Aufgabe 1

a)
$$\int_2^4 dx\,(x^2+4x+42)$$
$$=\left[\frac{x^3}{3}+2x^2+42x\right]_2^4$$
$$=\frac{64}{3}+32+168-\frac{8}{3}-8-84$$
$$=\frac{56}{3}+108=\frac{56+324}{3}=\frac{380}{3}=126\tfrac{2}{3}$$

b)
$$\int_{-\frac{1}{\alpha}}^{0} dx\, e^{\alpha x}=\left[\frac{1}{\alpha}e^{\alpha x}\right]_{-\frac{1}{\alpha}}^{0}$$
$$=\frac{1}{\alpha}-\frac{1}{\alpha}e^{-1}=\frac{1}{\alpha}\left(1-\frac{1}{e}\right)$$

c)
$$\int_1^5 \frac{1}{x}\,dx=\left[\ln(x)\right]_1^5=\ln(5)-\ln(1)$$
$$=\ln(5)$$

Aufgabe 2

a) $\int dt\, \dot{x}(t) = x(t) + A$

b) $\int dt\, \dot{x}(t)\, x(t) = \frac{1}{2}(x(t))^2 + B$

c) $\int dq \frac{1}{a+bq} = \frac{1}{b} \ln(a+bq) + C$

Aufgabe 3

a) $\int_0^a dx \frac{1}{x^{1-a}} = \int_0^a dx\ x^{a-1}$

$= \left[\frac{1}{a} x^a\right]_0^a = \frac{a^a}{a} - 0 = a^{a-1}$ //

b) $\int_0^1 dx\ (1-x^2)^2 = \int_0^1 dx (1 - 2x^2 + x^4)$

$= \left[x - \frac{2}{3}x^3 + \frac{x^5}{5}\right]_0^1 = 1 - \frac{2}{3} + \frac{1}{5} =$

$= \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$ //

c) $\int_0^1 dx \sqrt{1+2x} = \int_0^1 dx\ (1+2x)^{1/2}$

$= \left[\frac{1}{3}(1+2x)^{3/2}\right]_0^1 = \frac{1}{3}\sqrt{3^3} - \frac{1}{3}$

$= \sqrt{3} - \frac{1}{3}$ //

Aufgabe 4

a) $$\frac{d}{dx}\sqrt[n]{x} = \frac{d}{dx}x^{\frac{1}{n}} = \frac{d}{dx}e^{\frac{1}{n}\ln(x)}$$

$$= e^{\frac{1}{n}\ln(x)} \cdot \frac{1}{nx} = \frac{x^{\frac{1}{n}}}{nx}$$

$$= \frac{1}{n}x^{\frac{1}{n}-1}$$

//

b) $$\frac{d}{dx}x^{\frac{p}{q}} = \frac{d}{dx}e^{\frac{p}{q}\ln(x)}$$

$$= e^{\frac{p}{q}\ln(x)} \cdot \frac{p}{q} \cdot \frac{1}{x} = \frac{p}{q}\,\frac{x^{\frac{p}{q}}}{x}$$

$$= \frac{p}{q} \cdot x^{\frac{p}{q}-1}$$

Aufgabe 5

Aufgabe 6