Elementary Particle Physics - Exercises	**Problem Sheet 05**	November 27, 2015

[Tutorials on Thursdays: 8-10 in C01-148 and 16-18 in U2-135]

Exercise 5.1: Helicity operator
The helicity operator is set to be $h(\vec{p}) \equiv \vec{e}_{\vec{p}}\vec{\Sigma}$, where

$$\vec{e}_{\vec{p}} \equiv \frac{\vec{p}}{|\vec{p}|} \text{ , and } \vec{\Sigma} \equiv \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

is given by the Pauli matrices σ^k. Show the following properties:

i) $h(\vec{p})^2 = I_4$ and ii) the operators $P_\pm \equiv \frac{1}{2}(I_4 \pm h(\vec{p}))$ are projection operators, that is: $(P_\pm)^2 = P_\pm$, $P_+ + P_- = I_4$, and $P_\pm P_\mp = 0$.

Exercise 5.2: Chirality
Show that the eigenvalue of the righthanded, respectively lefthanded spinor $\psi_{R/L} = P_{R/L}\psi$ equals ± 1 , whereat $P_{R/L}$ had been introduced in Exercise 3.1. This eigenvalue is called chirality.

Exercise 5.3: Phase space integration of the two-body decay
Let us consider the decay $A \to 1 + 2$ in the system of rest of the initial particle A into particles 1 and 2, with masses m, m_1, m_2 respectively, and the associated four-momenta $p^\mu = \left(m, \vec{0}\right)$, and $p_1^\mu = (E_{\vec{p}_1}, \vec{p}_1)$, $p_2^\mu = (E_{\vec{p}_2}, \vec{p}_2)$. Compute the decay rate from lecture

$$\Gamma_{A\to 1+2} = \frac{(2\pi)^4}{2m} \int \frac{d^3p_1}{(2\pi)^3 2E_{\vec{p}_1}} \frac{d^3p_2}{(2\pi)^3 2E_{\vec{p}_2}} |\mathcal{M}(\vec{p}_1, \vec{p}_2)|^2 \delta^{(4)}(p_1 + p_2 - p)$$

in the following steps:

1. Integrate with respect to p_1 exploiting the δ-function:
$$\Gamma_{A\to 1+2} = \frac{1}{2m(4\pi)^2} \int \frac{d^3p_2 |\mathcal{M}(-\vec{p}_2, \vec{p}_2)|^2}{\sqrt{m_1^2 + \vec{p}_2^2}\sqrt{m_2^2 + \vec{p}_2^2}}\, \delta\left(m - \sqrt{m_1^2 + \vec{p}_2^2} - \sqrt{m_2^2 + \vec{p}_2^2}\right)$$
2. Assume that $\mathcal{M}(-\vec{p}_2, \vec{p}_2) = \mathcal{M}(|\vec{p}_2|)$ only depends of the absolute value, so that you find in spherical coordinates:
$$\Gamma_{A\to 1+2} = \frac{1}{8\pi m} \int_0^\infty \frac{dr\, r^2 |\mathcal{M}(r)|^2}{\sqrt{m_1^2 + r^2}\sqrt{m_2^2 + r^2}}\, \delta\left(m - \sqrt{m_1^2 + r^2} - \sqrt{m_2^2 + r^2}\right)$$
3. By change of variables $r \to E = \sqrt{m_1^2 + r^2} + \sqrt{m_2^2 + r^2}$ show that we arrive at
$$\Gamma_{A\to 1+2} = \frac{r_0}{8\pi m^2} |\mathcal{M}(r_0)|^2 \Theta(m - m_1 - m_2) \text{ with } r_0 = ?$$

Exercise 5.4: Lorentz invariant measure
Show that the measure from phase space integration can be written in the following Lorentz invariant form given in lecture:

$$\int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} = 2\pi \int \frac{d^4p}{(2\pi)^4} \delta(p^2 - m^2)\ \Theta(p^0)\ .$$