Elementary Particle Physics - Exercises	**Problem sheet 03**	November 13, 2015

[Tutorials on Thursdays: 8-10 in C01-148 and 16-18 in U2-135]

Exercise 3.1: Dirac's γ-matrices

1. Starting from $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} I_4$, show that Tr$\gamma^\mu$= 0, *without* using an explicit representation for the γ-matrices.
2. Prove that $(\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$ with the help of the standard representation of the Dirac matrices γ^μ introduced in the lecture. What does this result imply for the Hermiticity of the Dirac matrices?
3. Let us define $\gamma_5 = \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$.

   What is the expression for γ^5 in standard representation? Prove the following properties:

   a) $\{\gamma^\mu, \gamma^5\} = 0$, b) $(\gamma_5)^2 = I_4$, c) $\gamma_5^\dagger = \gamma_5$ d) Tr$\gamma_5 = 0$.

   In particular, these properties imply that $P_{R/L} \equiv \frac{1}{2}(I_4 \pm \gamma_5)$ are Hermitian projection operators, Show that they satisfy

   $(P_{R/L})^2 = P_{R/L}$, $P_L P_R = 0 = P_R P_L$ and $P_R + P_L = I_4$.

Exercise 3.2: The Dirac equation from its Lagrange density

1. Show that the Dirac-adjoint spinor $\bar\psi \equiv \psi^\dagger\gamma^0$ satisfies the following equation,

$$\bar\psi(x)(i\gamma^\mu \overleftarrow{\partial}_\mu + m) = 0 \ ,$$

   where the arrow to the left means that the derivative acts to the left.
2. Find the Dirac equation for $\psi(x)$ and $\bar\psi(x)$ from the associated Euler-Lagrange equations.
3. Show that $\int d^3x\ \bar\psi(x)\gamma^0\psi(x)$ is a conserved quantity.

Exercise 3.3: Maxwell's equations from its Lagrange density

1. Derive the equations of motion for the (four-component-)vector potential $A^\mu(x)$ introduced in lecture from the Euler-Lagrange equations of the Lagrange density $\mathcal{L}_M = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$.
2. How should the scalar field $\chi(x)$ be chosen in the gauge transformation $A^\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\chi(x)$, such that the Lorenz gauge is satisfied?
3. Show that the following Lagrange density $\mathcal{L} = -\frac{1}{4}\tilde F_{\mu\nu}(x)F^{\mu\nu}(x)$ containing the dual field strength $\tilde F_{\mu\nu}(x) = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}(x)$ tensor is a total derivative. [hint: consider $K_\mu = \epsilon_{\mu\nu\rho\sigma}A^\nu\partial^\rho A^\sigma$ and its derivative].