

Aufgabe 1

$$\begin{aligned} \text{a) } \int_2^4 dx (x^2 + 4x + 42) &= \left[\frac{x^3}{3} + 2x^2 + 42x \right]_2^4 \\ &= \frac{64}{3} + 32 + 168 - \frac{8}{3} - 8 - 84 \\ &= \frac{56}{3} + 108 = \frac{56 + 324}{3} = \frac{380}{3} = 126 \frac{2}{3} // \end{aligned}$$

$$\begin{aligned} \text{b) } \int_{-\frac{7}{2}}^0 dx e^{2x} &= \left[\frac{7}{2} e^{2x} \right]_{-\frac{7}{2}}^0 \\ &= \frac{7}{2} - \frac{7}{2} e^{-7} = \frac{7}{2} \left(1 - \frac{7}{e} \right) // \end{aligned}$$

$$\begin{aligned} \text{c) } \int_7^5 \frac{7}{x} dx &= \left[\ln(x) \right]_7^5 = \ln(5) - \ln(7) \\ &= \ln\left(\frac{5}{7}\right) // \end{aligned}$$

Aufgabe 2

$$a) \int dt \dot{x}(t) = x(t) + A$$

$$b) \int dt \dot{x}(t) x(t) = \frac{1}{2} (x(t))^2 + B$$

$$c) \int dq \frac{1}{a+bq} = \frac{1}{b} \ln|a+bq| + C$$

Aufgabe 3

$$\begin{aligned} \text{a) } \int_0^a dx \frac{1}{x^{1-a}} &= \int_0^a dx x^{a-1} \\ &= \left[\frac{1}{a} x^a \right]_0^a = \frac{a^a}{a} - 0 = a^{a-1} // \end{aligned}$$

$$\begin{aligned} \text{b) } \int_0^7 dx (1-x^2)^2 &= \int_0^7 dx (1-2x^2+x^4) \\ &= \left[x - \frac{2}{3}x^3 + \frac{x^5}{5} \right]_0^7 = 7 - \frac{2}{3} + \frac{7}{5} = \\ &= \frac{5}{15} + \frac{3}{15} = \frac{8}{15} // \end{aligned}$$

$$\begin{aligned} \text{c) } \int_0^7 dx \sqrt{1+2x} &= \int_0^7 dx (1+2x)^{\frac{1}{2}} \\ &= \left[\frac{2}{3} (1+2x)^{\frac{3}{2}} \right]_0^7 = \frac{2}{3} \sqrt{3^3} - \frac{2}{3} \\ &= \sqrt{3} - \frac{2}{3} // \end{aligned}$$

Aufgabe 4

$$\begin{aligned} \text{a) } \frac{d}{dx} \sqrt[n]{x} &= \frac{d}{dx} x^{\frac{1}{n}} = \frac{d}{dx} e^{\frac{1}{n} \ln(x)} \\ &= e^{\frac{1}{n} \ln(x)} \cdot \frac{1}{nx} = \frac{x^{\frac{1}{n}}}{nx} \\ &= \frac{1}{n} x^{\frac{1}{n}-1} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{d}{dx} x^{\frac{p}{q}} &= \frac{d}{dx} e^{\frac{p}{q} \ln(x)} \\ &= e^{\frac{p}{q} \ln(x)} \cdot \frac{p}{q} \cdot \frac{1}{x} = \frac{p}{q} \frac{x^{\frac{p}{q}}}{x} \\ &= \frac{p}{q} \cdot x^{\frac{p}{q}-1} \end{aligned}$$

Aufgabe 5

Aufgabe 6