

Energy-momentum tensor correlators in hot Yang-Mills theory

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Mikko Laine, Mikko Vepsäläinen, AV, 1008.3263, 1011.4439

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York Schröder, Mikko Vepsäläinen, AV, Yan Zhu, 1109.6548

Yan Zhu, AV, 1210.nnnn

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- Understanding the properties of QGP
- Perturbative input

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- The setup
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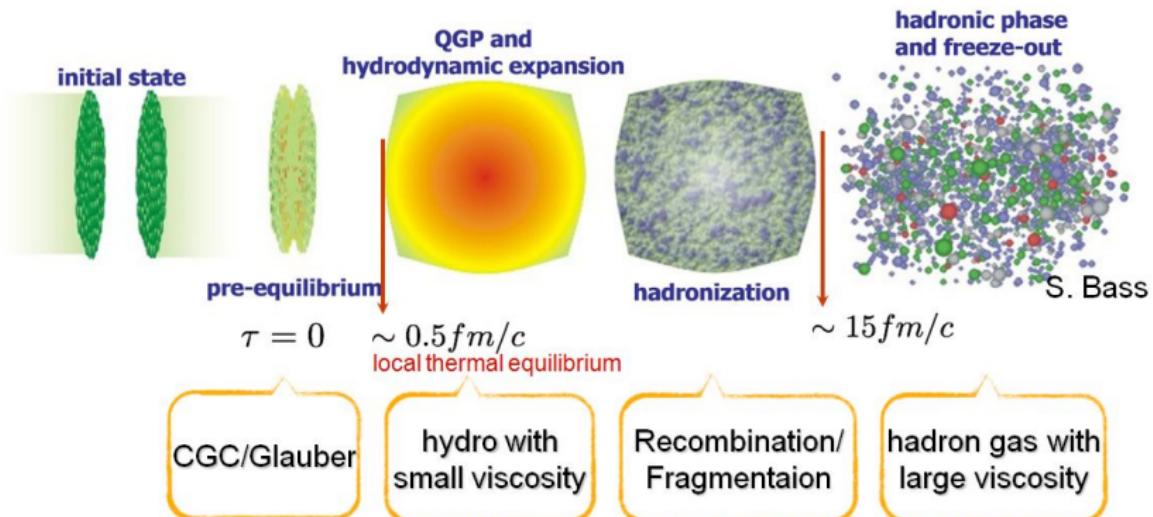
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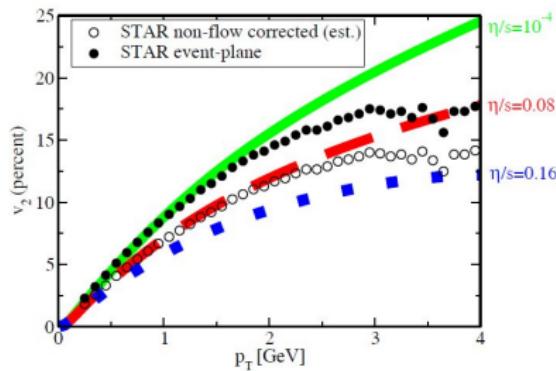
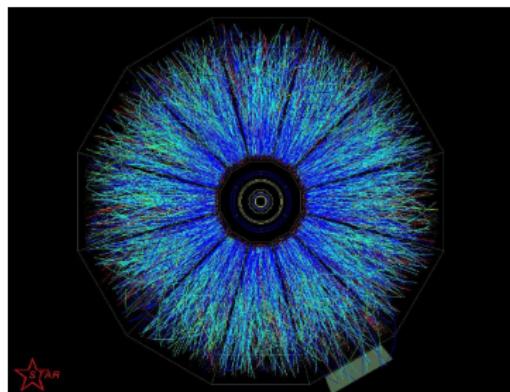
Background: Heavy ion collisions



Expansion of thermalizing plasma surprisingly well described in terms of a low energy effective theory — hydrodynamics

- UV physics encoded in **transport coefficients**: η, ζ, \dots

Expansion and thermalization of the fireball



RHIC observation: Hydrodynamics requires $\eta/s \lesssim 0.2$

- AdS/CFT: $\eta/s = \frac{1}{4\pi}$ in ‘two-derivative’ models

Obvious questions: What are η, ζ, \dots in QCD? Is the plasma ‘strongly coupled’? Is $\mathcal{N} = 4$ SYM really a good model for QGP?

Ultimate answer only from **non-perturbative** calculations in **QCD**

Motivation I: Transport coefficients in hot QCD

Kubo formulas: Transport coefficients obtainable from IR limit of **retarded Minkowski correlators** — viscosities from those of the energy momentum tensor $T_{\mu\nu}$:

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im } D_{12,12}^R(\omega, \mathbf{k} = 0) \equiv \lim_{\omega \rightarrow 0} \frac{\rho_{12,12}(\omega, \mathbf{k} = 0)}{\omega}$$

Problem: Lattice can only measure **Euclidean correlators**: Spectral density available only through inversion of

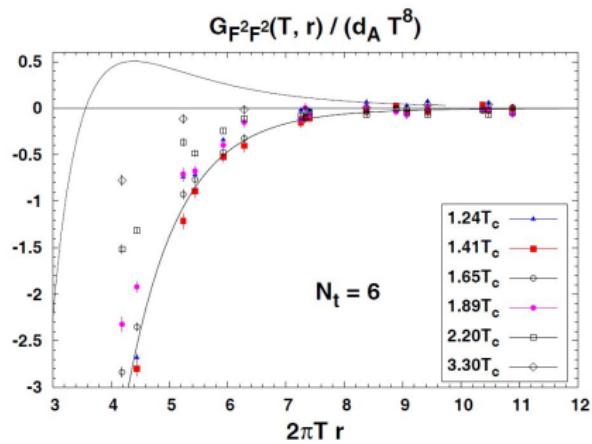
$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh \frac{(\beta - 2\tau)\omega}{2}}{\sinh \frac{\beta\omega}{2}}$$

\therefore To extract the IR limit of ρ , need to understand its behavior also at $\omega \gtrsim \pi T$ — very nontrivial challenge for lattice QCD, requiring perturbative input (cf. talk of Mikko Laine)

Motivation II: Correlators as measure of interaction

Spatial correlators measure screening in medium \Rightarrow Comparison between lattice QCD, pQCD and AdS/CFT results offers insights into structure and properties of the QGP

Iqbal, Meyer (0909.0582): Lattice data for correlators of $\text{Tr } F_{\mu\nu}^2$ in semi-quantitative agreement with strongly coupled $\mathcal{N} = 4$ SYM, while leading order pQCD result completely off. How about NLO?



Challenge for perturbation theory

Goal: Perturbatively evaluate Euclidean and Minkowskian correlators of $T_{\mu\nu}$ in hot Yang-Mills theory to

- ① Inspect Operator Product Expansions (OPEs) at finite temperature
- ② Use the spectral densities to dig up transport coefficients from Euclidean lattice data
- ③ Compare behavior of spatial correlators to lattice QCD and AdS/CFT to investigate the nature of the QGP

Challenge for perturbation theory

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Concretely: Specialize to scalar, pseudoscalar and shear operators

$$\theta \equiv c_\theta g_B^2 F_{\mu\nu}^a F_{\mu\nu}^a, \quad \chi \equiv c_\chi g_B^2 F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a, \quad \eta \equiv 2c_\eta T_{12} = -2c_\eta F_{1\mu}^a F_{2\mu}^a$$

and proceed from 1 to 3 working at **NLO**.

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Setting up the calculation

The plan: Work within finite- T SU(3) Yang-Mills theory

$$S_E = \int_0^\beta d\tau \int d^{3-2\epsilon}x \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \right\},$$

write down diagrammatic expansions for **Euclidean** correlators

$$G_\theta(x) \equiv \langle \theta(x)\theta(0) \rangle_c, \quad G_\chi(x) \equiv \langle \chi(x)\chi(0) \rangle, \quad G_\eta(x) \equiv \langle \eta(x)\eta(0) \rangle_c,$$

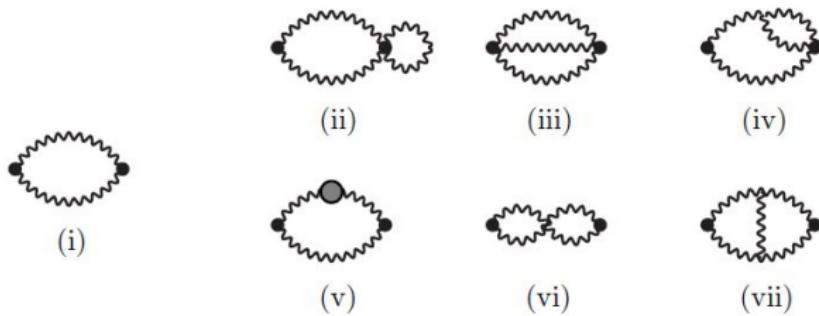
$$\tilde{G}_\alpha(P) \equiv \int_x e^{-iP \cdot x} G_\alpha(x),$$

$$\rho_\alpha(\omega) \equiv \text{Im } \tilde{G}_\alpha(p_0 = -i(\omega + i\epsilon), \mathbf{p} = 0),$$

and evaluate the necessary integrals.

Setting up the calculation

End up computing two-loop two-point diagrams in dimensional regularization, the black dots representing the operators:



NB: At $T \neq 0$, $4d$ integrals replaced by $3+1d$ **sum-integrals**

$$\int_Q \rightarrow \oint_Q \equiv T \sum_{q_0=2\pi nT} \int_q$$

Computational methods I: Identifying the masters

Step 1: Perform Wick contractions and perform Lorentz algebra
(typically with FORM)

Result: Expansion in terms of scalar ‘masters’

$$\frac{\tilde{G}_\theta(P)}{4d_A c_\theta^2 g_B^4} = (D-2) \left[-\mathcal{I}_a + \frac{1}{2} \mathcal{I}_b \right]$$

$$+ g_B^2 N_c \left\{ 2(D-2) \left[-(D-1)\mathcal{I}_a + (D-4)\mathcal{I}_b \right] + (D-2)^2 \left[\mathcal{I}_c - \mathcal{I}_d \right] \right. \\ \left. + \frac{22-7D}{3} \mathcal{I}_f - \frac{(D-4)^2}{2} \mathcal{I}_g + (D-2) \left[-3\mathcal{I}_e + 3\mathcal{I}_h + 2\mathcal{I}_i - \mathcal{I}_j \right] \right\},$$

$$\mathcal{I}_a \equiv \oint_Q \frac{P^2}{Q^2}, \quad \mathcal{I}_b \equiv \oint_Q \frac{P^4}{Q^2(Q-P)^2}, \quad \mathcal{I}_a \equiv \oint_{Q,R} \frac{1}{Q^2 R^2}, \quad \mathcal{I}_b \equiv \oint_{Q,R} \frac{P^2}{Q^2 R^2(R-P)^2}, \quad \dots$$

$$\mathcal{I}_h \equiv \oint_{Q,R} \frac{P^4}{Q^2 R^2(Q-R)^2(R-P)^2}, \quad \mathcal{I}_i \equiv \oint_{Q,R} \frac{(Q-P)^4}{Q^2 R^2(Q-R)^2(R-P)^2},$$

$$\mathcal{I}_i \equiv \oint_{Q,R} \frac{4(Q \cdot P)^2}{Q^2 R^2(Q-R)^2(R-P)^2}, \quad \mathcal{I}_j \equiv \oint_{Q,R} \frac{P^6}{Q^2 R^2(Q-R)^2(Q-P)^2(R-P)^2}$$

Computational methods II: Evaluating the masters

The optimal method for dealing with the master integrals depends on the particular problem:

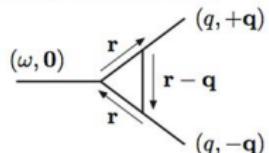
- OPEs (cf. $T = 0$ case in the talk of Max Zoller)
 - Explicit evaluation of Matsubara sums via ‘cutting’ methods
 - Expansion of the $3d$ integrals in powers of T^2/P^2
- Spectral functions: $\rho(\omega) \equiv \text{Im } \tilde{G}(p_0 = -i(\omega + i\epsilon), \mathbf{p} = 0)$
 - Matsubara sum again via cutting rules, then take explicitly the imaginary part $\Rightarrow \delta$ -function constraints for the 3-momenta
 - Most complicated part of the calculation: Dealing with the remaining spatial momentum integrals
- Time-averaged spatial correlators
 - Set $p_0 = 0$ and carry out $3d$ Fourier transform
 - Calculation most conveniently handled directly in coordinate space, where the Matsubara sum trivializes

Example: Integral j in the spectral density

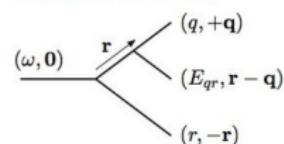
After performing the Matsubara sum and taking the imaginary part, obtain the \mathcal{I}_j contribution to the bulk spectral density ($E_{qr} \equiv |\mathbf{q} - \mathbf{r}|$):

$$\begin{aligned}\rho_{\mathcal{I}_j}(\omega) = & \int_{\mathbf{q}, \mathbf{r}} \frac{\omega^6 \pi}{4qr E_{qr}} \left\{ \right. \\ & \frac{1}{8q^2} \left[\delta(\omega - 2q) - \delta(\omega + 2q) \right] \times \\ & \times \left[\left(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)} \right) (1+2n_q)(n_{qr}-n_r) \right. \\ & \left. + \left(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)} \right) (1+2n_q)(1+n_{qr}+n_r) \right] \\ & + \frac{1}{8r^2} \left[\delta(\omega - 2r) - \delta(\omega + 2r) \right] \times \\ & \times \left[\left(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)} \right) (1+2n_r)(n_{qr}-n_q) \right. \\ & \left. + \left(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)} \right) (1+2n_r)(1+n_{qr}+n_q) \right] \\ & + \left. \left[\delta(\omega - q - r - E_{qr}) - \delta(\omega + q + r + E_{qr}) \right] \frac{(1+n_{qr})(1+n_q+n_r)+n_q n_r}{(q+r+E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \right. \\ & + \left. \left[\delta(\omega - q - r + E_{qr}) - \delta(\omega + q + r - E_{qr}) \right] \frac{n_{qr}(1+n_q+n_r)-n_q n_r}{(q+r-E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \right. \\ & + \left. \left[\delta(\omega - q + r - E_{qr}) - \delta(\omega + q - r + E_{qr}) \right] \frac{n_r(1+n_q+n_{qr})-n_q n_{qr}}{(q-r+E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})} \right. \\ & \left. \left. + \left[\delta(\omega + q - r - E_{qr}) - \delta(\omega - q + r + E_{qr}) \right] \frac{n_q(1+n_r+n_{qr})-n_r n_{qr}}{(q-r-E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})} \right\}\end{aligned}$$

Factorized int./
Virtual correction



Phase space int./
Real correction



Example: Integral j in the spectral density

After quite some work: Final result in terms of 1d and 2d (numerically evaluated) integrals

$$\begin{aligned}
 & \frac{(4\pi)^3 \rho_{\mathcal{I}_j}(\omega)}{\omega^4 (1 + 2n_{\frac{\omega}{2}})} = \\
 & \int_0^{\frac{\omega}{4}} dq n_q \left[\left(\frac{1}{q - \frac{\omega}{2}} - \frac{1}{q} \right) \ln \left(1 - \frac{2q}{\omega} \right) - \frac{\frac{\omega}{2}}{q(q + \frac{\omega}{2})} \ln \left(1 + \frac{2q}{\omega} \right) \right] \\
 & + \int_{\frac{\omega}{4}}^{\frac{\omega}{2}} dq n_q \left[\left(\frac{2}{q - \frac{\omega}{2}} - \frac{1}{q} \right) \ln \left(1 - \frac{2q}{\omega} \right) - \frac{\frac{\omega}{2}}{q(q + \frac{\omega}{2})} \ln \left(1 + \frac{2q}{\omega} \right) - \frac{1}{q - \frac{\omega}{2}} \ln \left(\frac{2q}{\omega} \right) \right] \\
 & + \int_{\frac{\omega}{2}}^{\infty} dq n_q \left[\left(\frac{2}{q - \frac{\omega}{2}} - \frac{2}{q} \right) \ln \left(\frac{2q}{\omega} - 1 \right) - \frac{\frac{\omega}{2}}{q(q + \frac{\omega}{2})} \ln \left(1 + \frac{2q}{\omega} \right) + \left(\frac{1}{q} - \frac{1}{q - \frac{\omega}{2}} \right) \ln \left(\frac{2q}{\omega} \right) \right] \\
 & + \int_0^{\frac{\omega}{2}} dq \int_0^{\frac{\omega}{4} - |q - \frac{\omega}{4}|} dr \left(-\frac{1}{qr} \right) \frac{n_{\frac{\omega}{2}-q} n_{q+r}(1 + n_{\frac{\omega}{2}-r})}{n_r^2} \\
 & + \int_{\frac{\omega}{2}}^{\infty} dq \int_0^{q - \frac{\omega}{2}} dr \left(-\frac{1}{qr} \right) \frac{n_{q-\frac{\omega}{2}}(1 + n_{q-r})(n_q - n_{r-\frac{\omega}{2}})}{n_r n_{-\frac{\omega}{2}}} \\
 & + \int_0^{\infty} dq \int_0^q dr \left(-\frac{1}{qr} \right) \frac{(1 + n_{q+\frac{\omega}{2}}) n_{q+r} n_{r+\frac{\omega}{2}}}{n_r^2}.
 \end{aligned}$$

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Summary of current NLO results

	OPEs	Spectral density	Coord. space
Scalar	[1]	[2]	[3]
Pseudoscalar	[1]	[2]	[3]
Shear	[4]	[5]	–

[1] Mikko Laine, Mikko Vepsäläinen, AV, 1008.3263

[2] Mikko Laine, AV, Yan Zhu, 1108.1259

[3] Mikko Laine, Mikko Vepsäläinen, AV, 1011.4439

[4] York Schröder, Mikko Vepsäläinen, AV, Yan Zhu, 1109.6548

[5] Yan Zhu, AV, 1210.nnnn

Note: Inclusion of fermions possible in all cases (cf. work of Chetyrkin, Zoller).

Wilson coefficients for OPEs

For $P \gg T$, perform large momentum expansion of Euclidean correlators to obtain $T \neq 0$ corrections to the OPEs

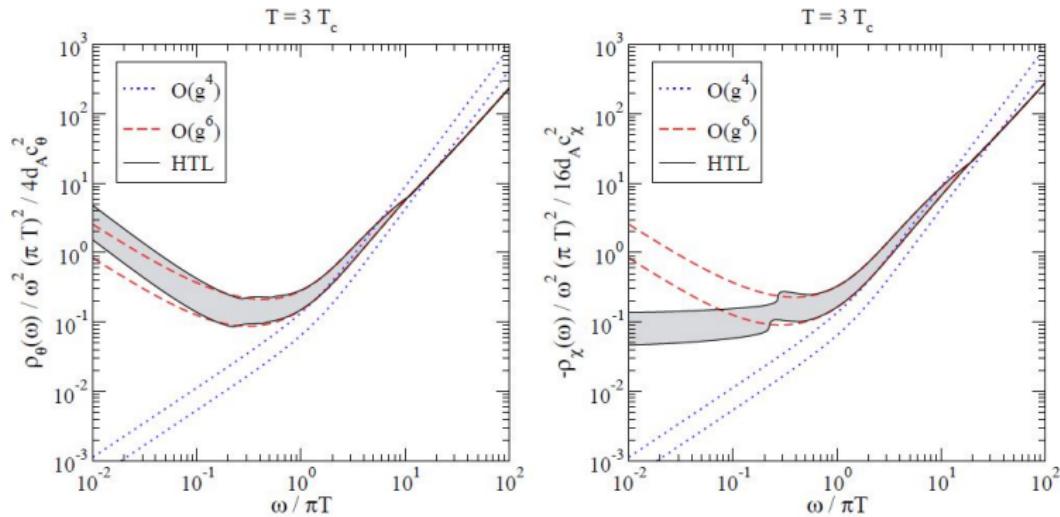
$$\begin{aligned} \frac{\Delta \tilde{G}_\theta(P)}{4c_\theta^2 g^4} &= \frac{3}{P^2} \left(\frac{p^2}{3} - p_n^2 \right) \left[1 + \frac{g^2 N_c}{(4\pi)^2} \left(\frac{22}{3} \ln \frac{\bar{\mu}^2}{P^2} + \frac{203}{18} \right) \right] (e + p)(T) \\ &\quad - \frac{2}{g^2 b_0} \left[1 + g^2 b_0 \ln \frac{\bar{\mu}^2}{\zeta_\theta P^2} \right] (e - 3p)(T) + \mathcal{O}\left(g^4, \frac{1}{P^2}\right) \\ \frac{\Delta \tilde{G}_\chi(P)}{-16c_\chi^2 g^4} &= \frac{3}{P^2} \left(\frac{p^2}{3} - p_n^2 \right) \left[1 + \frac{g^2 N_c}{(4\pi)^2} \left(\frac{22}{3} \ln \frac{\bar{\mu}^2}{P^2} + \frac{347}{18} \right) \right] (e + p)(T) \\ &\quad + \frac{2}{g^2 b_0} \left[1 + g^2 b_0 \ln \frac{\bar{\mu}^2}{\zeta_\chi P^2} \right] (e - 3p)(T) + \mathcal{O}\left(g^4, \frac{1}{P^2}\right) \\ \frac{\Delta \tilde{G}_\eta(P)}{4c_\eta^2} &= - \left\{ 1 + \frac{3}{P^2} \left(\frac{p^2}{3} - p_n^2 \right) - \frac{1}{3} \frac{g^2 N_c}{(4\pi)^2} \left[22 + \frac{41}{P^2} \left(\frac{p^2}{3} - p_n^2 \right) \right] \right\} (e + p)(T) \\ &\quad + \frac{4}{3g^2 b_0} \left[1 - g^2 b_0 \ln \zeta_\eta \right] (e - 3p)(T) + \mathcal{O}\left(g^4, \frac{1}{P^2}\right) \end{aligned}$$

Note the appearance of non-Lorentz-invariant operator $(e + p)$.

Spectral densities

Bulk channel spectral densities:

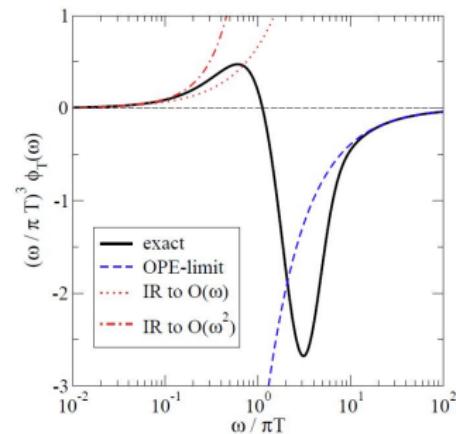
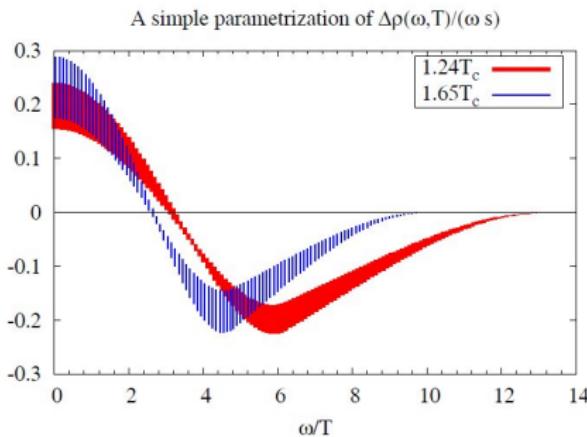
$$\begin{aligned}\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} &= \frac{\pi\omega^4}{(4\pi)^2} (1 + 2n_{\frac{\omega}{2}}) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[\frac{22}{3} \ln \frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8\phi_T(\omega) \right] \right\} + \mathcal{O}(g^8) \\ \frac{-\rho_\chi(\omega)}{16d_A c_\chi^2} &= \frac{\pi\omega^4}{(4\pi)^2} (1 + 2n_{\frac{\omega}{2}}) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[\frac{22}{3} \ln \frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8\phi_T(\omega) \right] \right\} + \mathcal{O}(g^8)\end{aligned}$$



Spectral densities

Bulk channel spectral densities:

$$\begin{aligned}\frac{\rho_\theta(\omega)}{4d_A c_\theta^2} &= \frac{\pi\omega^4}{(4\pi)^2} \left(1 + 2n_{\frac{\omega}{2}}\right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[\frac{22}{3} \ln \frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8\phi_T(\omega) \right] \right\} + \mathcal{O}(g^8) \\ \frac{-\rho_\chi(\omega)}{16d_A c_\chi^2} &= \frac{\pi\omega^4}{(4\pi)^2} \left(1 + 2n_{\frac{\omega}{2}}\right) \left\{ g^4 + \frac{g^6 N_c}{(4\pi)^2} \left[\frac{22}{3} \ln \frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8\phi_T(\omega) \right] \right\} + \mathcal{O}(g^8)\end{aligned}$$

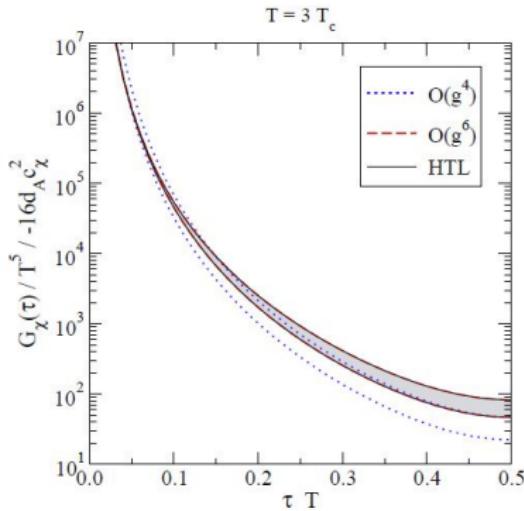
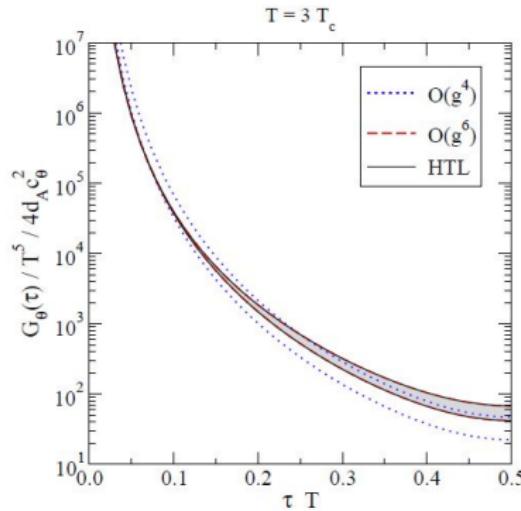


H. B. Meyer, 1002.3343

Spectral densities

In the imaginary time correlator, theoretical uncertainties (renormalization scale running) considerably suppressed

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh \frac{(\beta - 2\tau)\omega}{2}}{\sinh \frac{\beta\omega}{2}}$$

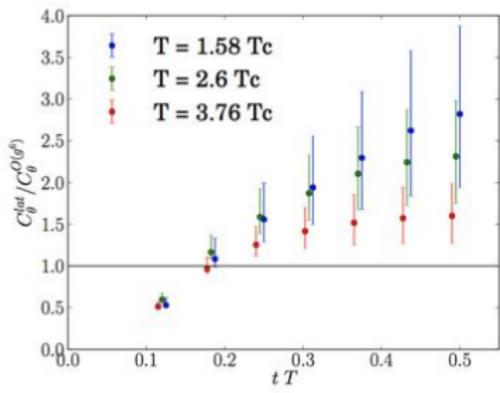


Spectral densities

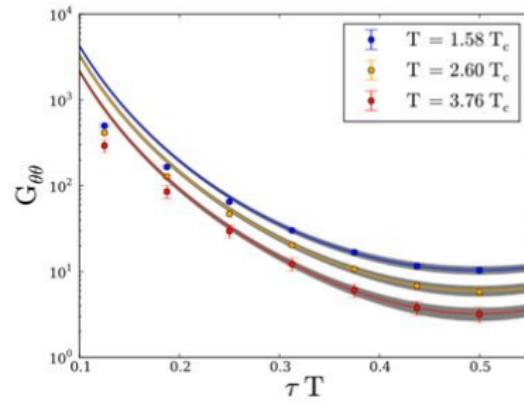
For the imaginary time correlator, direct comparison with lattice results possible

$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh \frac{(\beta - 2\tau)\omega}{2}}{\sinh \frac{\beta\omega}{2}}$$

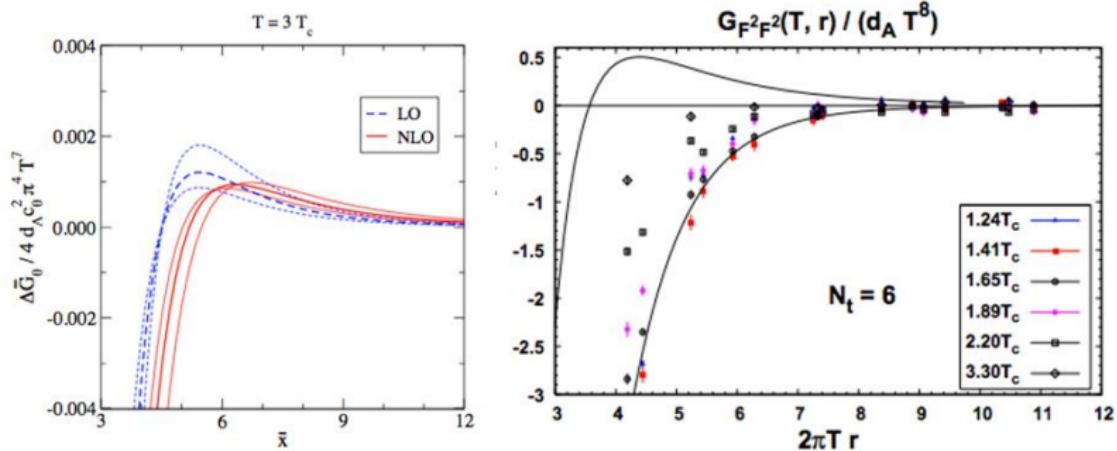
Chuan Miao(CPOD2011), H. B. Meyer



Chuan Miao, H.B. Meyer (Preliminary)



Time averaged coordinate space correlators in bulk channel



- Qualitatively, NLO results closer to lattice than LO ones
- However: We computed **time averaged** correlator, not equal time
- AdS computation of same correlator in large- N_c YM underway
(Kajantie, Krssak, AV)

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Conclusions

- Perturbative evaluation of various correlation functions in high-temperature QCD crucial for disentangling the properties of the quark gluon plasma
 - Spectral densities needed in extracting transport coefficients from lattice QCD data (cf. talk of Mikko Laine)
 - Spatial correlators useful way to test lattice, pQCD and holographic predictions
- NLO results derived for
 - OPEs in bulk and shear channels
 - Spectral density in the bulk channel
 - Time averaged spatial correlator in the bulk channel
- Current computational techniques involve plenty of handwork — automatization of calculations challenging at finite T
 - Room for new ideas/developments