Technicolor theories off and on the lattice

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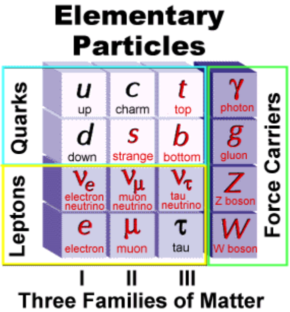
Contents:

- Need for beyond the Standard Model physics
- Why technicolor?
- Simulating technicolor-like theories on the lattice

Standard Model



The contents of the Standard Model:

- fermions: quarks, leptons
- gauge bosons: gluon (strong), $W^{\pm}$, Z (weak), photon (EM)
- Higgs

Higgs is special:

- *it is the linchpin of the standard model: provides the mechanism for the electroweak symmetry breaking*
- *it has not been seen*
- *it is a scalar*

$\Rightarrow$ *theoretical problems at very high scales:*
hierarchy problem, vacuum stability, unitarity bound ...

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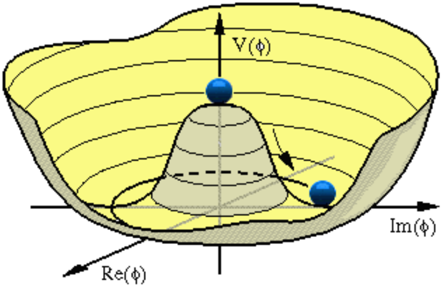
Higgs field $\mapsto$ EW symmetry breaking

- Higgs field

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \phi_i \in C$$

  transforms under weak SU(2)
- Higgs potential leads to spontaneous symmetry breaking:

$$V(\Phi) = V_0 - \mu^2|\Phi|^2 + \lambda|\Phi|^4$$



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Higgs field $\mapsto$ EW symmetry breaking

- Conventionally choose (unitary gauge choice).

$$\Phi \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v \geq 0$$

  Minimum at $v \approx 246\text{GeV}$

- Note: Higgs mass parameter μ^2 the only dimensionful parameter in the SM (at the classical level)
- The Goldstone bosons associated with the spontaneous symmetry breaking are "eaten" by the gauge bosons, which acquire a mass:

$$|D_\mu \Phi|^2 = |(\partial_\mu - igA_\mu^a T^a)\Phi|^2 \sim \frac{g^2 v^2}{4}(A_\mu^a)^2 \Rightarrow M_W \approx \frac{gv}{2}$$

- Higgs also gives masses to quarks and leptons (neutrinos?) through Yukawa couplings: if $\psi_L = (u,d)_L^T$, then

$$h_d \bar{\psi}_L \Phi d_R + h.c. \sim h_d v \bar{d} d \Rightarrow M_d \approx h_d v$$

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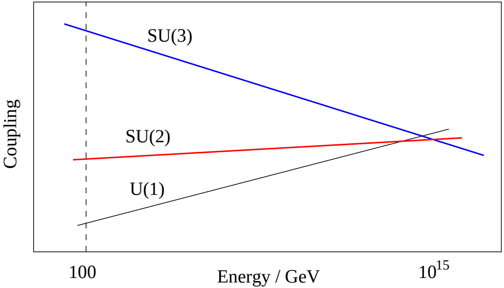
Problems with the Higgs

- The fermion and gauge sector of the Standard Model is consistent without any fine-tuning up to GUT ($\sim 10^{15}$ GeV) or Planck scale.



- Not so for the Higgs!
  - Naturalness problem (fine-tuning): why EW-scale is so much smaller than GUT (or Planck) scale?
  - Vacuum stability
  - Triviality bound
  - Flavour problem: huge range of fermion masses $\to$ Yukawa couplings.
- These problems arise only when the Standard Model is pushed to work substantially above the EW scale

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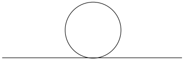
Naturalness problem:

- Loop corrections to Higgs 2-pt. function diverge as Λ^2, where Λ is the cutoff scale:



$$= \lambda \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 + m^2} \sim \lambda \Lambda^2$$

- This divergence can be cancelled by counterterm, or tuning the bare m^2 so that the divergence vanishes.
- If now $\Lambda \sim M_{\rm Planck} \sim 10^{19}$ GeV, the bare mass^2 needs to be fine-tuned to relative accuracy of $(M_{\rm EW}/M_{\rm Planck})^2 \approx 10^{-34}$
(up to $M_{\rm GUT}$, accuracy is $\sim 10^{-24}$)
- In other words, if EW theory is valid up to scale Λ, the "natural" value for the Higgs mass is Λ.
- Generic feature of scalar field theories

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Going beyond the SM

- The problems with the Standard Model are due to the scalar Higgs
- Most beyond-the-standard model theories aim to ameliorate these; e.g.
  - Supersymmetry: pair scalars with fermions so that loops cancel. No large radiative corrections, no fine tuning.
  - Large extra dimensions: cutoff near the EW scale
  - Technicolor: get rid of the scalars altogether, only fermions and gauge fields.
  - Many others
- In technicolor, Higgs is a composite, bound two fermion "meson" state, and Higgs condensate = fermion chiral condensate.

EW symmetry breaking in QCD

- The quark condensate in QCD breaks weak SU(2) gauge symmetry, even without Higgs!
- Left-handed quarks transform as weak SU(2) doublets:

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \text{whereas } u_R,\ d_R \text{ are SU(2) singlets.}$$

- Now e.g. chiral condensate $\langle \bar{d}_L d_R + h.c. \rangle$ breaks weak SU(2) like the Higgs field $\Phi \propto (0, v)^T$.
- Goldstone bosons: π-mesons. If the weak SU(2) were not already broken by Higgs, π-mesons in QCD would become longitudinal W, Z polarization states!
- The amount of symmetry breaking by QCD is small and usually neglected:

$$M_W = gF_\pi/2 \sim 30\,MeV$$

  where $F_\pi \approx 93\,\mathrm{MeV}$ is the pion decay constant, which characterises the magnitude of the chiral condensate.
- Technicolor: copy the idea to the Weak scale (100 GeV).

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Original technicolor (TC)

[Weinberg, Farhi, Susskind]

- SU(N) gauge field + n_f massless fermions, *techniquarks* Q
- Techniquarks have both technicolor and EW charge (exactly like quarks in the SM)
- Chiral symmetry breaking in technicolor $\longrightarrow$ Electroweak symmetry breaking
- Scale:
  $M_{\rm TC} \sim F_\pi^{\rm TC} \sim \Lambda_{\rm EW} \sim 246\text{GeV}$
- This gives $M_W = gF_\pi^{\rm TC}/2 \approx 80\text{GeV}$
- After chiral symmetry breaking:
  - $\Rightarrow$ scalar $\bar{Q}Q$ -meson $\leftrightarrow$ Higgs
  - $\Rightarrow$ pseudoscalars $\leftrightarrow$ W,Z -longitudinal modes
  - $\Rightarrow$ exotic technihadrons (observable!)
- Describes well the W, Z+Higgs sector (depending on the model, may have too many Goldstone bosons)
- Elegant, "proven" mechanism in the Standard Model
- Does *not* explain fermion masses (Yukawa). For that, we need additional structure $\rightarrow$ *Extended technicolor*

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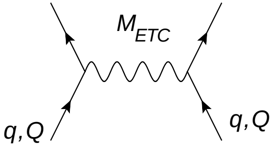
Extended technicolor (ETC)

- In addition to the "pure" technicolor, introduce a new higher-energy interaction which couples to Standard Model fermions q (quarks, leptons) and techniquarks (Q): extended technicolor (ETC)
- The most common option is massive gauge boson, with mass $M_{\rm ETC} > M_{\rm TC}$:



[Eichten,Lane,Holdom,Appelquist,Sannino,Luty. . .]

- At energies smaller than $M_{\rm ETC}$ this gives effective 4-fermion couplings, e.g.

$$\frac{g_{\rm ETC}^2 \bar{Q} Q \bar{q} q}{M_{\rm ETC}^2}$$

When techniquarks form a condensate, this becomes a fermion mass term with

$$m_q = \frac{g_{\rm ETC}^2}{M_{\rm ETC}^2} \langle \bar{Q} Q \rangle_{\rm ETC}$$

- Here $\langle \bar{Q} Q \rangle_{\rm ETC}$ means the techniquark condensate evaluated at the ETC scale.

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Extended technicolor (ETC)

Other types of 4-fermi terms:

- $$\frac{1}{M_{\mathrm{ETC}}^2}\bar{q}q\bar{q}q$$

  This is a non-Standard Model quark-quark coupling, giving rise to extra flavour changing neutral currents (FCNC's).
  Experimental constraints ($K_L K_S$ mass difference) yield a lower bound on the ETC scale of $M_{\mathrm{ETC}} \gtrsim 10^3 M_{\mathrm{TC}}$

- $$\frac{1}{M_{\mathrm{ETC}}^2}\bar{Q}Q\bar{Q}Q$$

  gives explicit χSB in the techniquark sector.

$\langle\bar{Q}Q\rangle_{\mathrm{ETC}}$: condensate evaluated at the ETC scale
$\langle\bar{Q}Q\rangle_{\mathrm{TC}}$: condensate at TC (EW) scale

Extended technicolor

Constraints:

I) FCNC's: $M_{\rm ETC} \gtrsim 1000 - 10000 \times M_{\rm TC} (M_{\rm TC} \approx \Lambda_{EW})$

II) For EWSB we must have $\langle \bar{Q}Q \rangle_{\rm TC} \propto M_{\rm TC}^3 \approx M_{\rm EW}^3$

III) On the other hand, $\langle \bar{Q}Q \rangle_{\rm ETC} \propto m_q M_{\rm ETC}^2$ (top quark!)

- Thus, $\langle \bar{Q}Q \rangle_{\rm ETC} \approx m_q \dfrac{M_{\rm ETC}^2}{M_{\rm TC}^3} \langle \bar{Q}Q \rangle_{\rm TC}$
- Using RG evolution

$$\langle \bar{Q}Q \rangle_{\rm ETC} = \langle \bar{Q}Q \rangle_{\rm TC} \exp \left[\int_{M_{\rm TC}}^{M_{\rm ETC}} \frac{\gamma(g^2)}{\mu} d\mu \right]$$

  where $\gamma(g^2)$ is the mass anomalous dimension.
- In weakly coupled theory $\gamma \propto g^2$ is small, and $\langle \bar{Q}Q \rangle$ is $\sim$ constant.
- In QCD-like theory g^2 is large only in a narrow energy range
- *Thus, it is not possible to satisfy the constraints I), II), II) in a QCD-like theory.*

Walking coupling

- On the other hand, if the coupling is approximately constant, $g^2 \approx g_*^2$, over the range from TC to ETC, then we can solve

$$\langle \bar{Q}Q \rangle_{\rm ETC} \approx \left(\frac{M_{\rm ETC}}{\Lambda_{\rm TC}} \right)^{\gamma(g_*^2)} \langle \bar{Q}Q \rangle_{\rm TC}$$

  (condensate enhancement)
- Inserting II) and III) we obtain the requirement

$$\gamma(g_*^2) \approx 2$$

- This limit can be relaxed somewhat in a more detailed computation.
- A coupling which approximately stops its evolution for some interval in energy is called a walking coupling.

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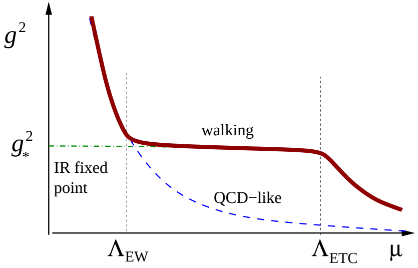
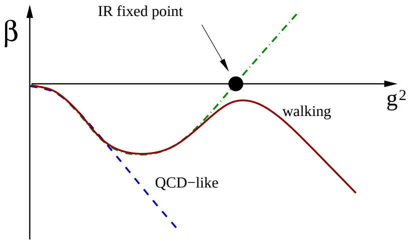
Walking coupling and IR fixed point





- In a walking theory the β-function

$$\beta = \mu \frac{dg}{d\mu}$$

  reaches almost zero near g_*^2. (The shape is scheme-dependent, however.)
- If the β-function hits zero there is an infrared (IR) fixed point, where the system becomes *conformal* (scale invariant).
- At the conformal point the system cannot have any mass scales. All correlation functions are powerlike.

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Perturbative β-function

2-loop universal β-function for SU(N_c) gauge theory with N_f fermions:

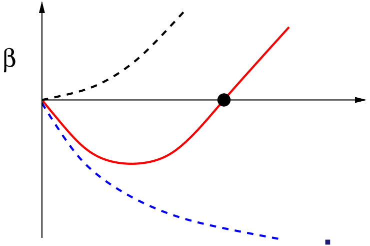
$$\beta(g) = -\mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

where the coefficients are

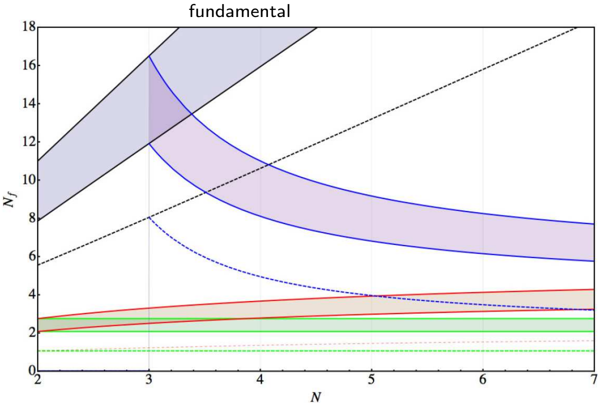
$$\beta_0 = \frac{11}{3} C_r - \frac{4}{3} T_r N_f, \qquad \beta_1 = \frac{34}{3} C_r^2 - \frac{20}{3} C_r T_r N_f - 4 C_r T_r N_f$$

When N_f is varied, generically 3 different behaviours seen:

- confinement and χSB at small N_f
- IR fixed point (conformal window) at medium N_f [Banks,Zaks]
- Asymptotic freedom lost at large N_f



Conformal window in SU(N) gauge



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

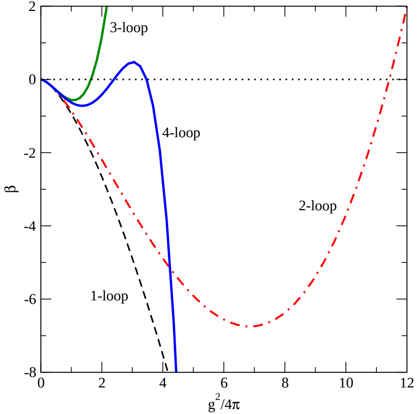
2-index symmetric

adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- In higher reps it is easier to satisfy EW constraints [Sannino,Tuominen,Dietrich] → lot of recent activity!

Existence of the IRFP essentially non-perturbative

Example: Perturbative β-function of SU(2) gauge with $N_f = 6$ fundamental rep fermions



[4-loop MS: Ritbergen, Vermaseren, Larin]

Results from lattice: existence of IRFP inconclusive, maybe at $g_*^2/4\pi \sim 1.2$ ($g^2 \sim 15$)

[Karavirta et al, to be published]

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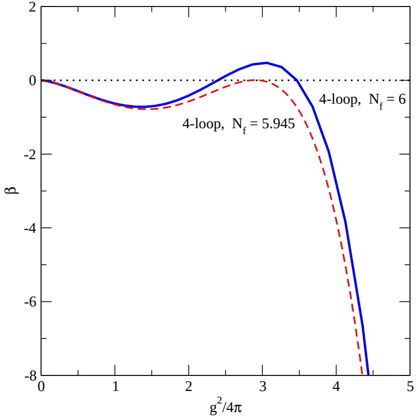
Appearance of the IRFP

IRFP can come from $g^2 = \infty$ or from below:



[4-loop MS: Ritbergen, Vermaseren, Larin]

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What do we want?

Take SU(N) gauge theory with N_f fermions in some representation.

- Measure $\beta(g^2)$-function
- Measure $\gamma(g^2)$
- Classify QCD-like / walking / conformal
- We want to find a theory which
  - is walking or
  - is just within conformal window (easy to deform into walking)
  - has large anomalous exponent γ near FP
  - Compatible with EW precision measurements (S,T,U -parameters) $\rightarrow$ small N_f preferred!
- Favourite candidates: SU(2) or SU(3) gauge theory with $N_f = 2$ adjoint or 2-index symmetric representation fermions. [Sannino,Tuominen,Dietrich]
- "Hadron" spectrum, chiral symmetry breaking pattern

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Models studied on the lattice

- SU(3) + $N_f = 8$–16 fundamental rep:
  - $N_f = 8$: χSB [Appelquist et al; Deuzeman et al; Fodor et al; Jin et al]
  - $N_f = 9$: χSB [Fodor et al]
  - $N_f = 10$: unclear [Yamada et al]
  - $N_f = 12$: conflicting results [Hasenfratz; Fodor et al; Appelquist et al; Deuzeman et al]
  - $N_f = 16$: conformal [Damgaard et al; Heller; Hasenfratz; Fodor et al]
- SU(2) + fundamental rep fermions:
  - $N_f = 2$: χSB [many]
  - $N_f = 4$: χSB [Karavirta et al (to be published)]
  - $N_f = 6$: unclear [Del Debbio et al, Karavirta et al (to be published)]
  - $N_f = 8$: conformal [Iwasaki et al]
  - $N_f = 10$: conformal [Karavirta et al (to be published)]
- SU(2) + $N_f = 2$ adjoint rep: (*Minimal walking technicolor*) conformal [Catterall et al; Bursa et al; Hietanen et al; De Grand et al]
- SU(3) + $N_f = 2$ 2-index symmetric rep: unclear [de Grand et al; Sinclair and Kogut; Fodor et al]

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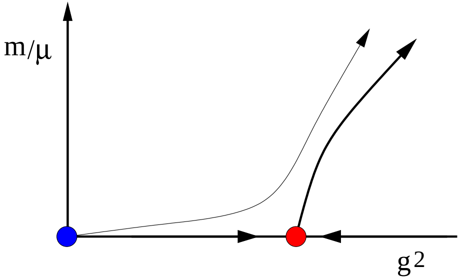
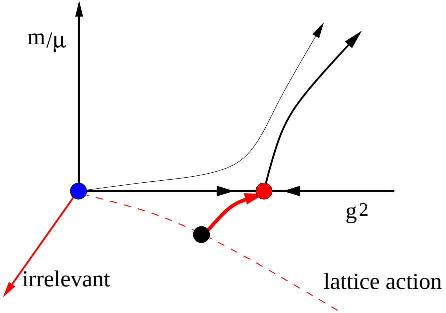
RG flow in the conformal case

- Relevant parameters at UV: g^2 and m_Q



- Only m_Q is relevant at the IRFP.
- Scaling near IRFP: masses of physical particles $M \propto (m_Q)^{1/(1+\gamma)}$

RG flow on the lattice



- Irrelevant operators (cutooff effects) die out as a/L, $(a/L)^2 \ldots (L$: IR scale)
- Evolution of g^2 along the physical axis very slow

$\Rightarrow$ irrelevant operators can (and do!) mask the physical evolution

- Need either:
  - Very large lattices (large L/a) – impractical
  - Very high quality lattice action – small cutoff effects

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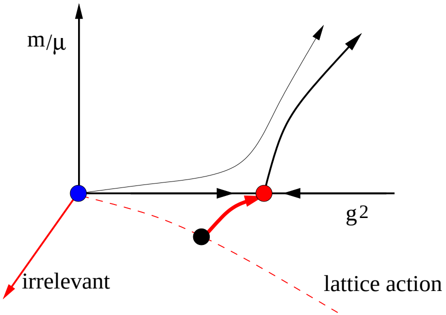
RG flow on the lattice



- Near the IRFP, the continuum limit cannot be taken at weak bare coupling:
- Even when the lattice spacing $a \to a/100$, the bare coupling barely decreases
- Present experience: we really need highly improved lattice actions which work at strong coupling
  - Non-perturbative (thin-link) clover – not sufficient?
  - nHYP smeared clover
  - perfect gauge? perfect fermions?
  - do staggered quarks work?

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Case study:

Minimal walking technicolor

Case study: Minimal walking technicolor (MWTC)

- SU(2) with $N_f = 2$ *adjoint* representation techniquarks
- Study on the lattice using (unimproved) Wilson fermions
  [Catterall, Sannino; Del Debbio, Patella, Pica; Hietanen et al, Bursa et al]
- What is studied?
  - Measure the evolution of the coupling directly using the Schrödinger functional method
  - Particle spectrum: do we observe chiral symmetry breaking (QCD) or do all modes become massless as $m_q \to 0$ (no χSB, possibly conformal)
  - Mass anomalous exponent γ (from spectrum or directly using SF methods)
  - Improvement of the lattice action

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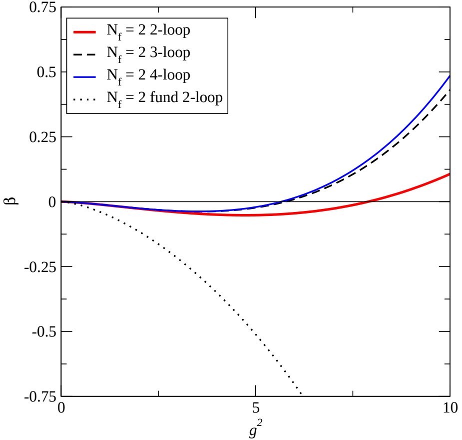
Minimal technicolor

- Perturbative β-function compared with fundamental rep: very slow evolution!



Lattice model:

- SU(2) gauge action in fundamental rep.
- massless fermions in adjoint rep.

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} i \gamma_\mu D_\mu \psi$$

- On the lattice:
  - gauge fields U in the fundamental rep.
  - For the fermion action, these transformed into adjoint rep

    $$V^{ab} = 2\,\mathrm{Tr}[U^\dagger \lambda^a U \lambda^b]$$

    $a, b = 1, 2, 3.$
  - We use standard Wilson action (these results); now non-perturbatively O(a) improved Wilson-clover action (future results)
  - For comparison, we also do analysis with $N_f = 2$ fundamental quarks

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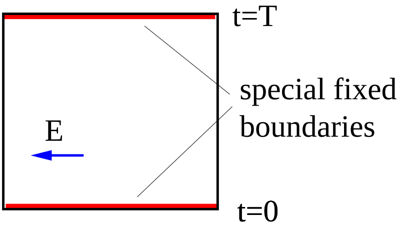
Evolution of the coupling

Schrödinger functional: Generate a *background* chromoelectric field using non-trivial fixed boundary conditions, parametrised by a twist angle η

At the classical level, we have

$$\frac{dS_{\text{class.}}}{d\eta} = \frac{A}{g^2}$$



where $A(\eta)$ is a known constant.

At the quantum level, we define the coupling through

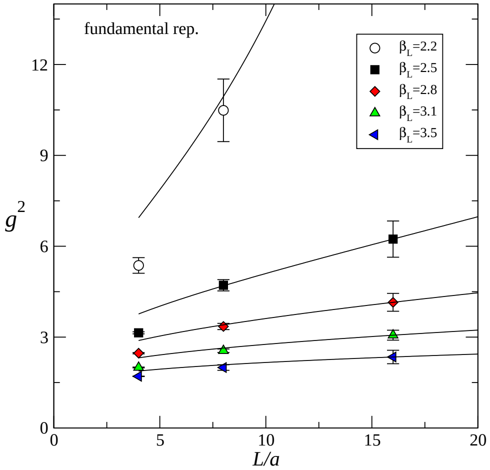
$$\begin{aligned} \frac{1}{g^2} &= \frac{1}{A}\frac{dS}{d\eta} \\ &= \text{const.} \times \langle(\text{boundary plaq.})\rangle \end{aligned}$$

- Evaluates g^2 directly at scale $\mu = 1/L$, the lattice size
- Can use $m_Q = 0$
- Has been used very succesfully in QCD by the Alpha collaboration

Evolution of the coupling: QCD-like

Test with $N_f = 2$ **fundamental representation**, QCD-like test case:

- L/a grows, $k \sim a/L$ decreases, $g^2(L)$ increases: *asymptotic freedom*, OK!
- Large $\beta_L \rightarrow$ small lattice spacing $\rightarrow$ small volume
- Continuous line: coupling evaluated from the 2-loop β-function (integration constant fixed to measurement at $L/a = 16$)
- Not a continuum limit, but shows consistency



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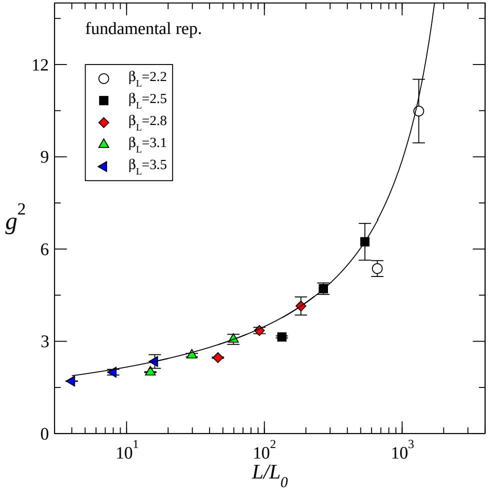
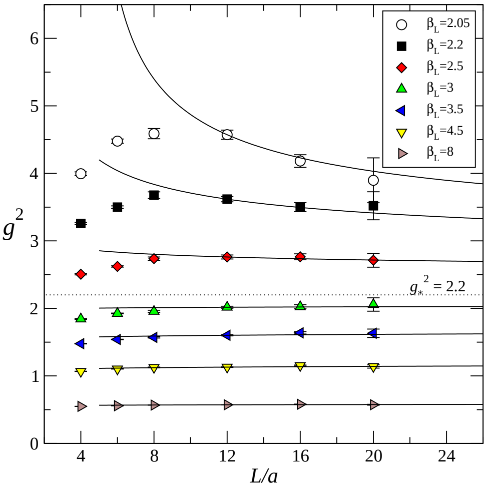


Evolution of the coupling: MWTC

In **adjoint representation**:

- At small $g^2(L)$: increases with L (asymptotic freedom)
- At large $g^2(L)$: *decreases* as L increases
  $\Rightarrow$ β-function positive here!
- Large cutoff effects at small L/a – discard
- As $L/a \to \infty$, apparently $g^2(L) \to g_*^2 \approx 2 \ldots 3$.
  $\Rightarrow$ conformal behaviour!?
- Continuous line: coupling evaluated with fitted β-function ansatz (to be described)

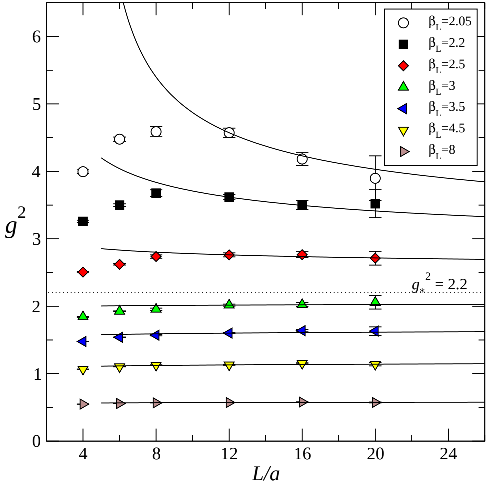


β-function

Assuming that the lattice effects on the large-volume data are small, we can describe the features of the β-function by fitting an ansatz:

$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

Here b_1, b_2 are perturbative constants and b_3 and δ are fit parameters. (Parametrising the location of the fixed point and the slope of the β-function there).
The ansatz is fitted to the data at $L/a = 12, 16, 20$:



β-function

Fit result:

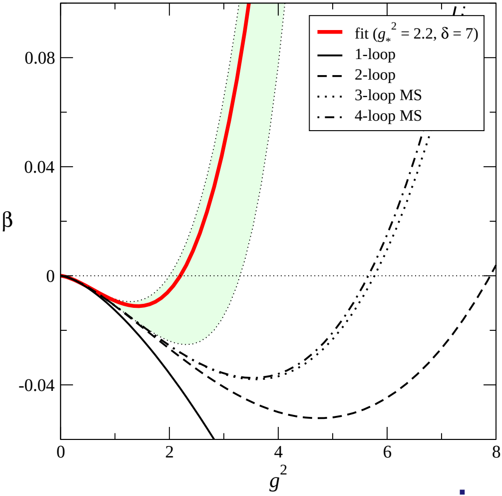
$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

FP is at substantially smaller coupling than indicated by 2-loop P.T.

Recent results w. nHYP-improved fermions indicate IRFP at $g^2 \sim 3 - -5$. [De Grand et al.]

In MS-schema, β-function is known to 4-loop order: [Ritbergen, Vermaseren, Larin]

Not directly comparable to lattice (beyond 2 loops), because of different schema! But quantifies perturbative uncertainty.



Particle spectrum:

- If QCD-like χSB: as $m_Q a \to 0$,
  - $m_\pi \propto m_Q^{1/2}$
  - other states have finite mass.
- If IR fixed conformal point: when $m_Q a \to 0$, all states become massless with the same exponent.
- If walking behaviour: at high energy $\sim$ conformal, at small χSB.

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Minimal walking technicolor

O(a) improvement

$O(a)$ improvement of the action

- Wilson fermions have large $O(a)$ cutoff-effects. These are cancelled by adding a irrelevant "clover term" with a fine-tuned coefficient $c_{\rm SW}$.
- In the Schrödinger functional scheme also boundary term improvement must be computed

Schrödinger functional scheme action

$$S_i = S_u + \delta S_V + \delta S_{G,b} + \delta S_{F,b}$$

$$\begin{aligned}
\delta S_V &= \frac{ia^5}{4} c_{sw} \sum_{x_0=a}^{L-a} \sum_{\vec{x}} \bar{\psi}(x) \sigma_{\mu\nu} \hat{F}_{\mu\nu}(x) \psi(x) \\
\delta S_{G,b} &= \frac{1}{2g_0^2}(c_s - 1) \sum_{p_s} \mathrm{Tr}[1 - U(p_s)] \\
&\quad + \frac{1}{g_0^2}(c_t - 1) \sum_{p_t} \mathrm{Tr}[1 - U(p_t)] \\
\delta S_{F,b} &= a^4(\tilde{c}_s - 1) \sum_{\vec{x}} [\hat{O}_s(\vec{x}) + \hat{O}'_s(\vec{x})] \\
&\quad + a^4(\tilde{c}_t - 1) \sum_{\vec{x}} [\hat{O}_t(\vec{x}) - \hat{O}'_t(\vec{x})]
\end{aligned}$$

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Boundary terms

- The clover coefficient $c_{\rm SW}$ is determined non-perturbatively
- The boundary coefficients c_t, $\tilde{c}_t$ perturbatively
- c_s, $\tilde{c}_s$ are not needed

We obtain [Karavirta et al, for fundamental rep Lüscher, Weisz]

$$\tilde{c}_t = 1 - 0.0135(1) \times C_R g_0^2 + O(g_0^4)$$

Write $c_t = 1 + g_0^2 (c_t^{(1,0)} + N_F * c_t^{(1,1)}) + O(g_0^4)$

N_c	rep.	$c_t^{(1,0)}$	$c_t^{(1,1)}$
2	**2**	$-0.0543(5)$	0.0192(2)
2	**3**	$-0.0543(5)$	0.075(1)
3	**3**	$-0.08900(5)$	0.0192(4)
3	**8**	$-0.08900(5)$	0.113(1)
3	**6**	$-0.08900(5)$	0.0946(9)
4	**4**		0.0192(5)

[Sint et al, Karavirta et al]

These are in agreement with $c_t^{(1,1)} = 0.019141 \times (2T_R)$

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Boundary conditions for the clover coefficient

- Matche $c_{\rm SW}$ using Schrödinger functional method to generate a background chromoelectric field and "optimizing" the fermion mass defined through axial Ward identity:

$$M(x_0) = \frac{1}{2}\frac{\frac{1}{2}(\partial_0^* + \partial_0)f_A(x_0) + c_A a \partial_0^* \partial_0 f_P(x_0)}{f_P(x_0)}$$

- However: the standard diagonal ("Abelian") boundary matrices are not quite sufficient for higher reps:

$$U = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \Rightarrow V^{ab} = 2\,\mathrm{Tr}[U^\dagger \lambda^a U \lambda^b] \Rightarrow V = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- For adjoint fermion, there is a color component which does not see the background field: problem at long distances
- We maximise the asymmetry between the boundaries using the following "non-Abelian" boundary conditions:

$$U_i(t=0) = 1, \quad U_i(t=T) = \exp[i\theta\sigma_i]$$

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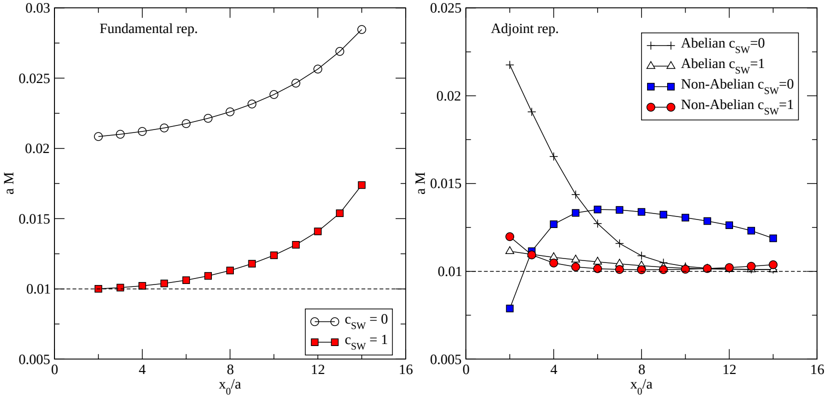
Boundary conditions: demonstrate at the classical level

$8^3 \times 16$ lattice, $m_0 a = 0.01$: Axial Ward identity against a classical background in SU(2) fundamental and adjoint rep:



With “Abelian” boundary conditions, no lever-arm to determine the value of $c_{\rm SW}$.