Hadron Structure - II

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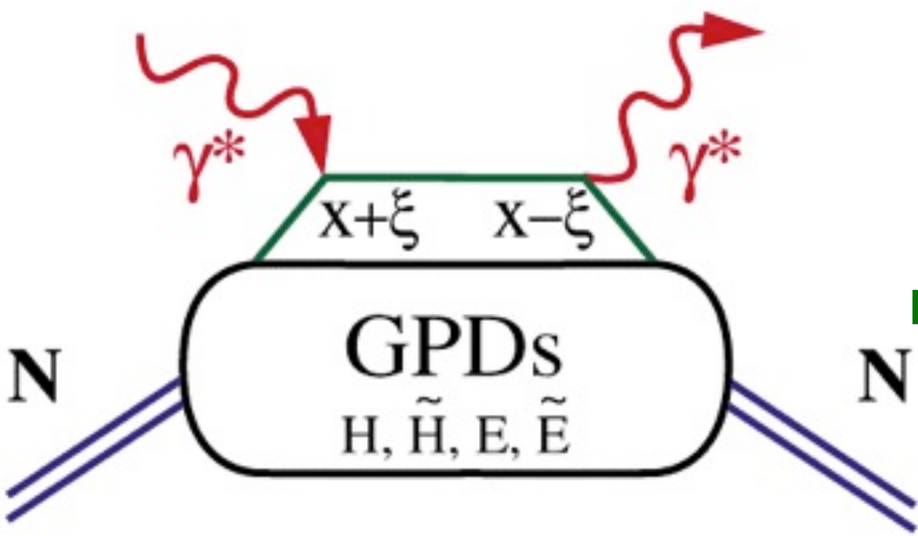
StrongNet 2011, Bielefeld

Plan of Lectures

- Lecture 2 - Hadron Structure I
  - What are we studying, and how do we encapsulate it?
  - Paradigm: electromagnetic form factor of pion
  - Nucleon EM form factors
  - Polarized and unpolarized structure functions
  - Three-dimensional imaging of hadrons: Generalized Parton Distributions
- Lecture 3: Hadron Structure - II
  - Recent advances: Transverse-Momentum-Dependent distributions
  - Flavor-singlet contributions: role of sea quarks and gluons
  - Structure of excited states: radiative transitions between mesons

Generalized Parton Distributions (GPDs)



D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

- Matrix elements of light-cone correlation functions

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) nP e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- Expand *O(x)* around light-cone

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \dots D^{\mu_n\}} \psi_q$$

LHPC, QCDSF, 2003

- *Off-forward* matrix element

Co-efficient of ξ^i

$$\langle P'|O_q^{\{\mu_1\cdots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$

$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t), \tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$$

GPDs and Orbital Angular Momentum

- **Form factors of energy momentum tensor - *quark and gluon angular momentum***

“$\bar{q}\gamma_\mu D_\nu q$”

$$\frac{1}{2} = \sum_q J^q + J^g$$

X.D. Ji, PRL 78, 610 (1997)

$$= \frac{1}{2}\left\{\sum_q \left(A^q_{20}(t=0) + B^q_{20}(t=0)\right) + A^g_{20}(t=0) + B^g_{20}(t=0)\right\}$$

$$\sum_q \left(\frac{1}{2}\Delta\Sigma^q + L^q\right)$$

gluon operators - see later

Decomposition

- **Gauge-invariant**
- **Renormalization-scale dependent**
- **Handle on Quark orbital angular momentum**

Mathur et al., *Phys.Rev. D62 (2000) 114504*

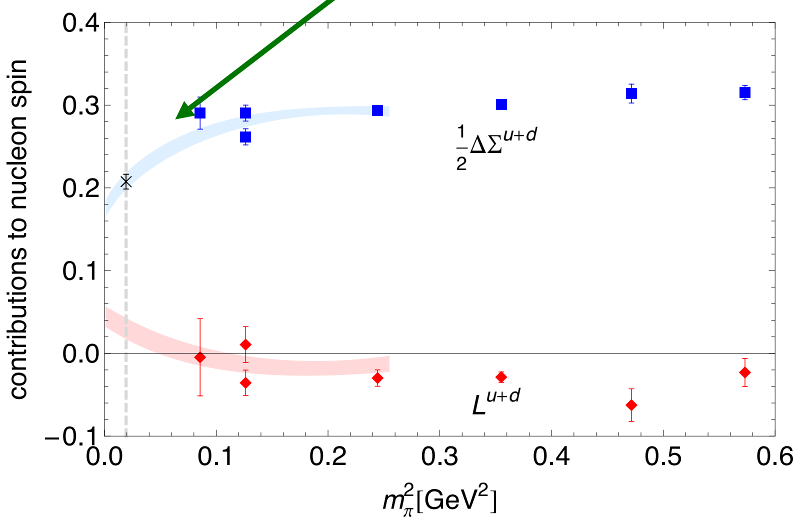
Origin of Nucleon Spin

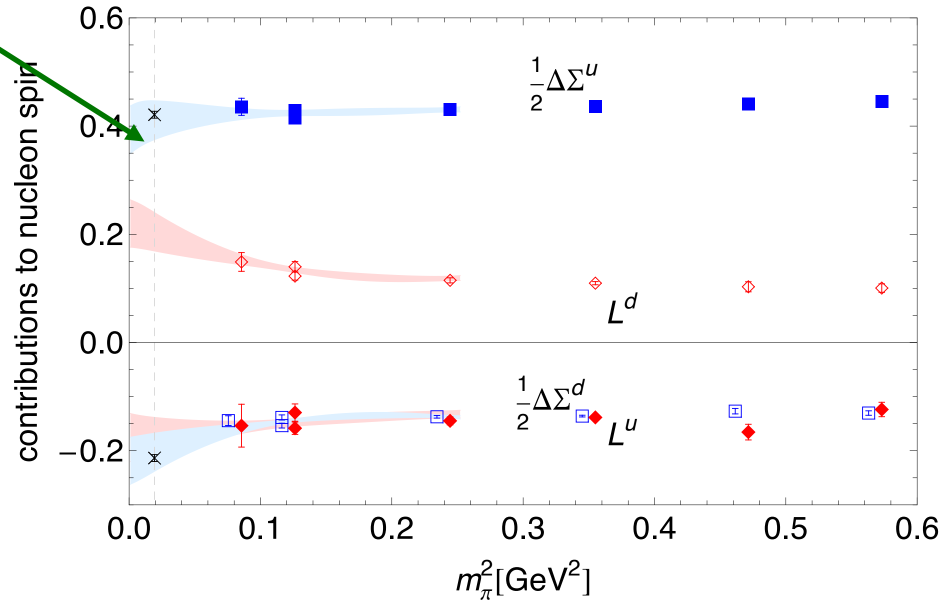
$$
\begin{aligned}
J^q &= 1/2\left(A^q_{20}(t=0) + B^q_{20}(t-0)\right) \\
\Delta\Sigma^q/2 &= \tilde{A}^q_{10}(t=0)/2 \\
\frac{1}{2} &= \frac{1}{2}\Delta\Sigma^{u+d} + L^{u+d} + J^g
\end{aligned}
$$

- **Total orbital angular momentum carried by quarks small**
- **Orbital angular momentum carried by individual quark flavours substantial.**

LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); arXiv.1001.3620

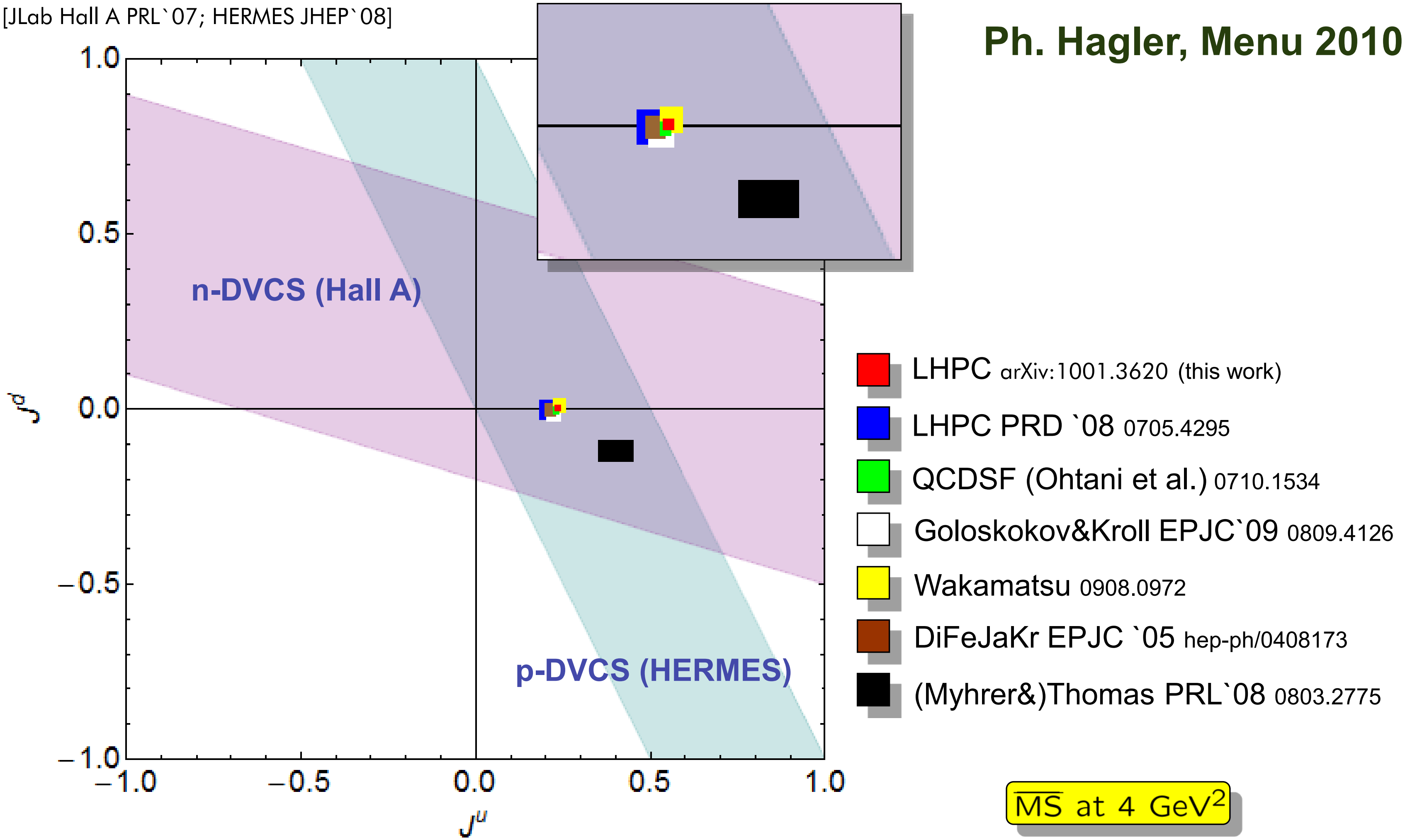
HERMES, PRD75 (2007)





Disconnected contributions neglected.

Origin of Nucleon Spin - II



Ph. Hagler, Menu 2010

Can we separate gluon orbital + spin?

- **Can we go further?**

JI's sum rule

$$\frac{1}{2} = J^Q + J^G$$

We also have:

Jaffe, Manohar, NPB337, 509

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

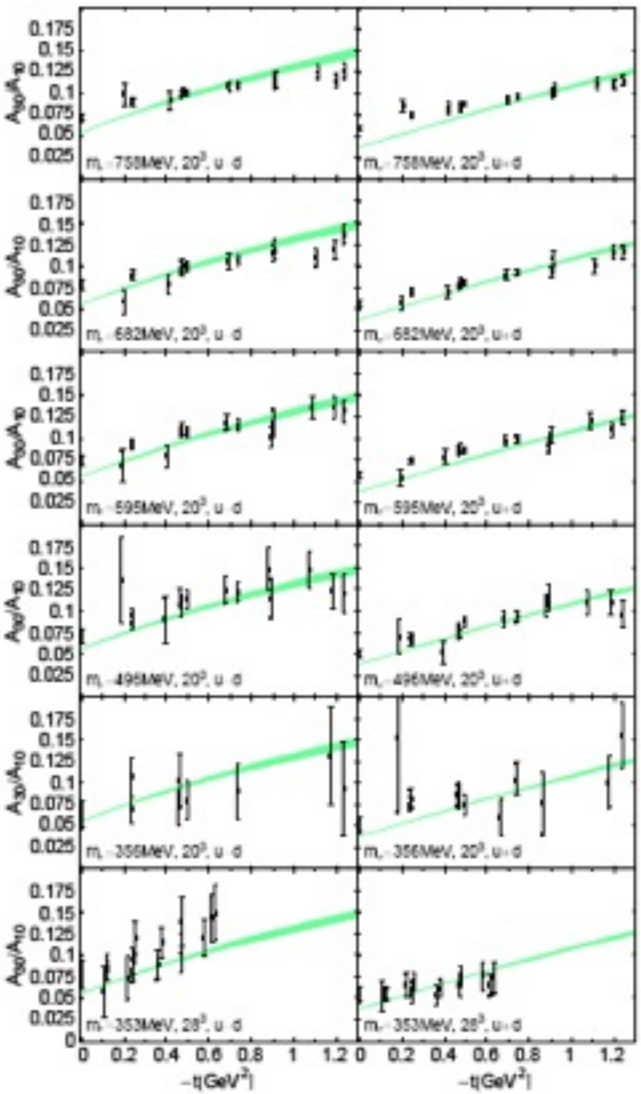
← **Light-cone decomposition**

Cannot further decompose

$$L^g = J^G - \Delta G$$

Jaffe, hep-ph/0008038
Burkardt, Miller, Nowak, arXiv: 0812.2208

Parametrizations of GPDs



Provide phenomenological guidance for GPD's

- *CTEQ, Nucleon Form Factors, Regge*

Comparison with *Diehl et al, hep-ph/0408173*

Ratio of form factors agrees with phenomenological model - without fits

Important Role for LQCD

Transverse Spin in Nucleon

Measuring generalized form factors corresponding to tensor current gives provides information on transverse spin of nucleon

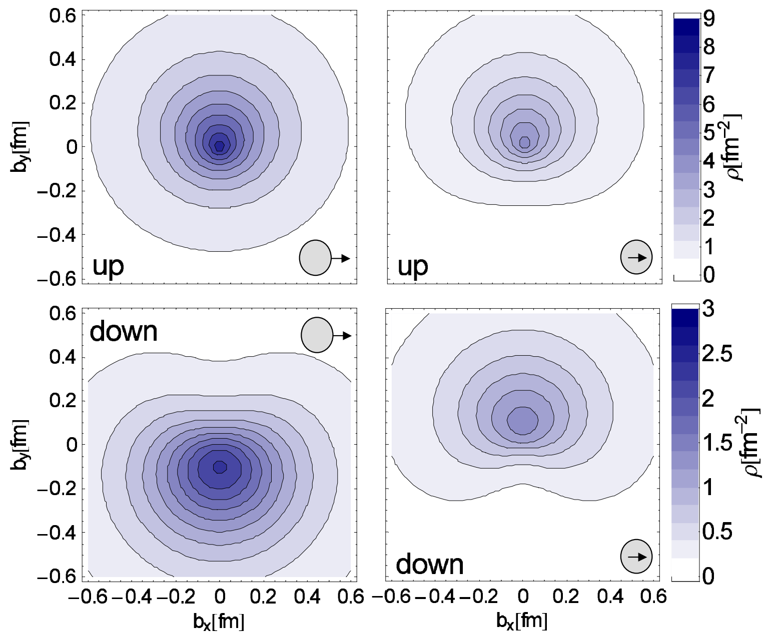
$$\langle P'\Lambda'|\mathcal{O}_T^{\mu\nu}|P\Lambda\rangle = \overline{u}(P',\Lambda')\Big\{\sigma^{\mu\nu}\gamma_5\Big(A_{T10}(t) - \frac{t}{2m^2}\widetilde{A}_{T10}(t)\Big) + \frac{\epsilon^{\mu\nu\alpha\beta}\Delta_\alpha\gamma_\beta}{2m}\overline{B}_{T10}(t) - \frac{\Delta^{[\mu}\sigma^{\nu]\alpha}\gamma_5\Delta_\alpha}{2m^2}\widetilde{A}_{T10}(t)\Big\}u(P,\Lambda)\ ,$$

QCDSF, PRL, 0612021

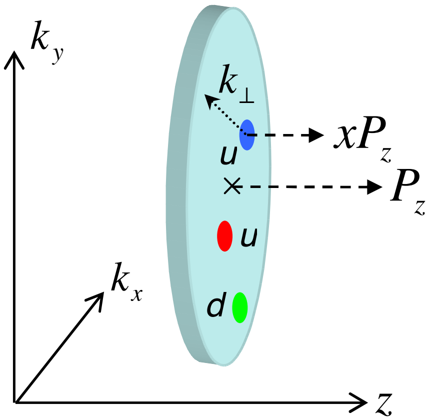
$$\mathcal{O}_T^{\mu\nu} = \bar{q}\sigma_{\mu\nu}\gamma_5 q$$

Lowest moment $B_{T10}(t)$

Impact parameter $b_T \leftrightarrow$ Fourier transform of t



Transverse-Momentum Distributions



B. Musch, PhD Thesis; Haegler, Musch, Negele, Schafer arXiv: 0908.1283

Introduce Momentum-space correlators

$$\Phi_\Gamma = \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\tilde{\Phi}_\Gamma(l;P,S)$$

$$= \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\langle P,S \mid \bar{q}(l)\Gamma\mathcal{U}q(0) \mid P,S\rangle$$

continuum

$$\mathcal{U} \equiv \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to ℓ

Choice of path - retain gauge invariance

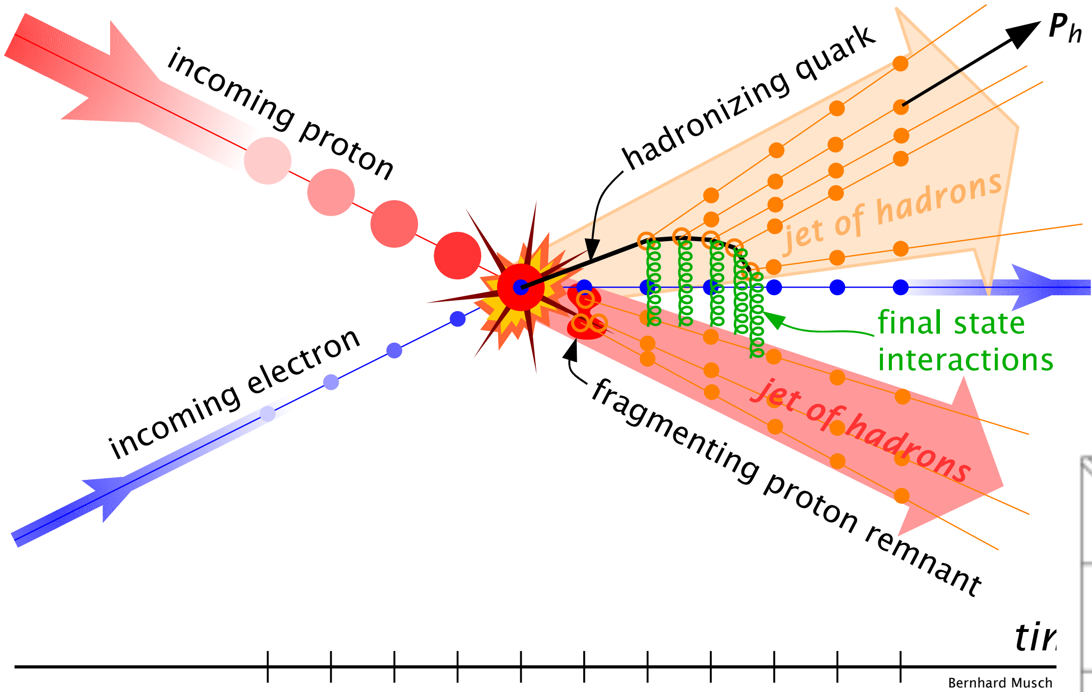
Real world!: path runs to infinity

Lattice: equal time slice

Transverse momentum distributions (TMDs)

from experiment, e.g., **SIDIS** (semi-inclusive deep inelastic scattering)

HERMES, COMPASS, JLab 6 GeV, JLab 12 GeV , ... , EIC



Cf: measured in Drell-Yan, eg at RHIC-spin

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Boer-Mulders

Sivers ← time-reversal odd

final state interactions!
explain large asymmetries otherwise forbidden!
signature of QCD!

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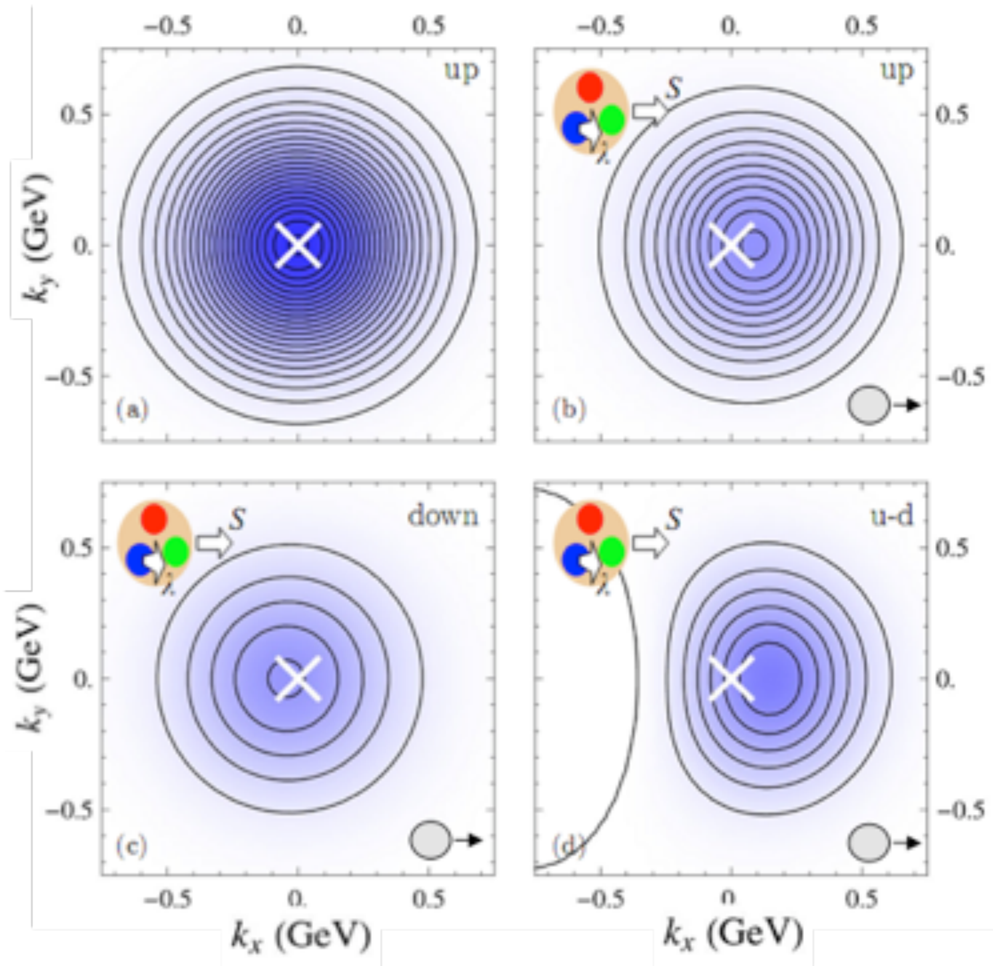
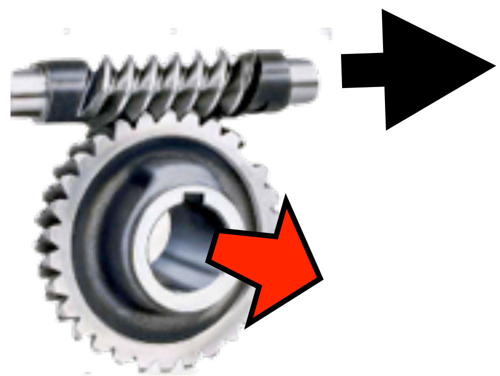
Slide: A. Bacchetta

Worm gears on the lattice





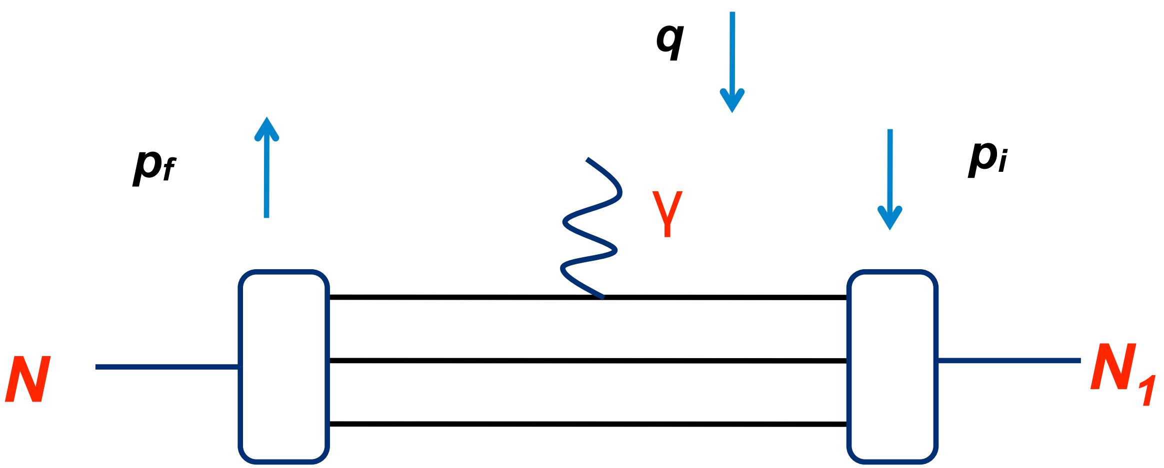
Flavor-Singlet Hadron Structure

Flavor-singlet Quantities

$$\langle p_f \mid V_\mu \mid p_i \rangle = \bar{u}(p_f)\left[\gamma_\mu F_1(q^2) + iq_\nu \frac{\sigma_{\mu\nu}}{2m_N} F_2(q^2)\right] u(p_i)$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2m_N)^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$



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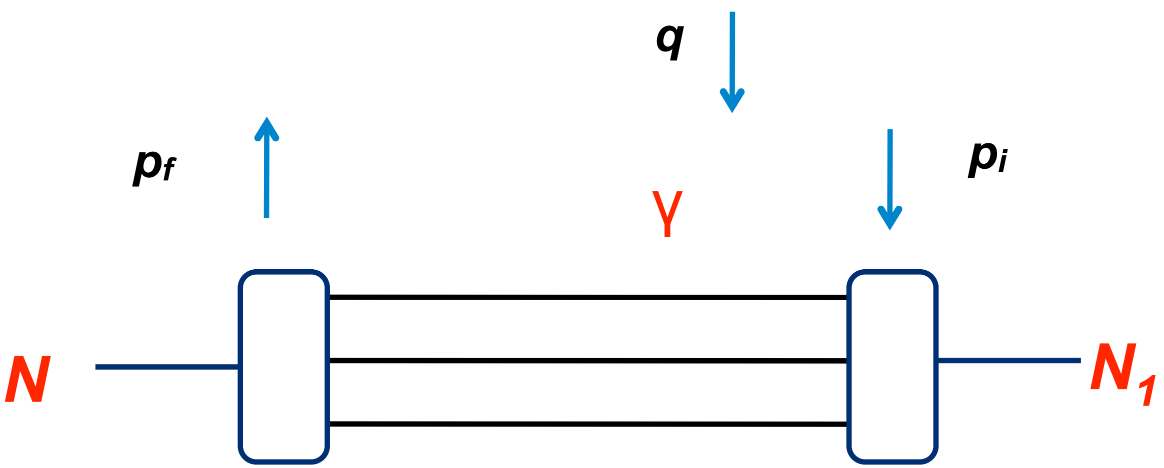

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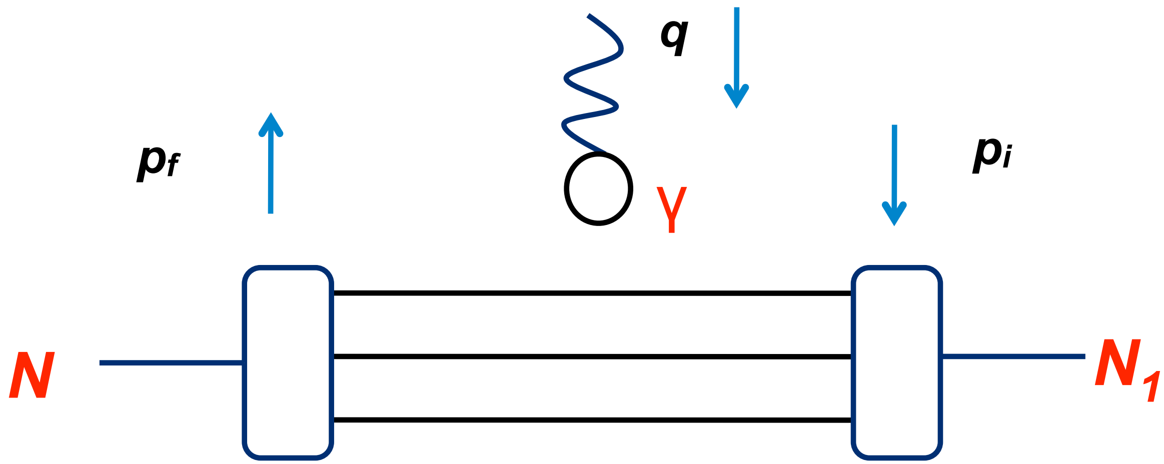

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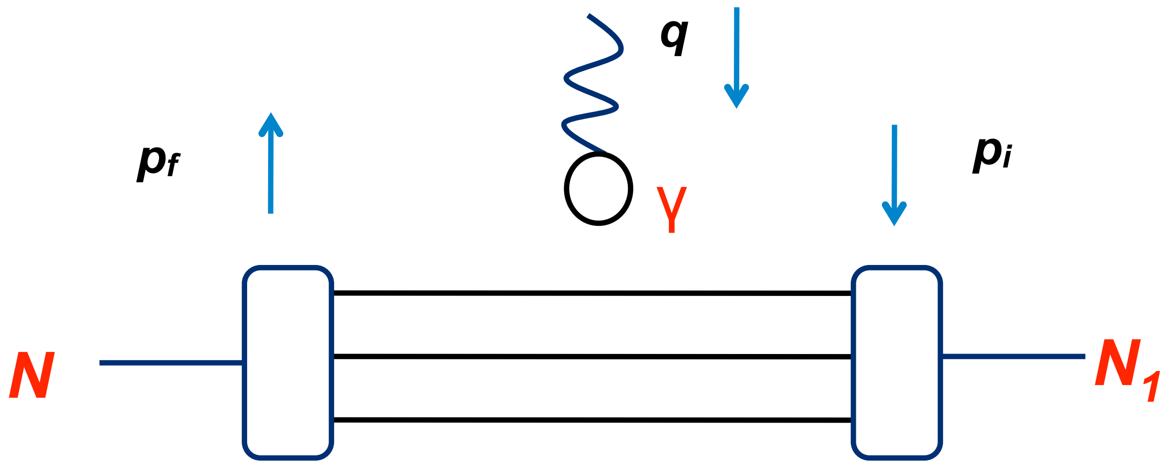
Isoscalar: p and n separately, or u and d separated contribution.



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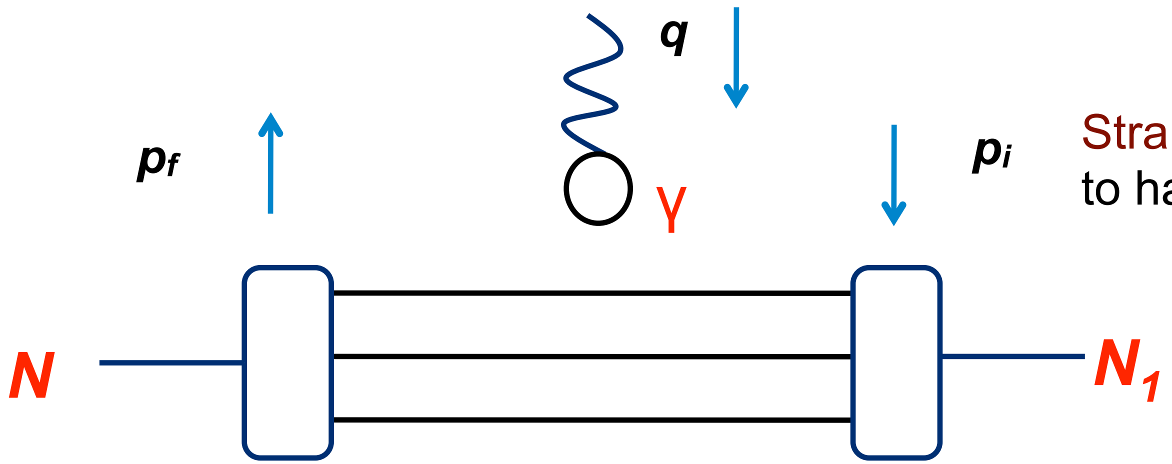
Isoscalar: p and n separately, or u and d separated contribution.

$$V_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$$



Strange-quark contribution to hadron structure

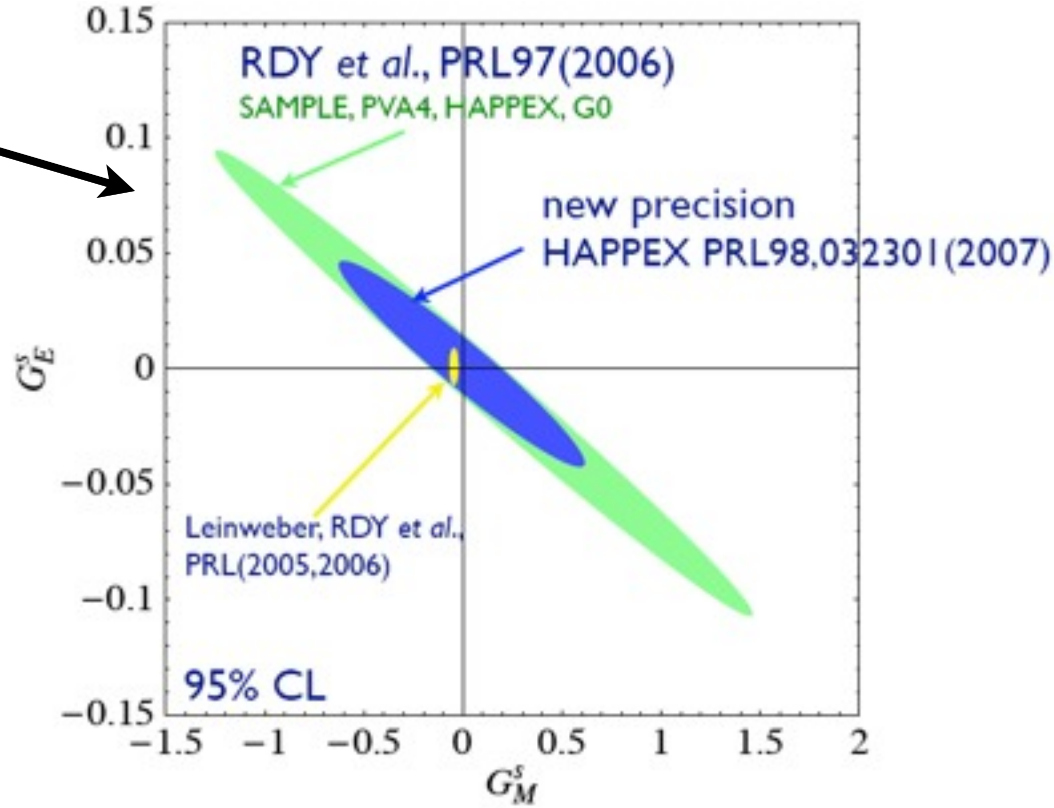
Flavor-singlet: Disconnected Contributions

Parity-violating electron scattering

$$G^{\gamma,p}_{E/M} = \frac{2}{3}G^u_{E,M} - \frac{1}{3}G^d_{E,M} - \frac{1}{3}G^s_{E,M}$$

$$G^{Z,p}_{E/M} = (1 - \frac{8}{3}\sin^2\theta_W)G^u_{E,M} - (1 - \frac{4}{3}\sin^2\theta_W)G^d_{E,M} - (1 - \frac{4}{3}\sin^2\theta_W)G^s_{E,M}$$

Expected to be small



Flavor-singlet: Disconnected Contributions

Parity-violating electron scattering

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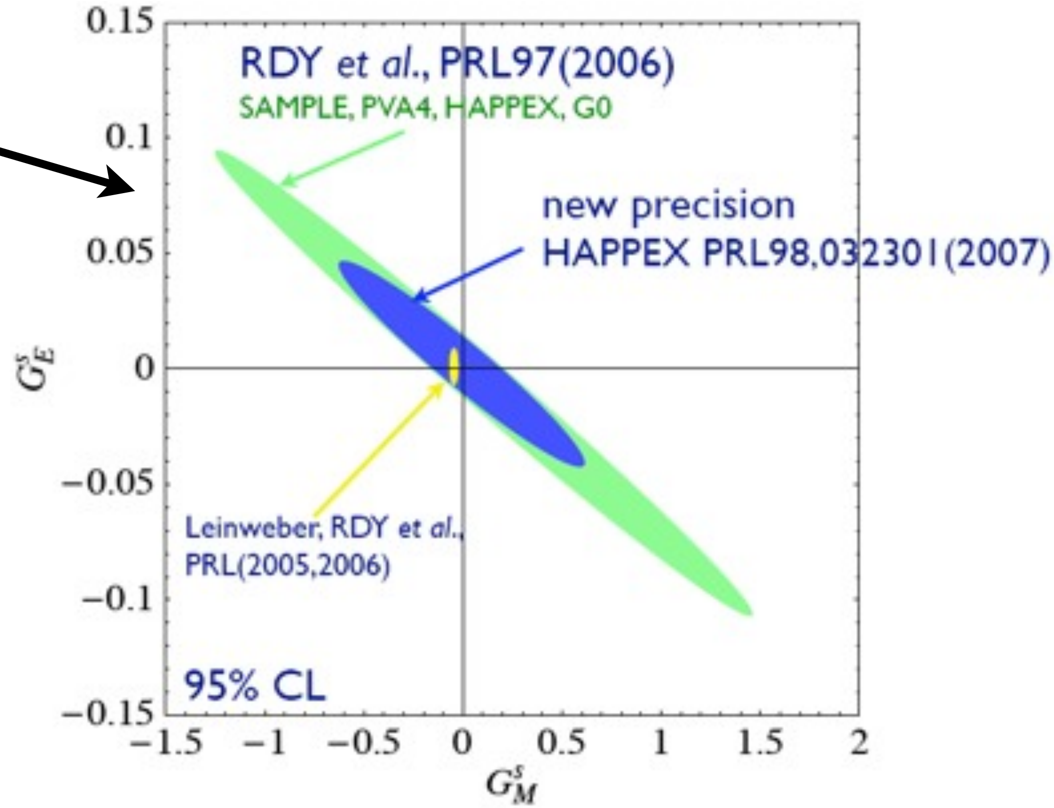


Spin carried by s-quark

$\Delta s = -0.085(13)(8)(9)$

HERMES: dominated by small x

Disconnected contributions

Three-point correlator looks like

$$\Gamma^{\rm disc}_{N\mu N}(t_f,t,0;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 \mid N(\vec{x},t_f)\bar{s}(\vec{y},t)\Gamma s(\vec{y},t)\bar{N}(\vec{0},0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

$$= \sum_{\vec{x}} \langle 0 \mid N(\vec{x},t_f) \left(\sum_{\vec{y}} \bar{s}(\vec{y},t)\Gamma s(\vec{y},t) e^{-i\vec{q}\cdot\vec{y}} \right) \bar{N}(\vec{0},0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}}$$

Need efficient means of evaluating $\sum_{\vec{y}} {\rm Tr}[M^{-1}(\vec{y},t;\vec{y},t)\Gamma]$

Straightforward way: introduce noise vectors such that $< \eta_i >= 0; \quad < \eta_i \eta_j >= \delta_{ij}$

Solve $MX = \eta$: then $< M^{-1}_{ij} >=< \eta_j X_i >$

Error both from Gauge Noise and from Stochastic noise

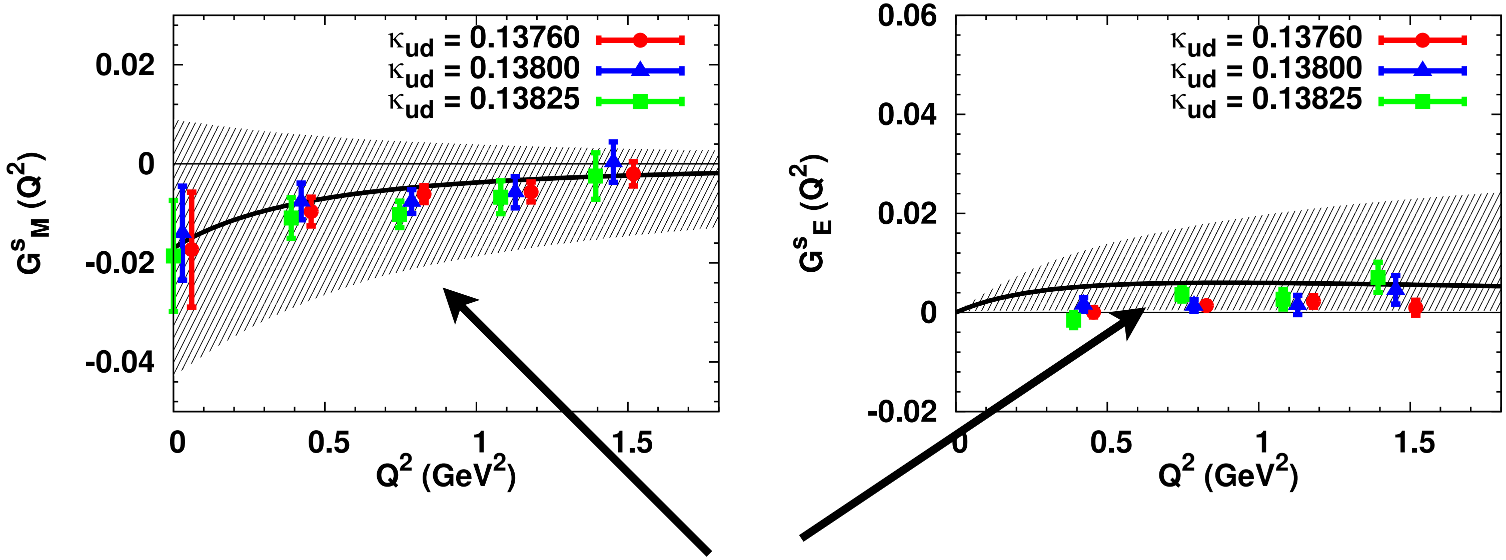
Noise-reduction methods

- Partitioning ("dilution") - sources have support on, say, 8 timeslices
- Hopping parameter expansion
- Different stochastic sources

s-quark contn. to EM Form Factor

Doi et al. (ChQCD Collaboration), arXiv:0910.2687, PRD79:094502,2009

2+1 Clover, pion mass > 600 MeV



Uncertainties: statistical, Q^2 dependence, chiral extrapolation

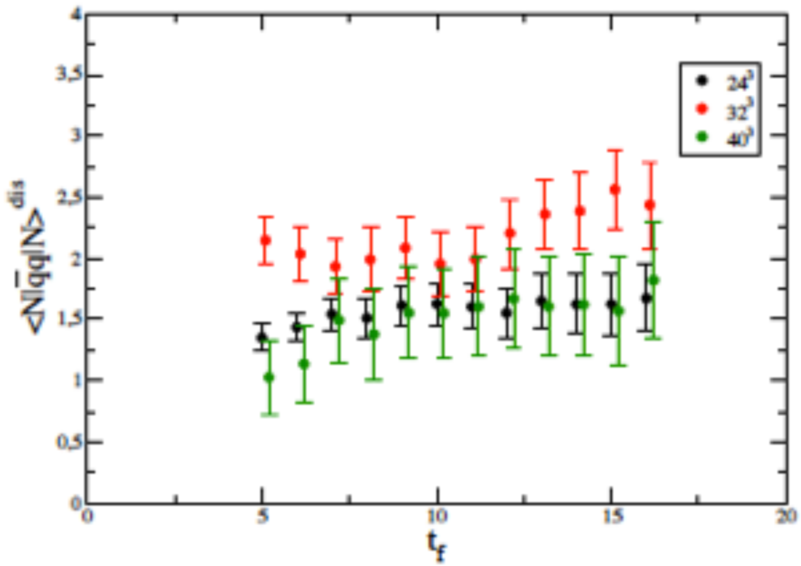
$$G^s_M(0) = -0.017(25)(07)$$

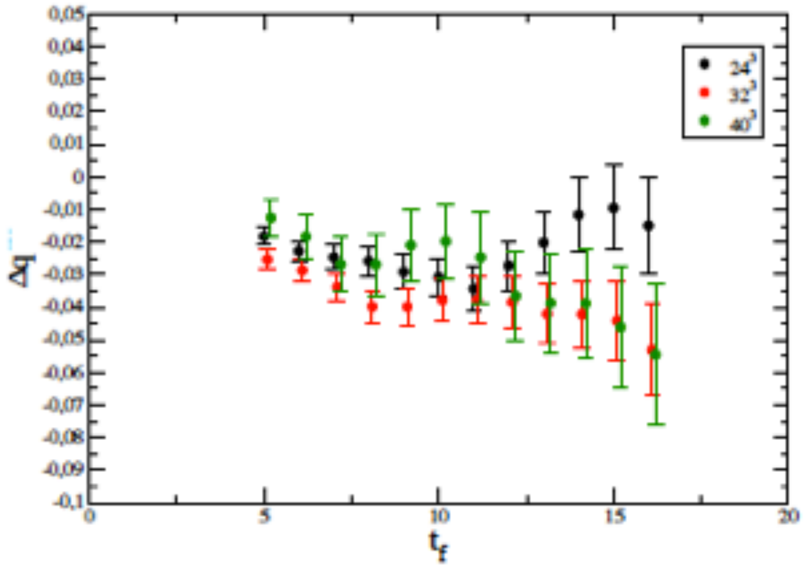
Δs and Sigma Correlator

Two-flavor NP clover; m_π = 270 MeV

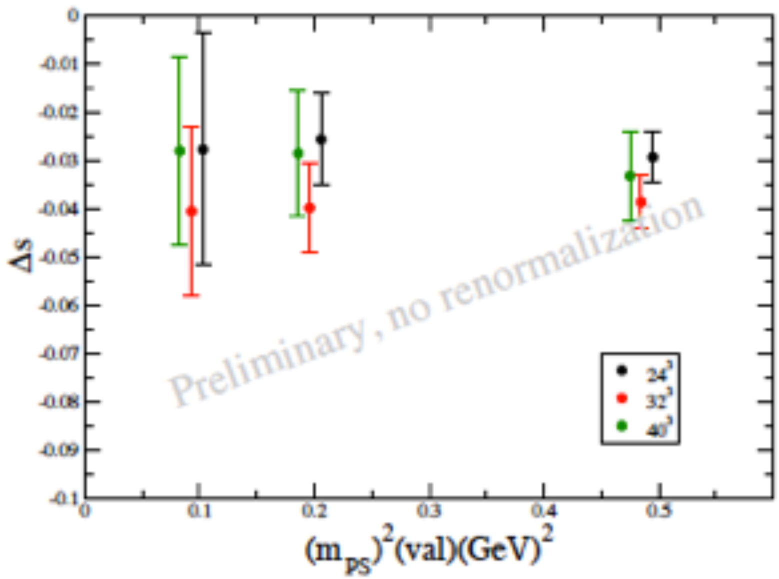
S. Collins et al, 2010 (StrongNET)

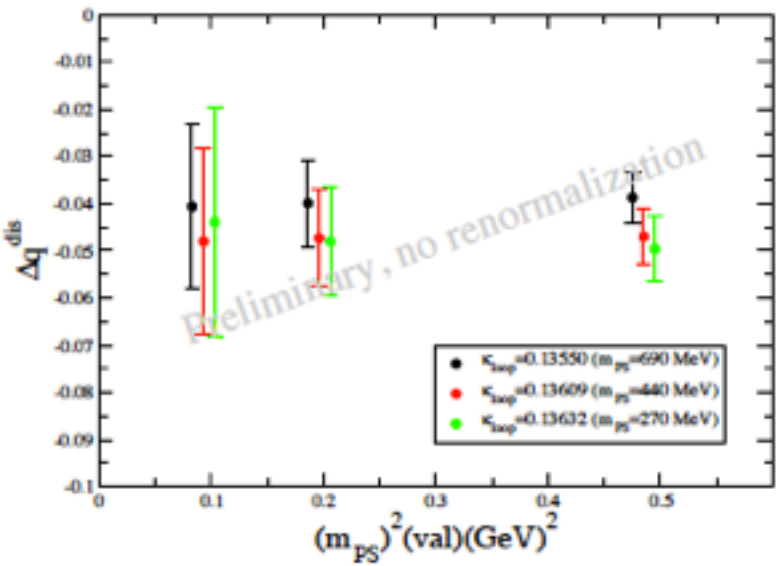
$$\mathrm{Tr}(M^{-1}\Gamma) = 2\kappa \mathrm{Tr}[(1-\kappa D)^{-1}\Gamma] = \mathrm{Tr}[(2\kappa + 2\kappa^2 D + \kappa^2 D^2 M^{-1})\Gamma]$$





Δs and Sigma Correlator





Quark and gluons mix under renormalization

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}$$

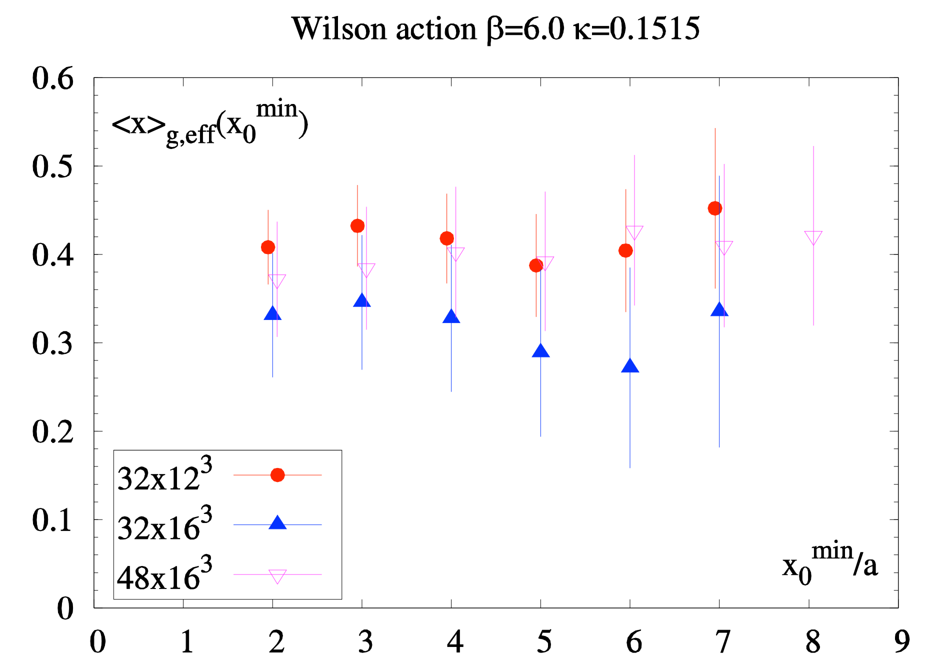
The local operators mix as follows:

$$O^{qS}_{\mu_1 \cdots \mu_N} = \frac{1}{2^N}\overline{\psi}\,\gamma_{[\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_N]}\,(1 \pm \gamma_5)\psi$$

$$O^{gS}_{\mu_1 \cdots \mu_N} = \sum_{\rho}\mathrm{Tr}\left[F_{[\mu_1\rho}\,\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_{N-1}}\,F_{\rho\mu_N]}\right]$$

Gluon Momentum Fraction in Pion

Use improved operator: E2 – B2: 40x increase in signal
HYP smeared, so loss of locality



H. Meyer, J. Negele, PRD (2008)

Quenched Wilson, m_π = 600 - 1100 MeV

$$\langle x \rangle_{\mathrm{glue}}(\mu = 2\ \mathrm{GeV}) = 0.37 \pm 8 \pm 12$$

Momentum sum rule: $\langle x \rangle_{\mathrm{glue}} + \langle x \rangle_{\mathrm{quarks}} = 0.99 \pm 8 \pm 12$

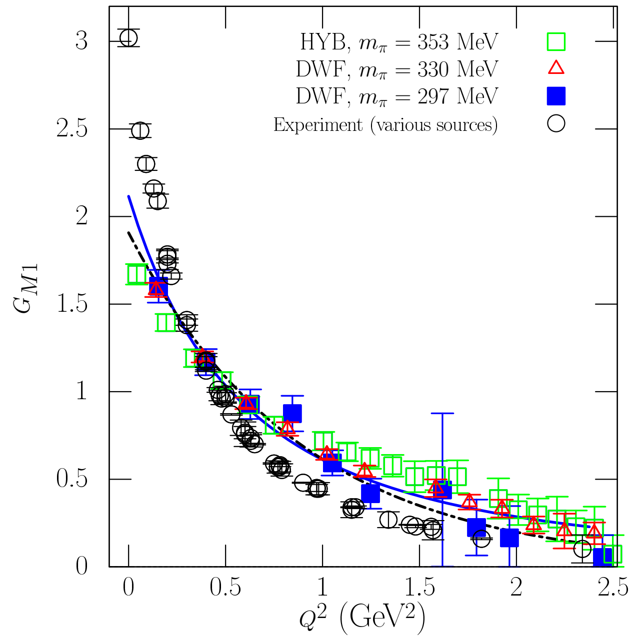
What about the proton?

Structure and EM Transitions to excited states

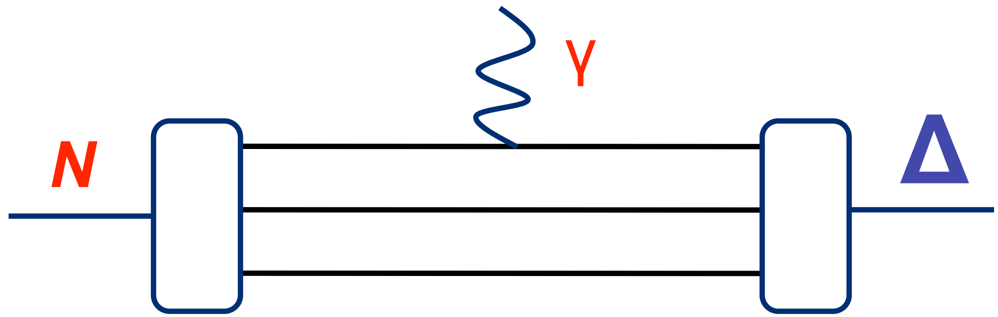
Form factors of excited states, and transition form factors to excited states, provide additional insight into nature of QCD. Precise electro-production data

Program of computations looking at Δ form factor, and Nγ → Δ transition form factors
N.B. Δ → Nπ is p-wave decay, suppressed at zero momentum.

Admits *three* multipoles: magnetic dipole, electric quadrupole and Coulomb quadrupole: G_{M1}, G_{E2}, G_{C2}



Alexandrou et al, DWF + DWF valence/Asqtad sea



Free of disconnected contributions

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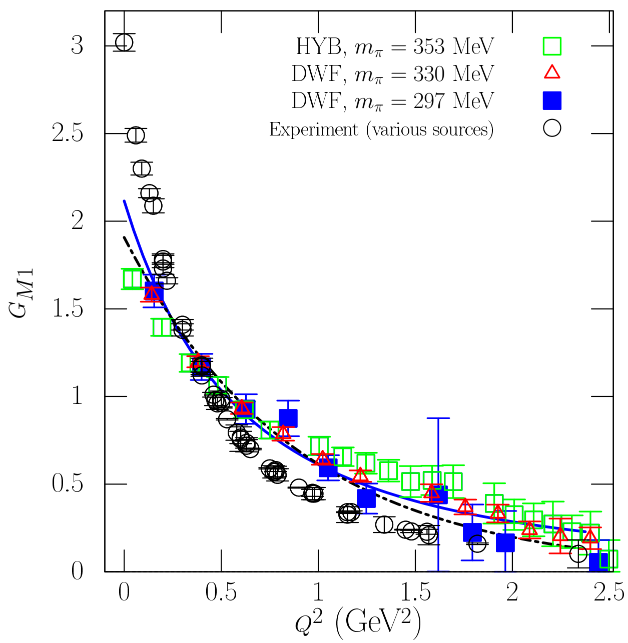

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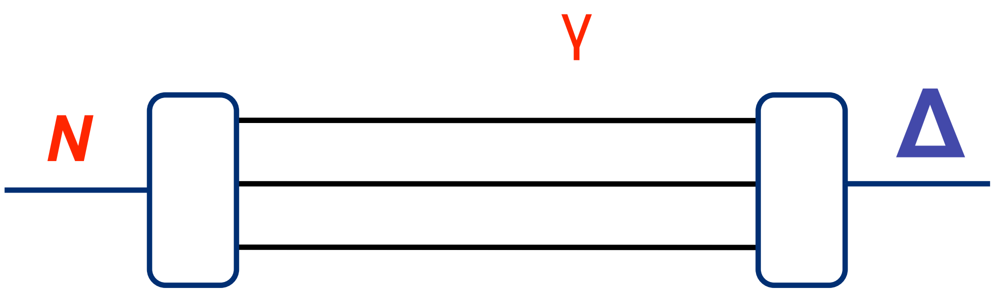
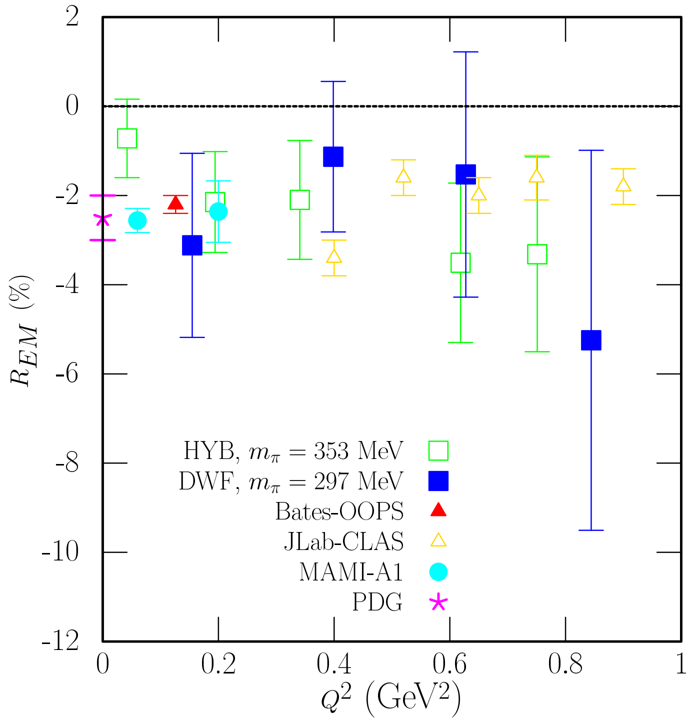


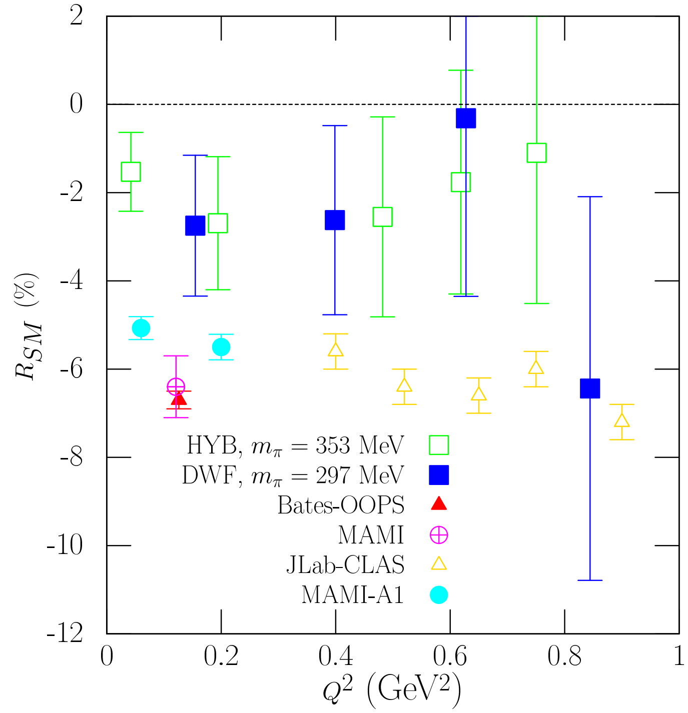
Free of disconnected contributions

N-Δ Transition Form Factor

$$R_{EM} = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$$

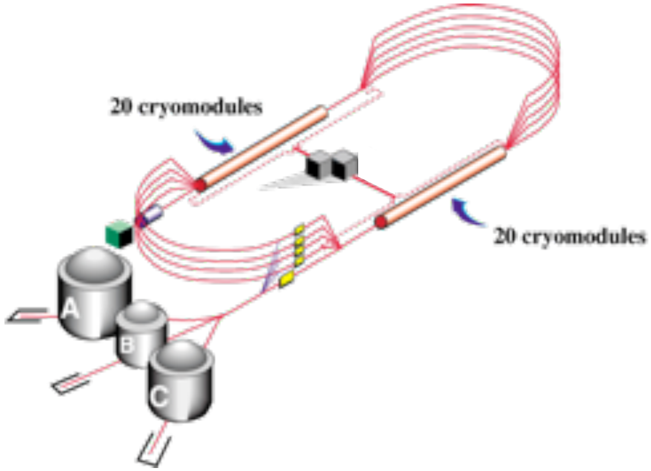
$$R_{SM} = -\frac{|\vec{q}\,|}{2m_\Delta}\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$$



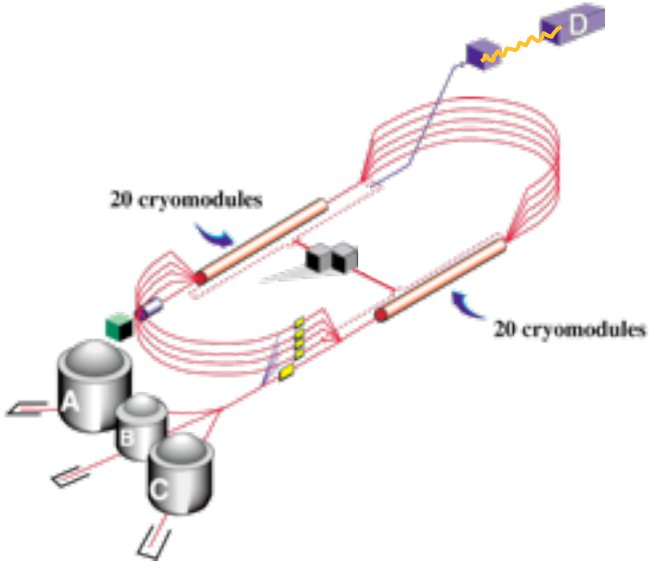


Non-zero values: sphericity in either N or Δ - zero quadrupole moment for spin-1/2 system

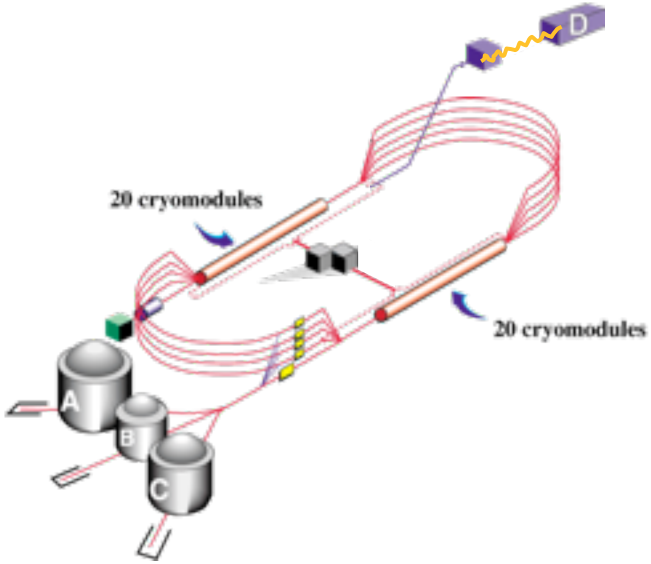
Radiative Transitions in Mesons

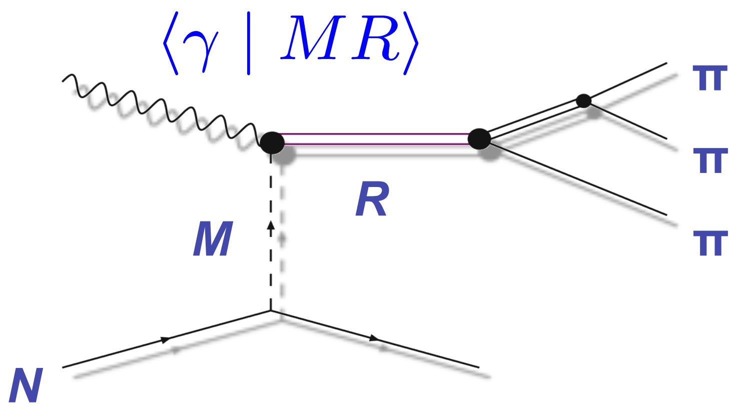


Radiative Transitions in Mesons



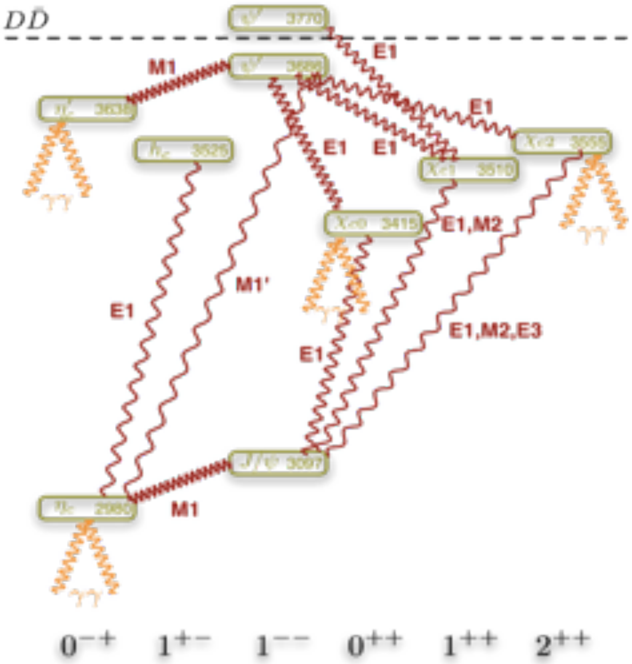
Radiative Transitions in Mesons





Radiative Transitions in Mesons - II

Look at radiative decays in charmonium - wealth of experimental data. Lots of transitions below threshold!

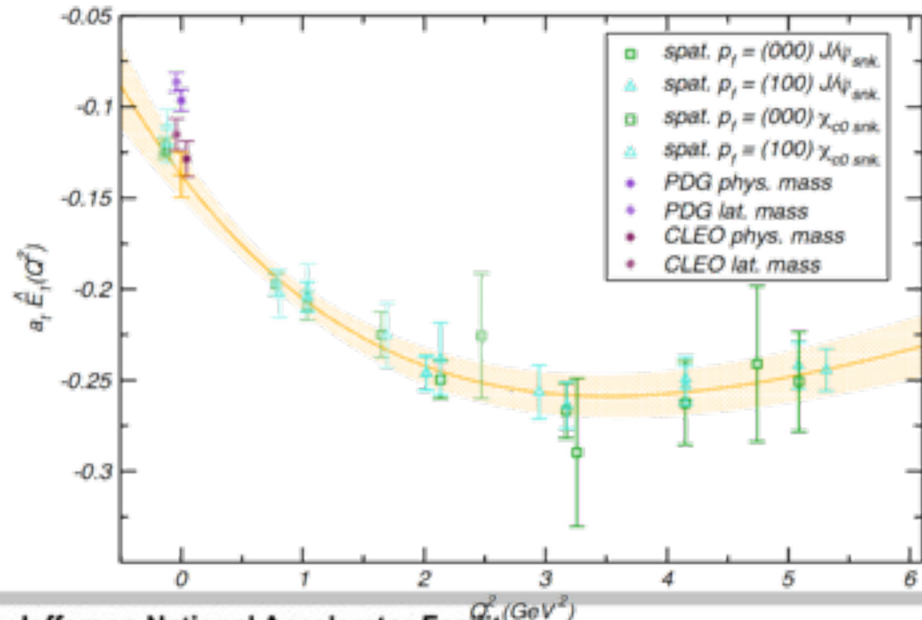


Dudek, Edwards, DGR - 2006
Chen et al (TMQCD), 2011

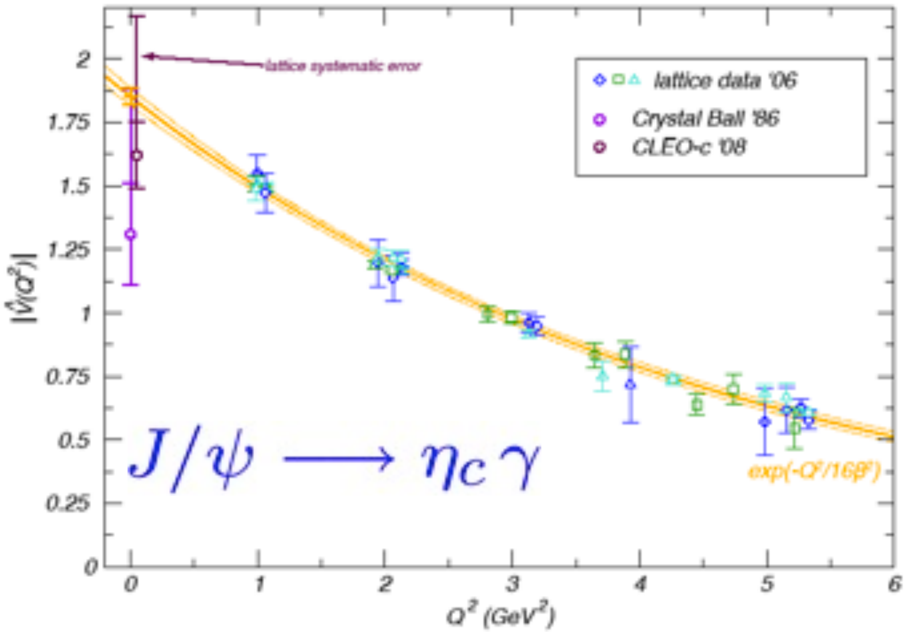
$$\Gamma(\chi_{c0} \to J/\psi\,\gamma) = \frac{1}{8\pi}\frac{|\vec{q}\,|}{m_S^2}\,2\,(2e_c)^2\,|\,E_1(0)\,|^2$$

Quenched, anisotropic Wilson-fermion action $a_t m_q < \mathcal{O}(1)$

Lattice spacing from static quark potential



Radiative Transitions in Mesons - III

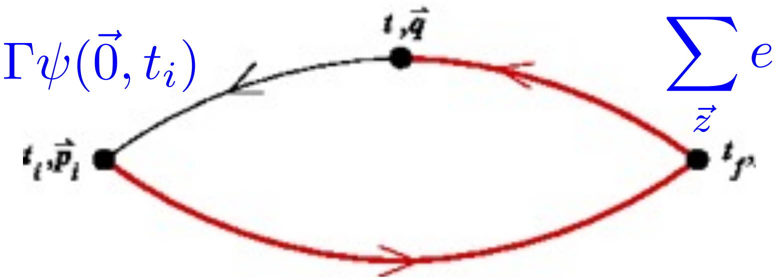


Experimental analysis by CLEO-c driven by lattice calculations

Transitions to excited states?

Dudek, Edwards, Thomas - 2009
Chen et al (TMQCD), 2011

Back to the first lecture: can we apply the variational method?



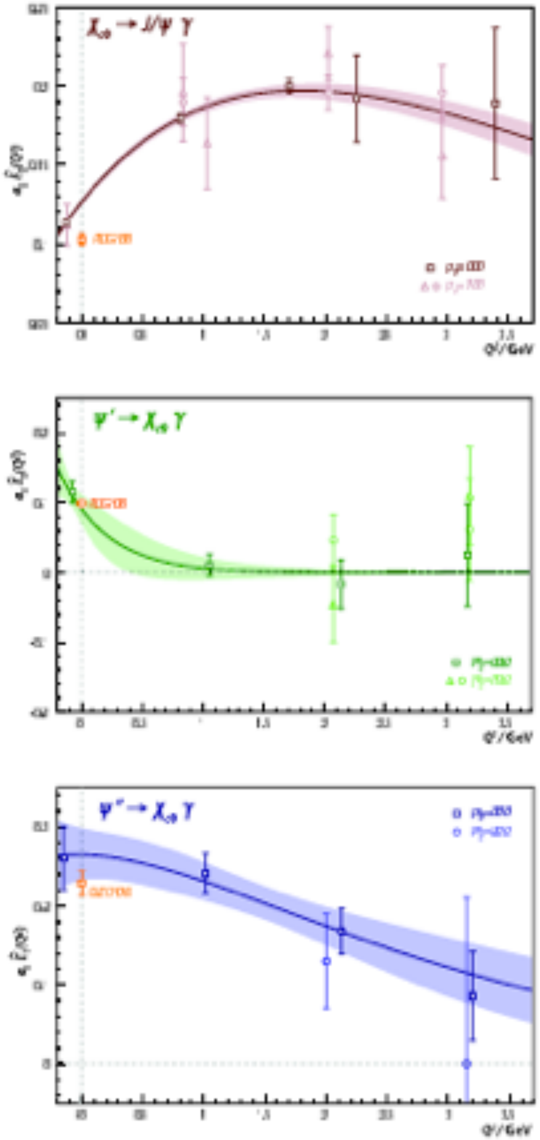
$$\Omega_j^n C_{\Gamma\mu j}(\vec{p}_i,\vec{p}_f;t_i,t,t_f) =$$

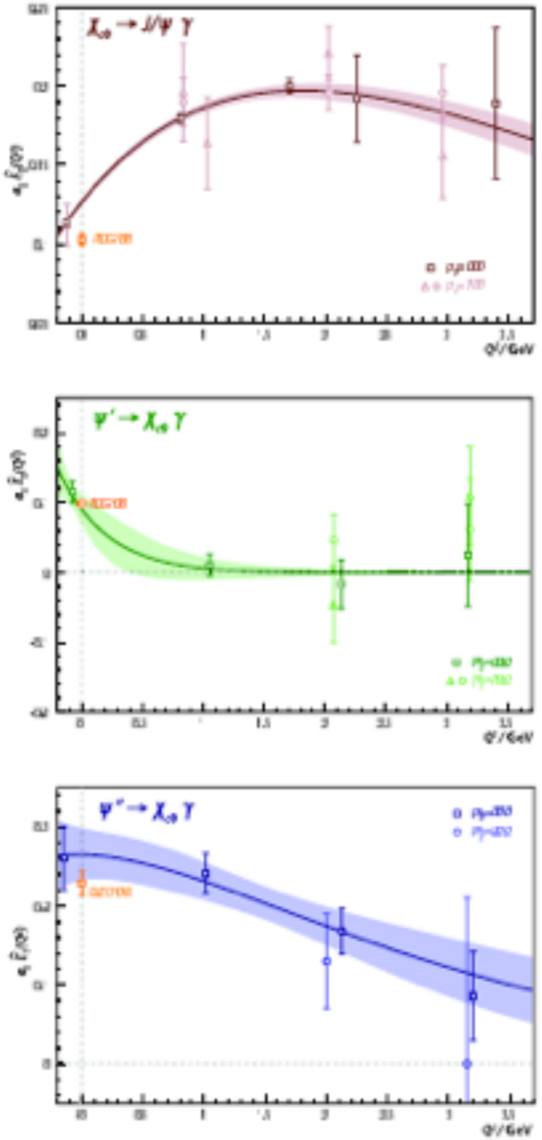
Project onto optimal operator at sink

$$\left\langle 0 \middle| \sum_{\vec{z}} e^{-i\vec{p}_f\cdot\vec{z}}\Omega_j^n\mathcal{O}_j(\vec{z},t_f)\cdot\sum_{\vec{y}} e^{i\vec{q}\cdot\vec{y}} j_\mu(\vec{y},t)\cdot\bar{\psi}(\vec{0},t_i)\Gamma\psi(\vec{0},t_i)\middle| 0\right\rangle$$

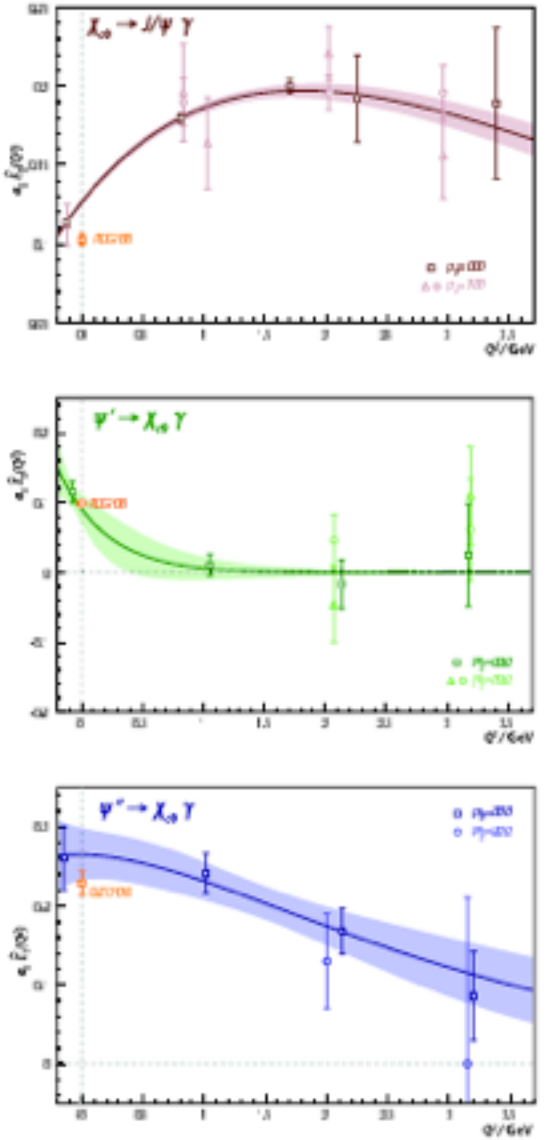
Compute radiative width - or infer photocoupling from expt.

$$\Gamma(A\to B\gamma) = \frac{1}{2J_A+1}\alpha\frac{16}{9}\frac{|\vec{q}|}{m_A^2}\sum_k\left|\hat{F}_k(0)\right|^2$$





Many of these radiative widths have been measured...



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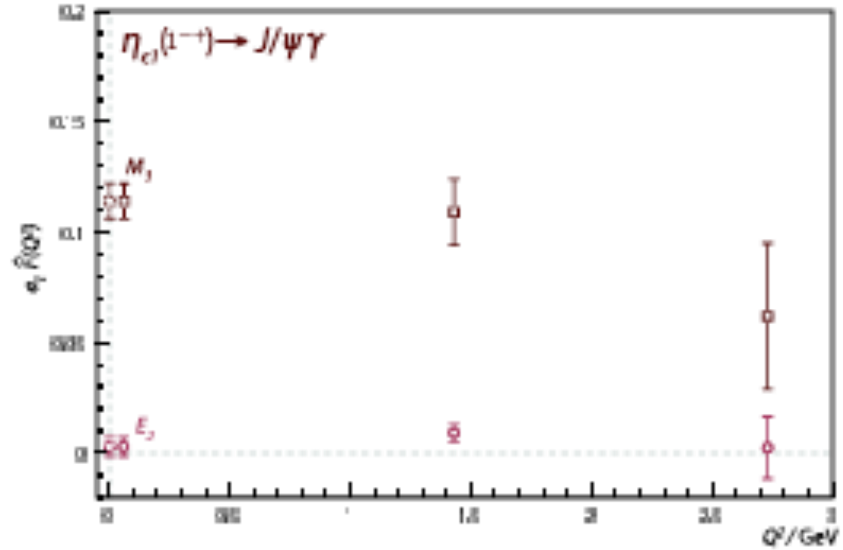

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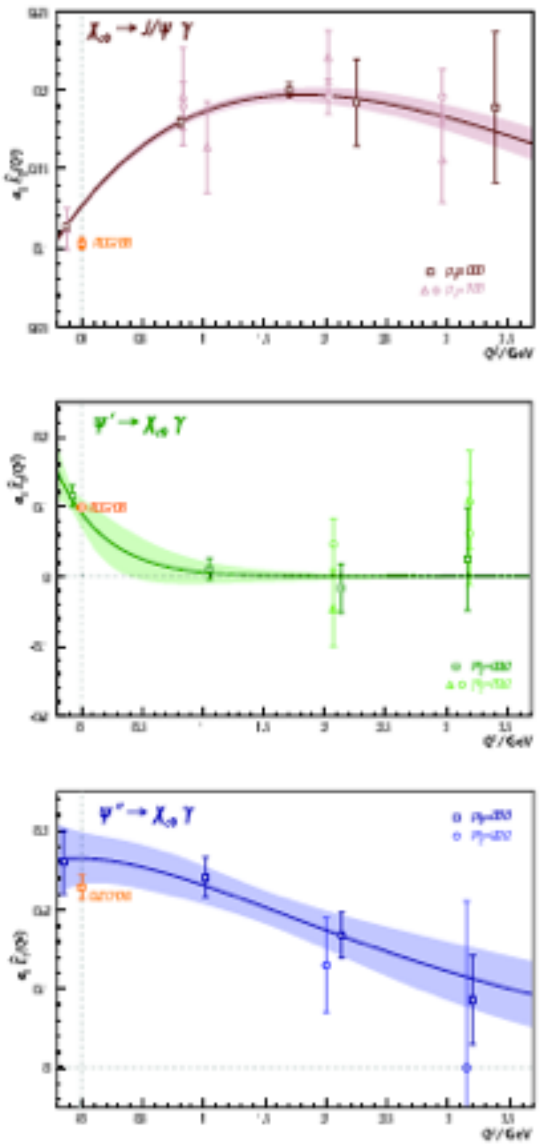
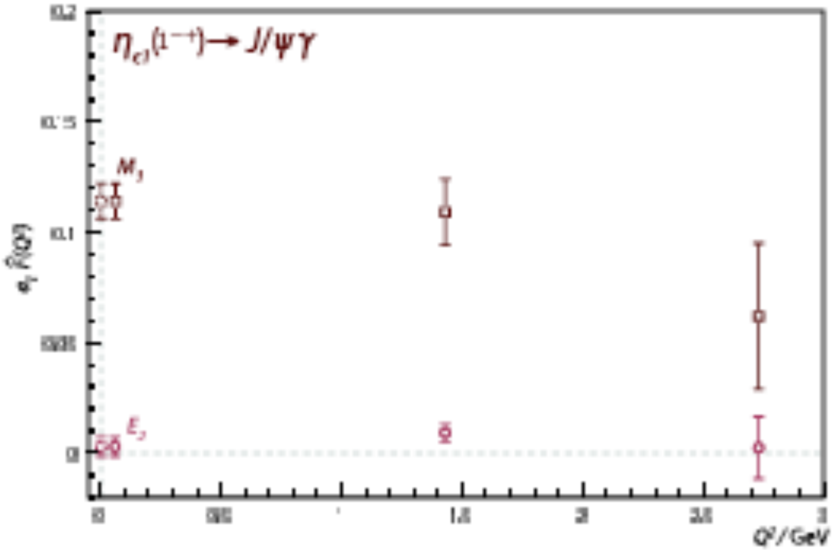




$$\Gamma(\eta_{c1} \to J/\psi\gamma) = 115(16)\ \mathrm{keV}$$

Large for M1 transition - large production of exotics at JLab if true in light-quark sector

Summary + Outlook

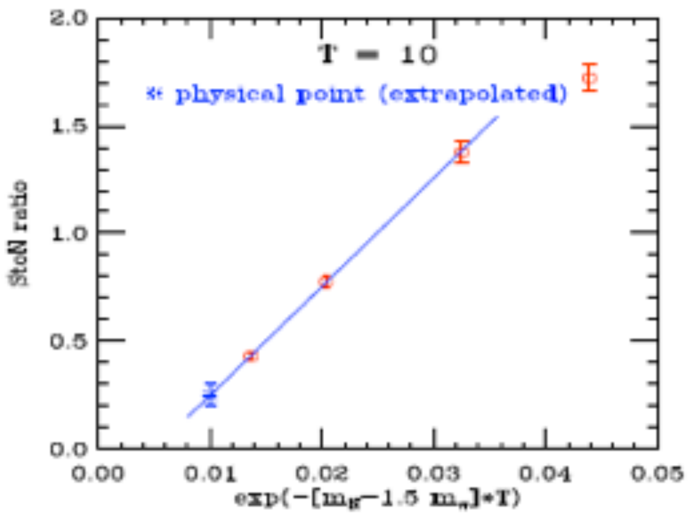
- Lattice QCD can describe describe both the spectrum of hadrons, but also their internal structure
- Spectroscopy: resonances unstable under the strong interactions - compute momentum-dependent phase shifts
- Precision hadron structure?
  - lighter quark masses, requiring large statistics
  - Control over systematic uncertainties: excited-state contributions, volume, renormalization
  - New ideas! Higher moments of PDFs. TMD's
- Lattice + expt. more powerful than either alone

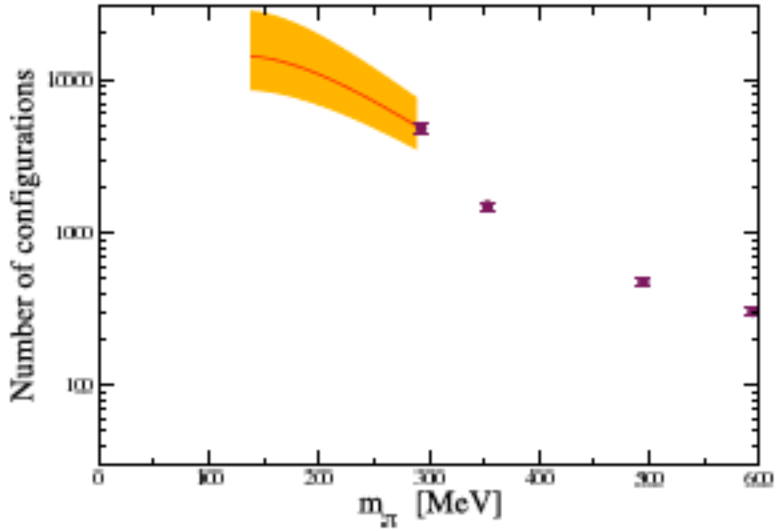
Statistics for Hadron Structure

$$\frac{\text{Signal}}{\text{Noise}} = \frac{\langle J(t)J(0)\rangle}{\frac{1}{\sqrt{N}}\sqrt{\langle |J(t)J(0)|^2\rangle - (\langle J(t)J(0)\rangle)^2}} \sim \frac{Ae^{-M_N t}}{\frac{1}{\sqrt{N}}\sqrt{Be^{-3m_\pi t} - Ce^{-2M_N t}}}$$

$$\sim \sqrt{N} D e^{-(M_N - \frac{3}{2}m_\pi)t}$$

LHPC, 2008





DWF data satisfies this expectation

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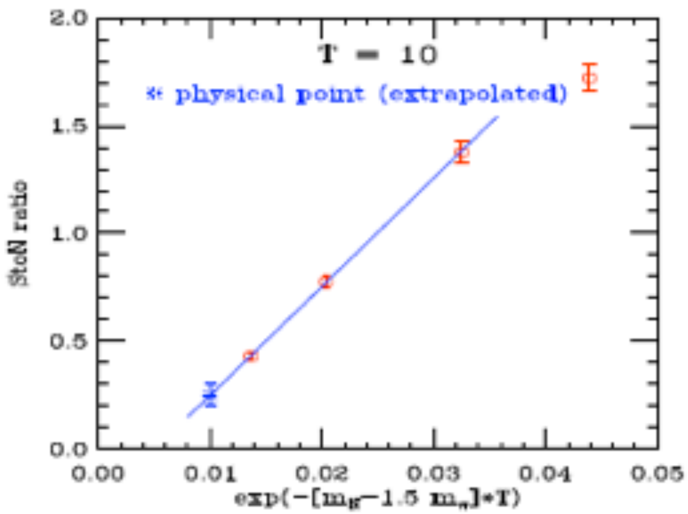

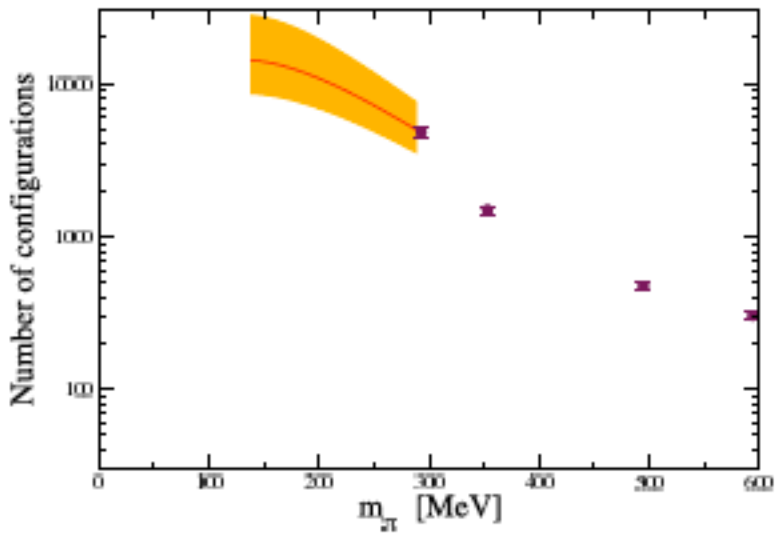

DWF data satisfies this expectation **10s of thousands configs at m_π = 140 MeV**

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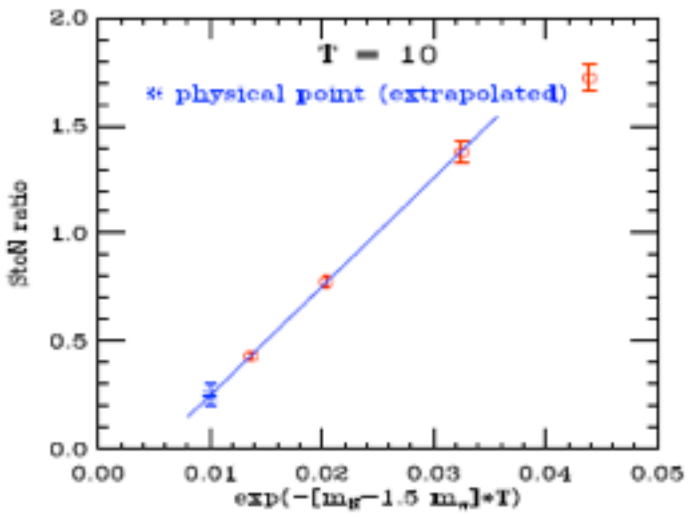

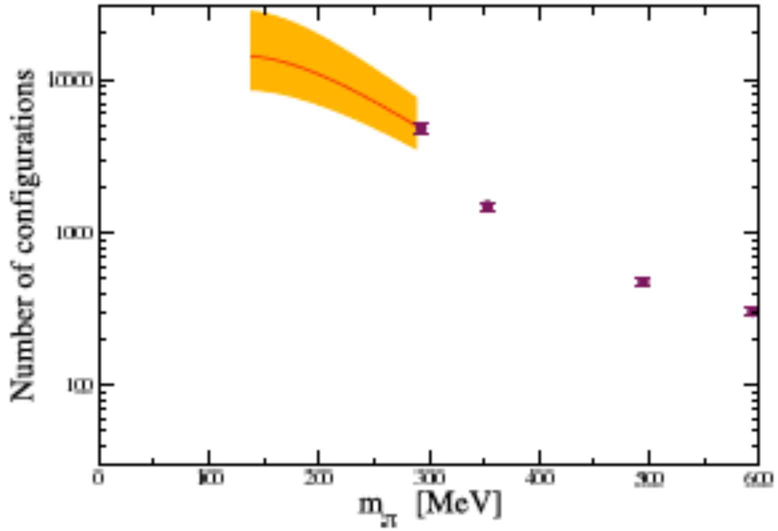

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Baryon structure much more demanding that mesons!

Lattice QCD Roadmap

Workshop on Extreme Computing, Jan. 2009