Elementary Particle Physics - Exercises	**Problem Sheet 06**	December 4, 2015

[Tutorials on Thursdays: 8-10 in C01-148 and 16-18 in U2-135]

Exercise 6.1: Decay Rate $\rho^0 \to \pi^+\pi^-$

Consider the result of the exercise 5.3

$$r_0 = \frac{1}{2m}\sqrt{m^4 + m_1^4 + m_2^4 - 2m^2m_1^2 - 2m^2m_2^2 - 2m_1^2m_2^2} \ .$$

Assume the following values for the masses, $m = m_\rho = 770$ MeV, $m_1 = m_2 = m_\pi = 140$ MeV and for the amplitude $\mathcal{M} = 2$ GeV.

(a) What do you obtain for the life time? Compare your result afterwards with the life time of the physical particle ρ, cf. http://pdg.lbl.gov

(b) Plot the decay rate as a function of m. What is its physical meaning?

Exercise 6.2: Rapidity

There are various possibilities for the choice of kinematic variables. For instance, when the ray direction is chosen to lie on the z axis, we define the rapidity y as

$$y \equiv \frac{1}{2}\ln\left(\frac{E + p_z}{E - p_z}\right).$$

1. Show that the four-momentum can be written as $p^\mu = (m_T\cosh(y), p_x, p_y, m_T\sinh(y))$, where $m_T = \sqrt{m^2 + p_x^2 + p_y^2}$ denotes the transversal mass.
2. Consider the rapidities of two particles. Starting from the addition formula for velocities, show how the rapidities can be added.

Exercise 6.3: Mandelstam variables

1. Show that the Mandelstam variables, defined as $s \equiv (Q_A + Q_B)^2$, $t \equiv (Q_A - P_1)^2$ and $u \equiv (Q_A - P_2)^2$, are not independent:
$$s + t + u = m_A^2 + m_B^2 + m_1^2 + m_2^2 \ .$$
2. Assume that one has $m_A = m_B = m_1 = m_2 = M$. What is the kinematically allowed domain in the (s, t)-plane?

Exercise 6.4:

Starting from the result given in lecture

$$\frac{d\sigma}{d\Omega} = \frac{|\vec{p}_1||\mathcal{M}|^2(|\vec{q}_A|, |\vec{p}_1|, \cos(\theta))}{(8\pi)^2|\vec{q}_A|(E_A + E_B)^2} \ ,$$

please derive an expression for $d\sigma/dt$ which depends only on the invariants $m_A^2, m_B^2, m_1^2, m_2^2, s, t$.