[Tutorials on Thursdays: 08-10 in C01-148 and 16-18 in U2-135]

Exercise 8.1: Identities for Dirac matrices
Starting from $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ prove the following identities - without using their explicit representation:

1. $\mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4\eta^{\mu\nu}$,
2. $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho] = 0$,
3. $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})$.

Exercise 8.2: Myon pair production
The amplitude $\mathcal{M}$ for Myon pair production $e^- + e^+ \to \mu^- + \mu^+$ with the associated four-momenta and spins (p_j, σ_j), $j = 1, 2, 3, 4$ of the ingoing electrons, respectively outgoing myons e^-, e^+, μ^-, μ^+ is given by

$$\mathcal{M} = \frac{-e^2}{(p_1 + p_2)^2}[\bar{v}(\vec{p}_2, \sigma_2)\gamma_\mu u(\vec{p}_1, \sigma_1)][\bar{u}(\vec{p}_3, \sigma_3)\gamma^\mu v(\vec{p}_4, \sigma_4)].$$

1. Draw the associated Feynman diagram.
2. Compute $\langle|\mathcal{M}|^2\rangle$ analogously to the computation from the lecture.
3. Derive the total cross section, which was already given to you in the lecture, by plugging $\langle|\mathcal{M}|^2\rangle$ into the formula found in Exercise 6.4 and integration:

$$\sigma(e^- + e^+ \to \mu^- + \mu^+) = \frac{4\pi}{3}\frac{\alpha_{EM}^2}{s}\sqrt{\frac{1 - 4m_\mu^2/s}{1 - 4m_e^2/s}}\left(1 + \frac{2m_\mu^2}{s}\right)\left(1 + \frac{2m_e^2}{s}\right).$$

Exercise 8.3: Number of Colours N_C
In the lecture we found the total cross section to be approximately $\sigma(e^+ + e^- \to q_i + \bar{q}_i) \approx Q_i^2 \frac{4\pi}{3}\left(\frac{\alpha_{EM}}{E}\right)^2 \Theta(\sqrt{s} - 2m_{q_i})$ and we could compute the following ratio:

$$R(E) \equiv \frac{\sigma(e^+ + e^- \to \text{hadrons})}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)}.$$

Derive the plateau values $\frac{p}{q}N_C$, which were given in lecture.
Are there any further plateaus with $\frac{p}{q} =$? On which energy scales (in GeV) are the associated plateaus valid?

Exercise 8.4: Deep-inelastic scattering and Bjorken-x

1. Determine how the new variables Q_E^2 and x given in lecture depend on the old variables E' and θ for given E.
2. Show that the following bound holds for Bjorken-x: $0 \leq x \leq 1$.