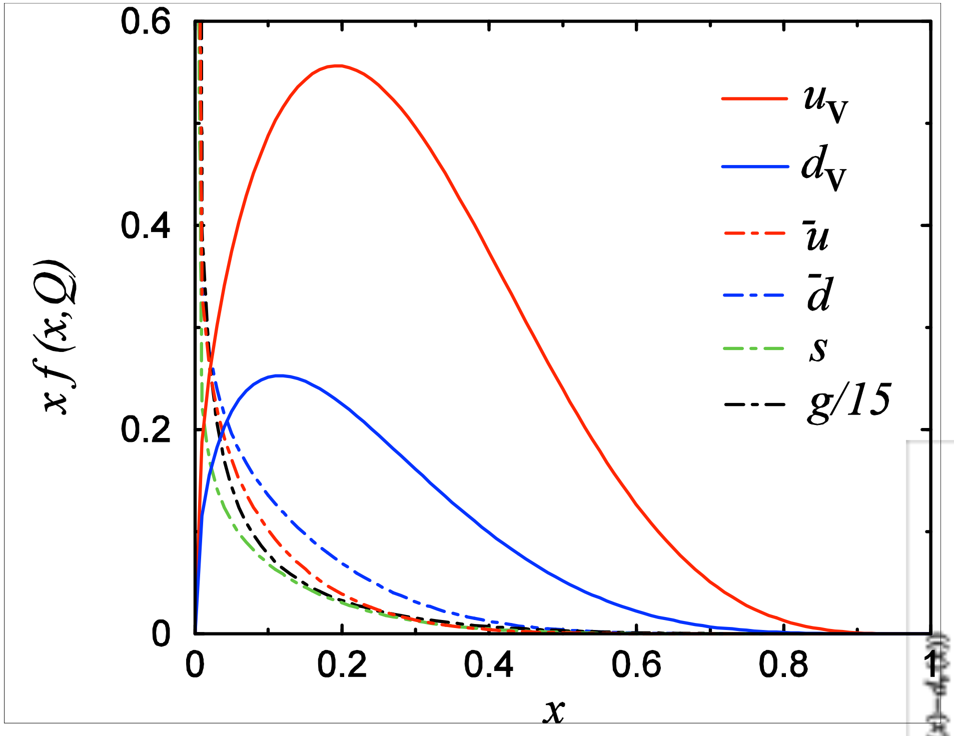

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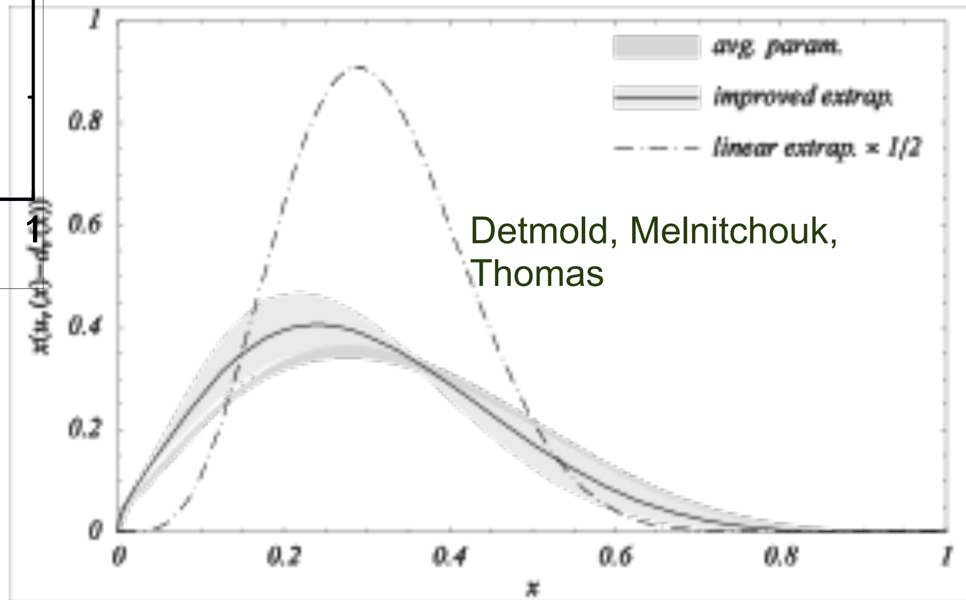
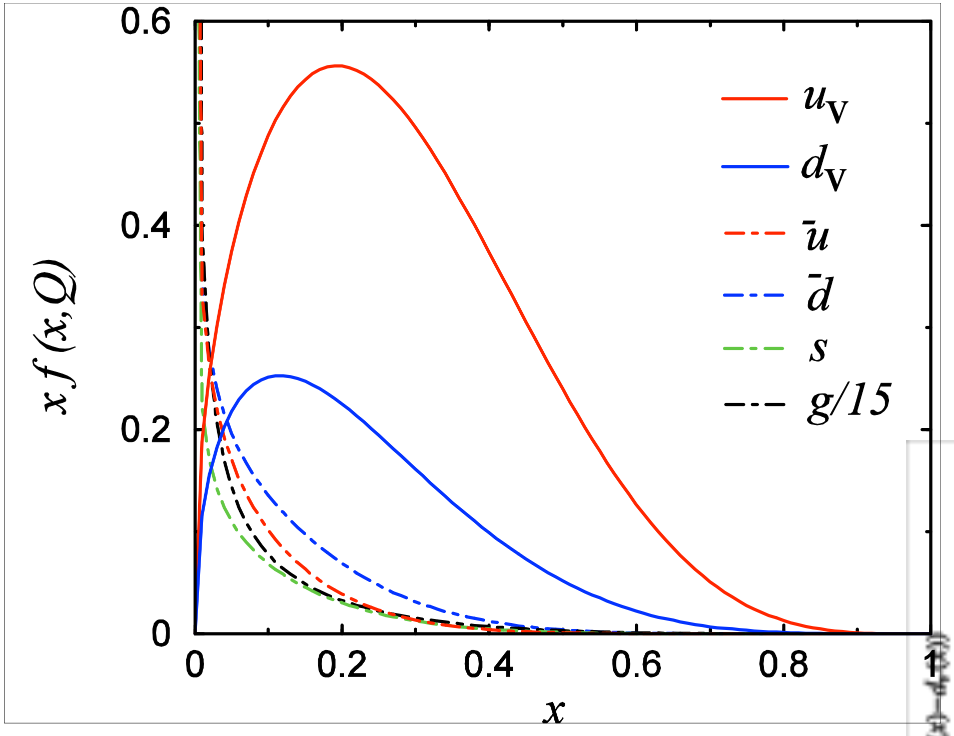


Detmold, Melnitchouk, Thomas

Moments of Parton Distributions



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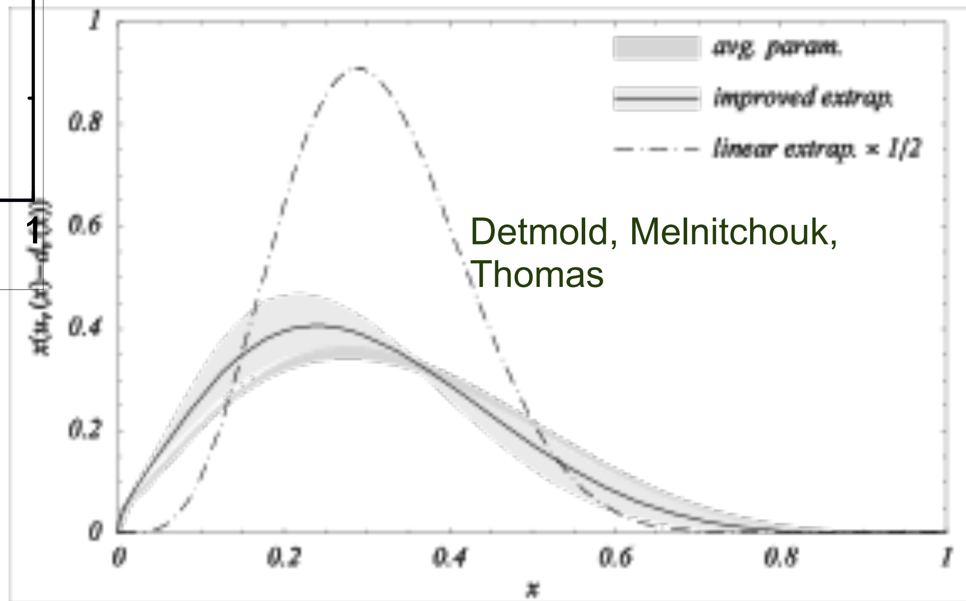
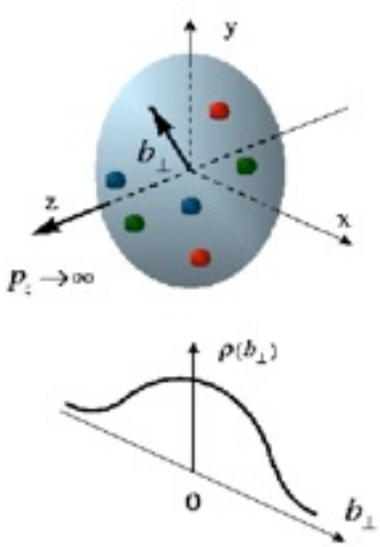


Need to assume parametrization

$$x(u_v(x) - d_v(x)) = a\,x^b(1-x)^c(1+\varepsilon\sqrt{x}+\gamma\, x)$$

Different Regimes in Different Experiments



Form Factors
transverse quark distribution in Coordinate space

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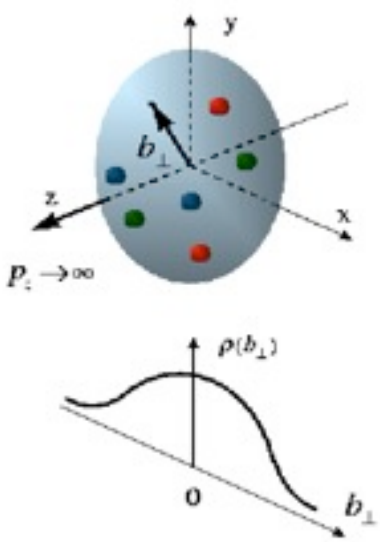

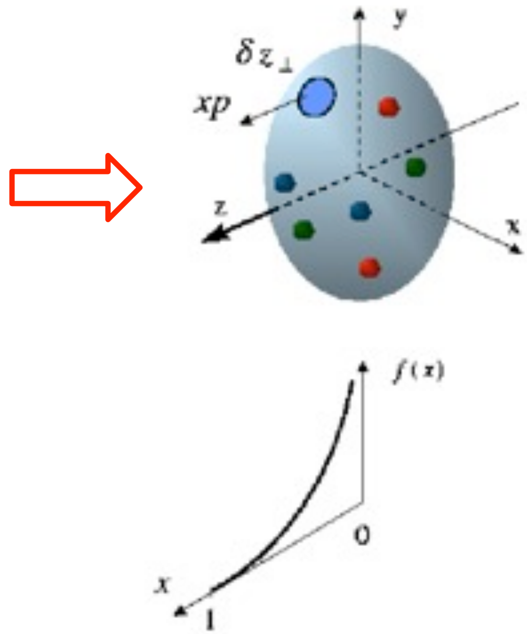

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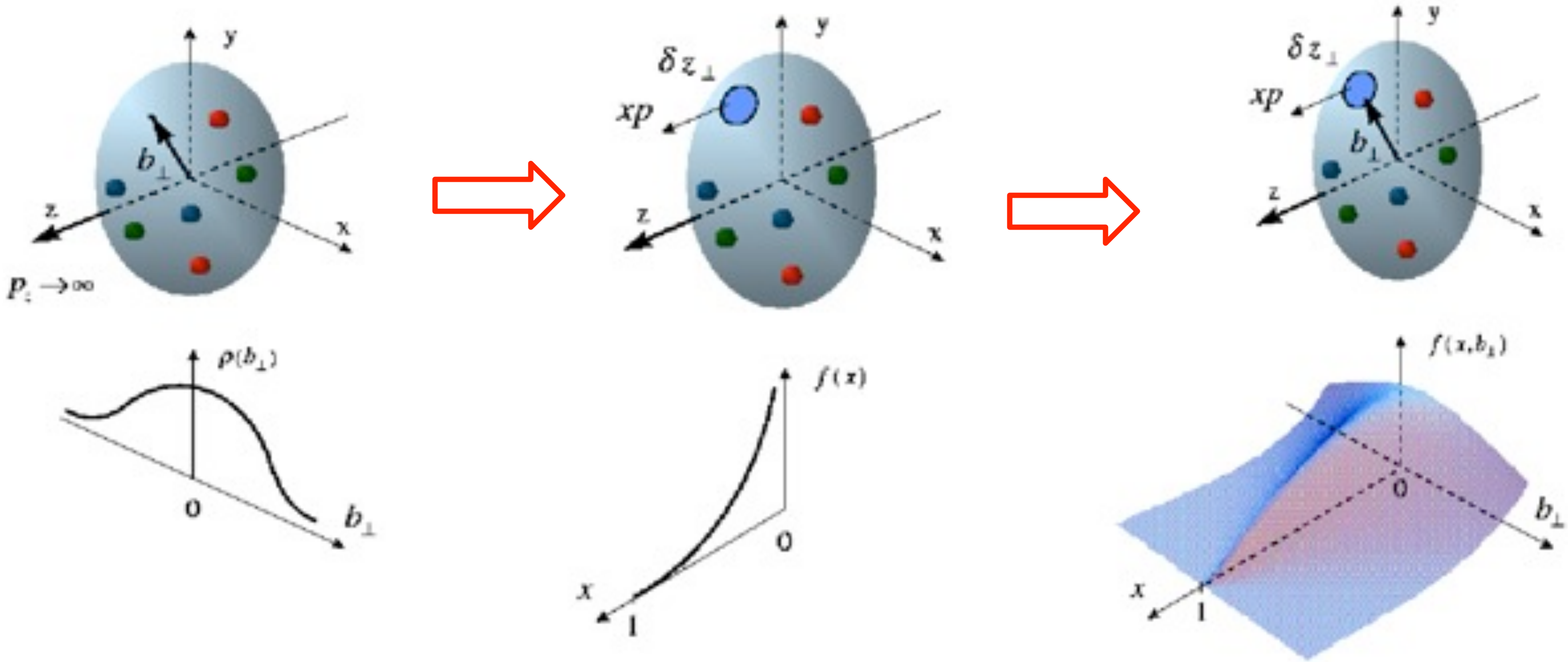

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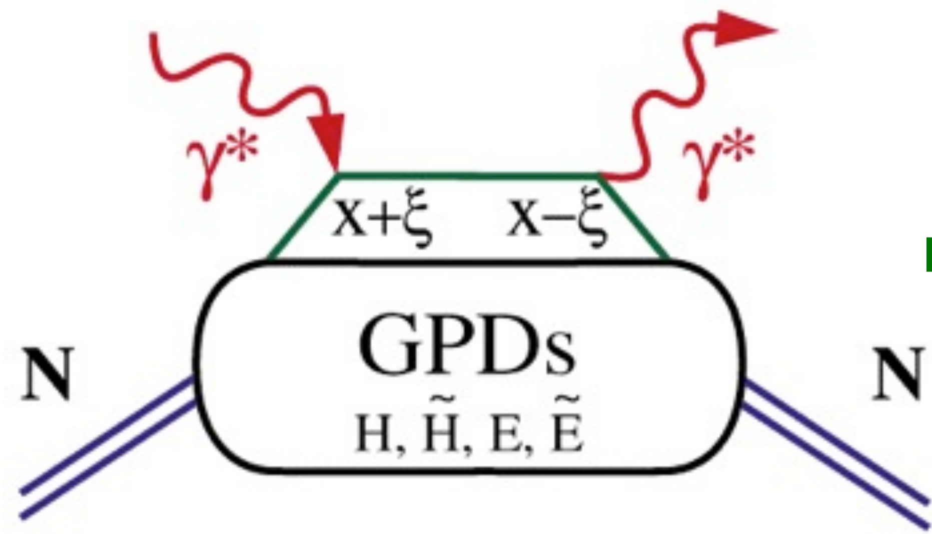
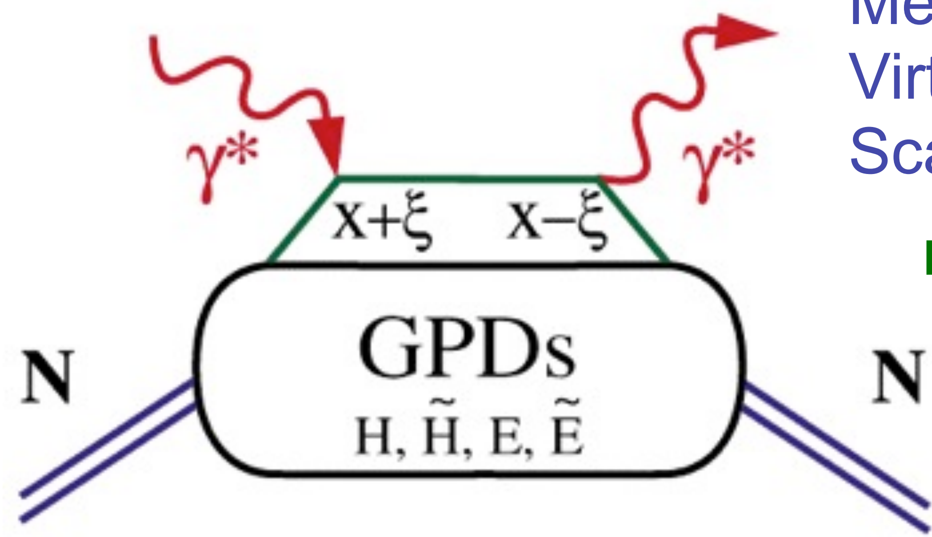
GPDs
Fully-correlated quark distribution in both coordinate and momentum space

Generalized Parton Distributions (GPDs)



D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

Generalized Parton Distributions (GPDs)



Measured in Deeply Virtual Compton Scattering

D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

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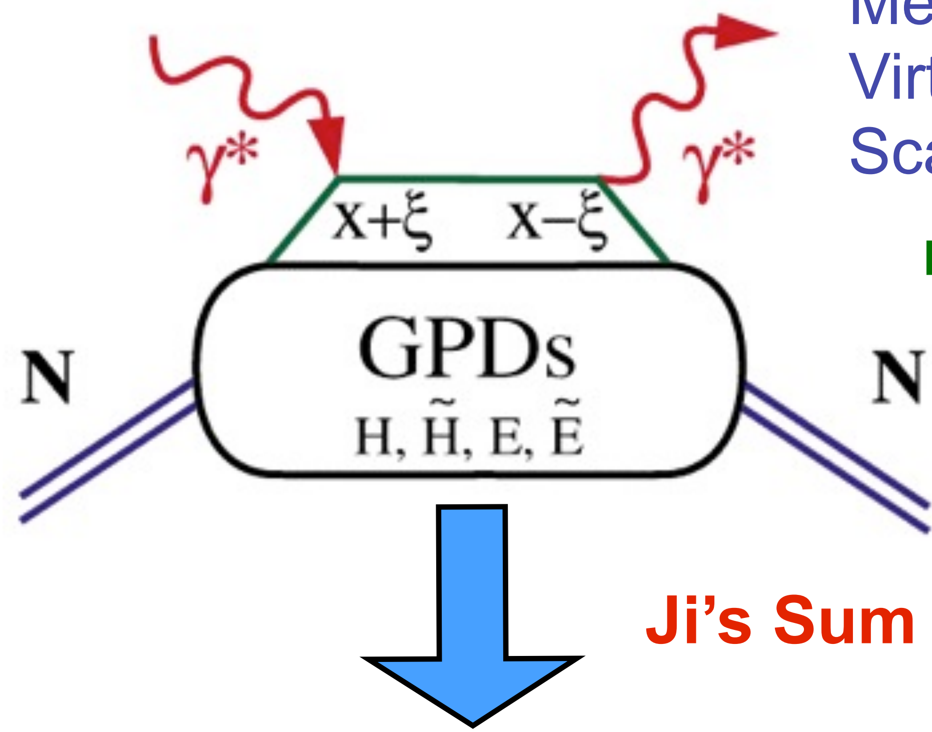

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Ji's Sum rule

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\,[H^q(x,\xi,0) + E^q(x,\xi,0)]$$

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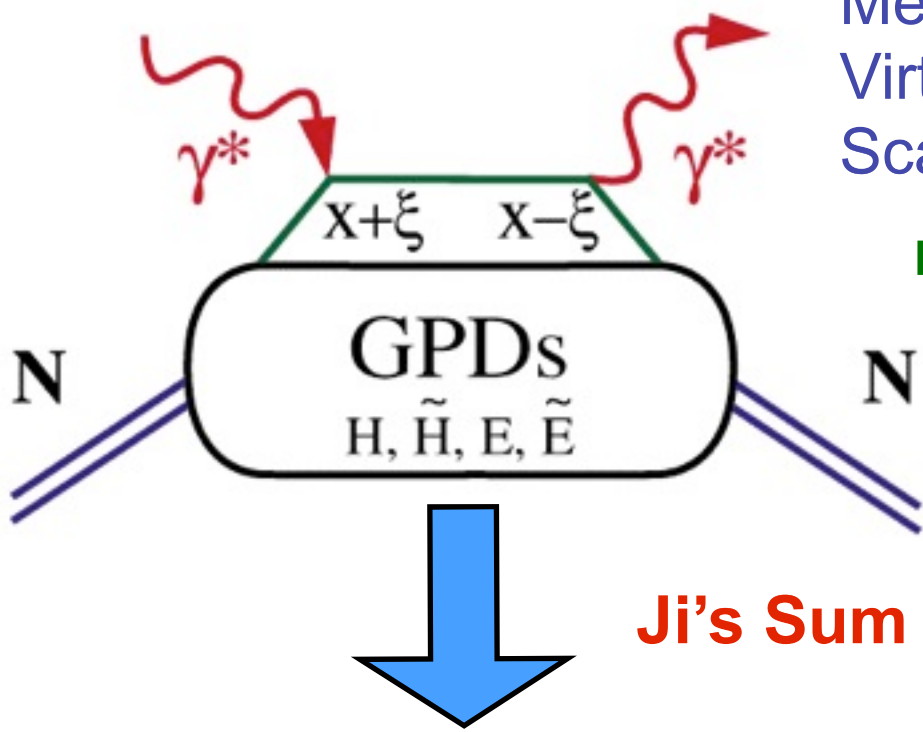

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ξ is *skewness*

Moments of GPD's

- Matrix elements of light-cone correlation functions

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) nP e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- Expand *O(x)* around light-cone

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots D^{\mu_n\}} \psi_q$$

- *Off-forward* matrix element

$$\langle P'|O_q^{\{\mu_1\cdots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$

$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t), \tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$$

↑

Co-efficient of ξ^i

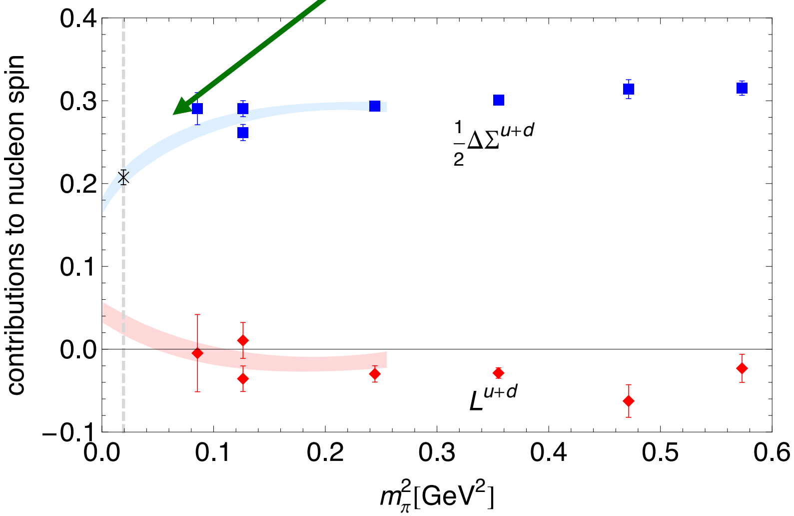
LHPC, QCDSF, 2003

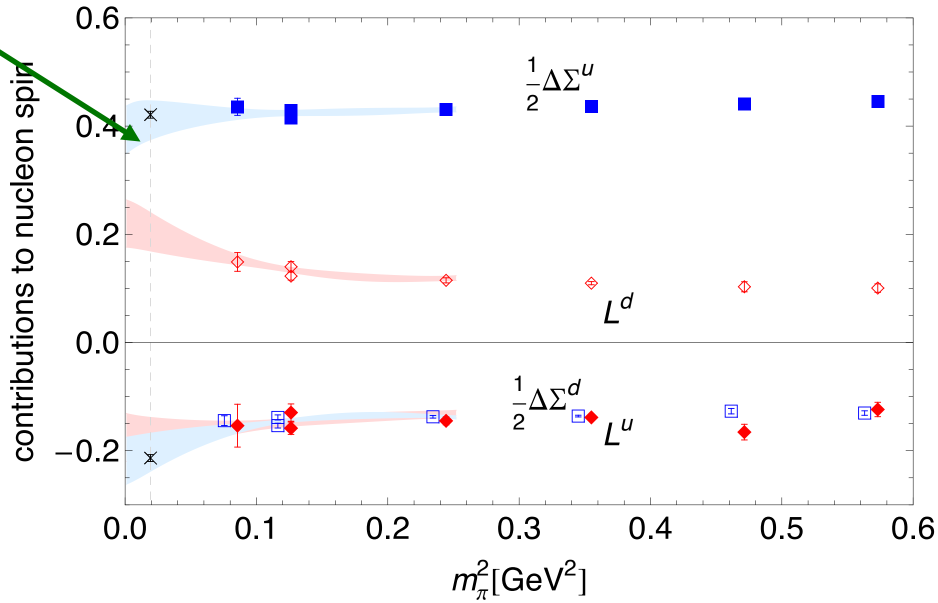
Origin of Nucleon Spin

$$J^q = 1/2\left(A^q_{20}(t=0) + B^q_{20}(t-0)\right)$$
$$\Delta\Sigma^q/2 = \tilde{A}^q_{10}(t=0)/2$$
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g$$

LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); arXiv.1001.3620

HERMES, PRD75 (2007)





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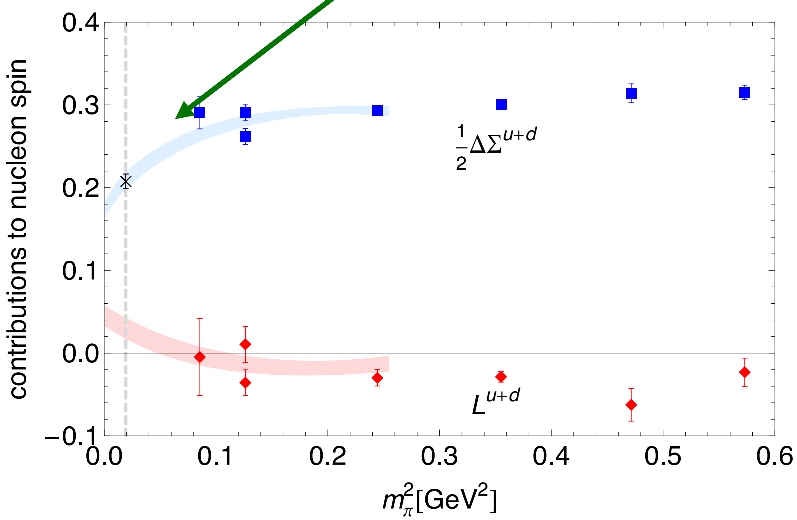
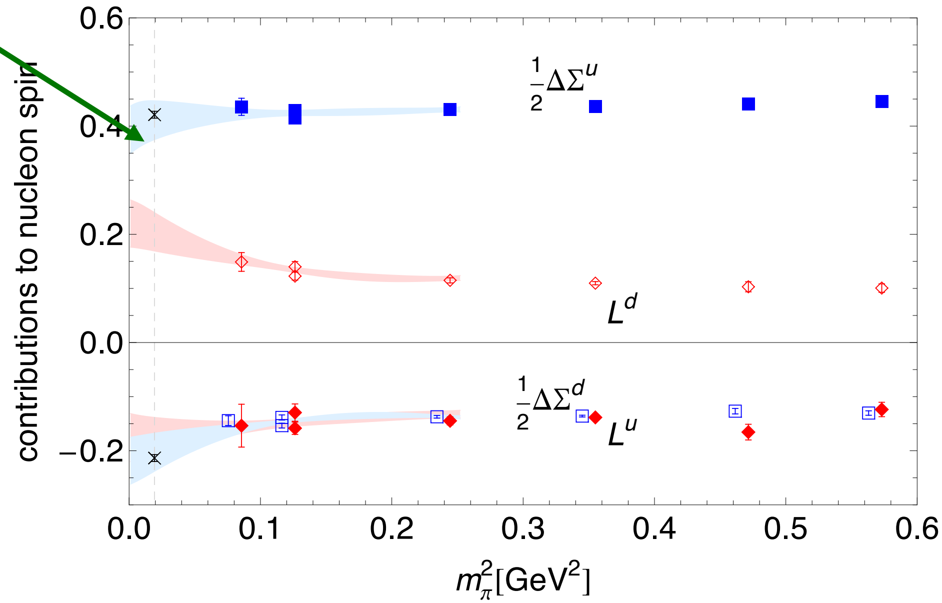
Total orbital angular momentum carried by quarks small

HERMES, PRD75 (2007)





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Total orbital angular momentum carried by quarks small

Orbital angular momentum carried by quark flavors substantial

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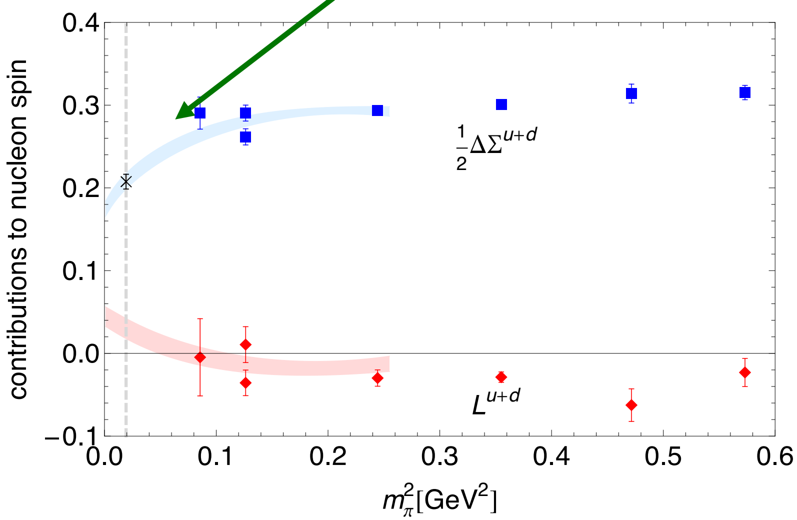
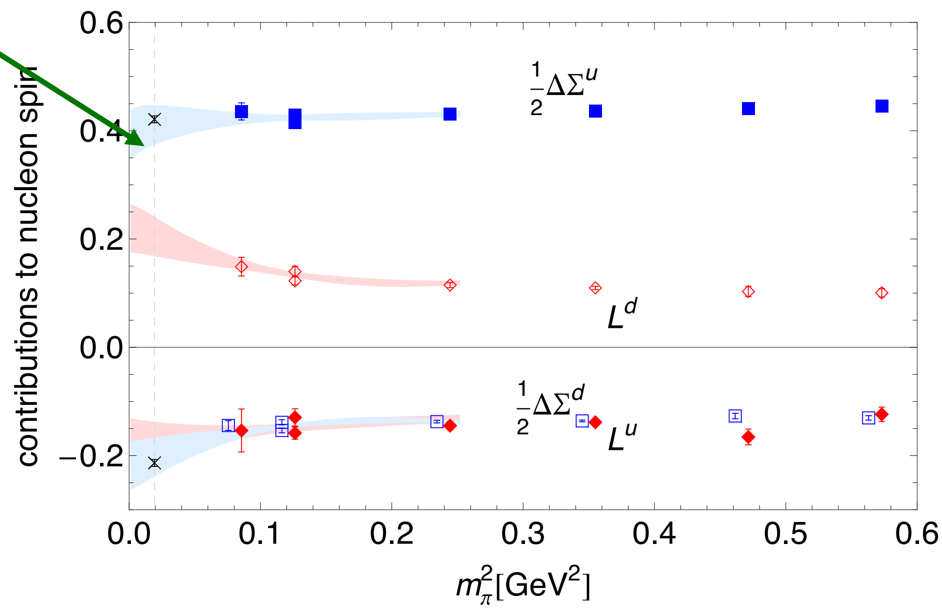
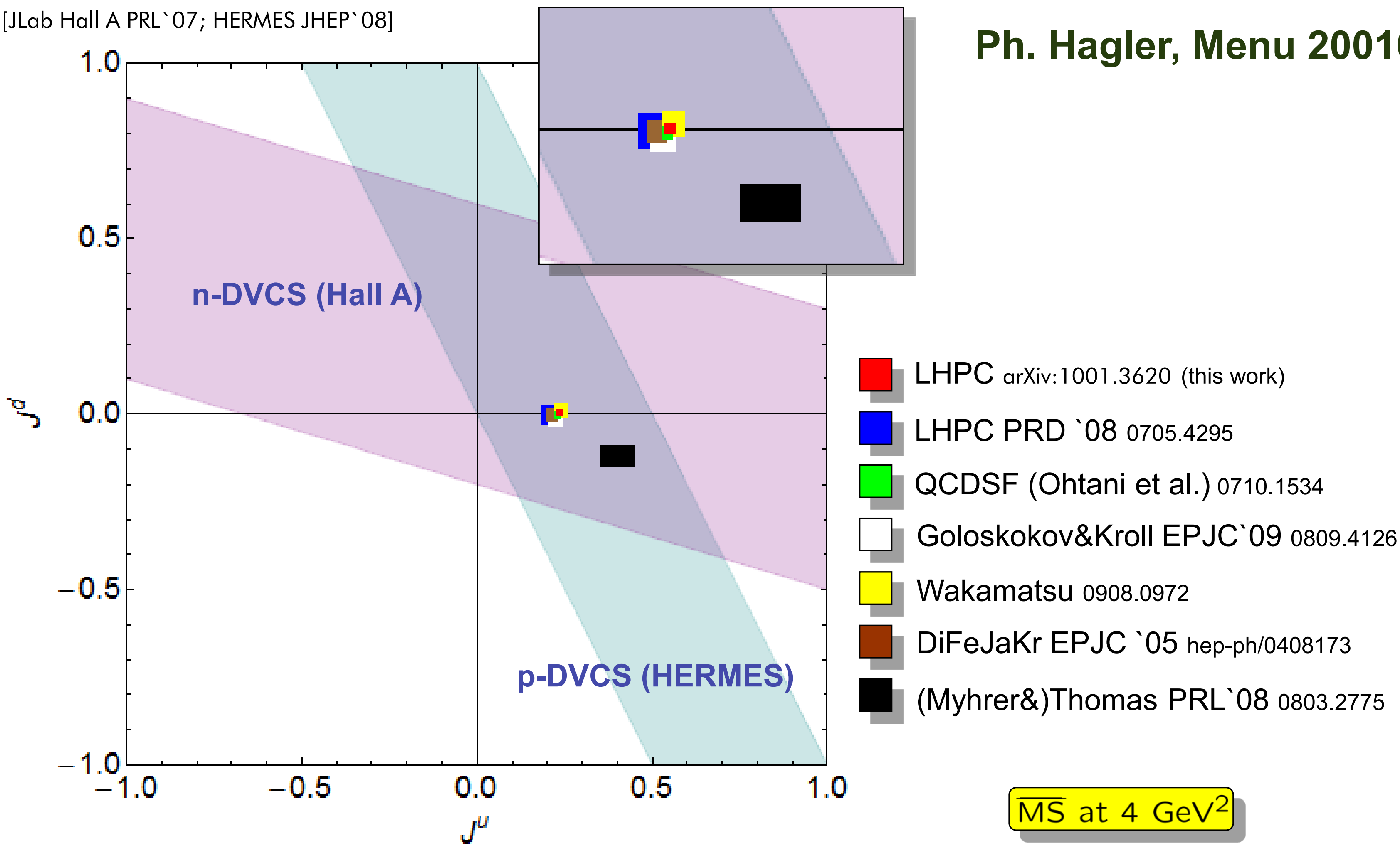




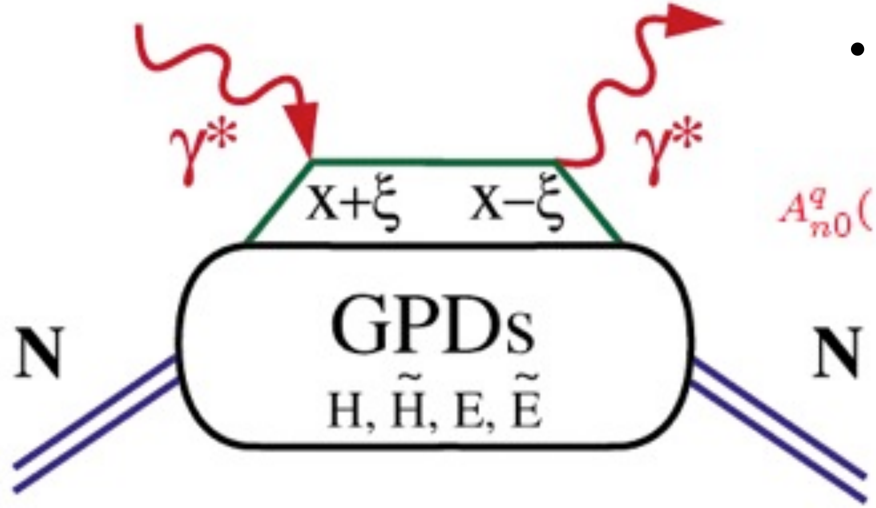
Origin of Nucleon Spin - II

Ph. Hagler, Menu 20010



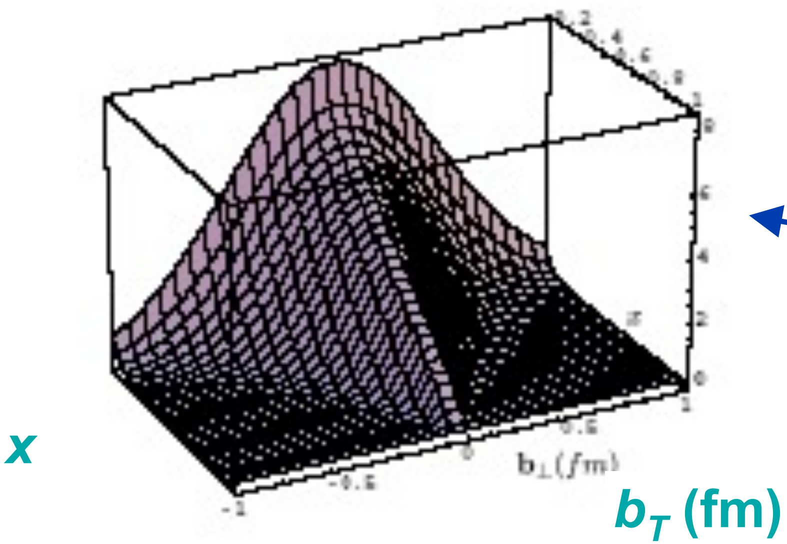
$\overline{MS}$ at 4 GeV^2

Transverse Distribution - I



- t-dependence ↔ impact parameter

$$A^q_{n0}(-\vec{\Delta}_\perp^2) = \int d^2b_\perp\, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\int_{-1}^{1} dx\, x^{n-1}\, q(x,\vec{b}_\perp)$$



Compare to phenomenological models

Decrease slope : decreasing transverse size as x ! 1

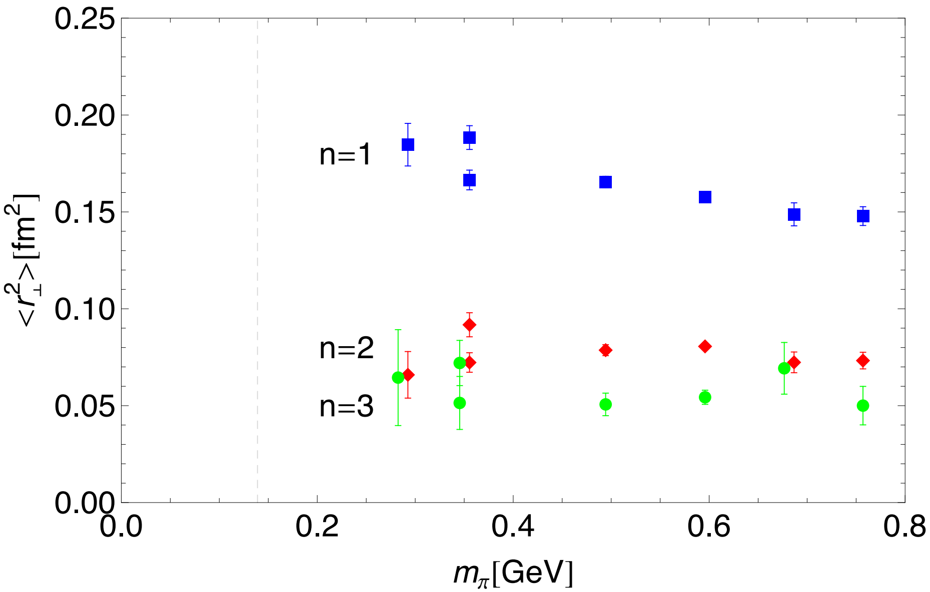
Burkardt

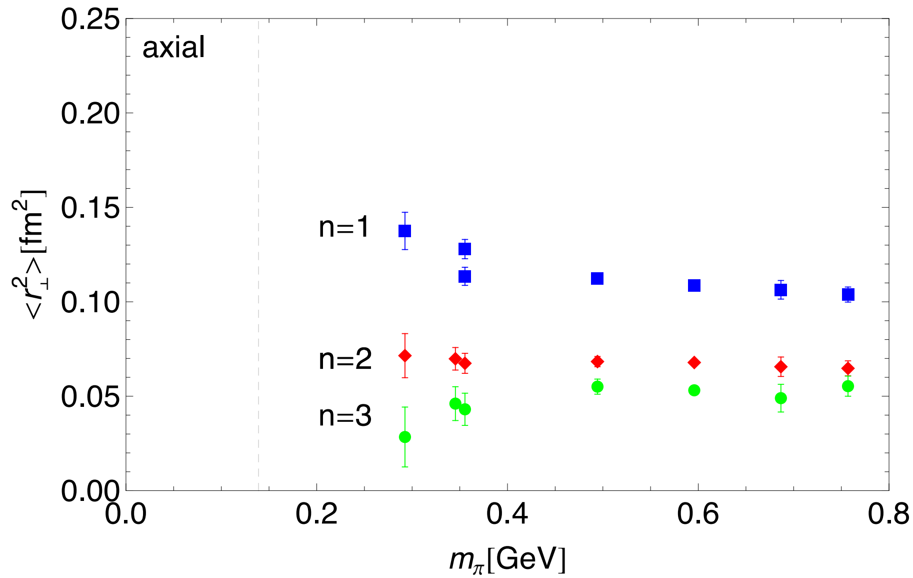
Transverse Distribution - II

Lattice results consistent with narrowing of transverse size with increasing x

LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008)

Flattening of GFFs with increasing n





Summary: Lecture II

- Lattice QCD can describe describe hadron structure in terms in terms of fundamental parton degrees of freedom
- Major effort: approach the physical light-quark masses to gain control over chiral behavior - Extrapolation to Interpolation
- Important role: lattice QCD + expt together determining eg GPDs in a way neither can alone
- Next time
  - New developments: TMDs
  - Flavor-singlet structure
  - Structure of excited states