[Tutorials on Thursdays: 08-10 in C01-148 and 16-18 in U2-135]

Exercise 9.1: Running coupling constant in QCD
The coupling constant in QCD $g_s(Q_E)$, which depends on the energy, is an approximate solution to the following differential equation:

$$Q_E \frac{d}{dQ_E} g_s^2 = -2b_0 g_s^4 \ , \quad \text{for} \quad Q_E \gg 1\text{GeV},$$

with $b_0 = (11N_c - 2N_f)/48\pi^2$. Find the general solution in the case $N_c = 3 = N_f$.

Exercise 9.2: The QCD scale Λ_{QCD}
The QCD scale is defined by

$$\Lambda_{QCD} \equiv \lim_{Q_E \to \infty} Q_E \exp\left[\frac{-1}{2b_0 g_s^2(Q_E)}\right]$$

when $g_s(Q_E)$ is the solution found in the previous exercise. Experimentally, we have found that for $Q_E = 91$GeV we arrive at $\alpha_s = g_s^2/4\pi \approx 0.12$. Which value do you obtain for Λ_{QCD}?

Exercise 9.3: Isospin
We consider elastic pion-nucleon scattering. There are six possible processes:

$$\pi^+ + p \to \pi^+ + p, \quad \pi^0 + p \to \pi^0 + p, \quad \pi^- + p \to \pi^- + p, \tag{1}$$
$$\pi^+ + n \to \pi^+ + n, \quad \pi^0 + n \to \pi^0 + n, \quad \pi^- + n \to \pi^- + n. \tag{2}$$

How many independent amplitudes exist, when we assert that isospin yields an exact symmetry? (Pions have isospin $I = 1$, nucleons have $I = 1/2$.)

Exercise 9.4: Kaon decay
In the lecture two neutrals kaons had been defined, $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$. Those particles have the following properties considering parity $\hat{P}$ and charge conjugation $\hat{C}$:

$$\hat{P}|K^0\rangle = -|K^0\rangle, \quad \hat{P}|\bar{K}^0\rangle = -|\bar{K}^0\rangle, \tag{3}$$
$$\hat{C}|K^0\rangle = +|\bar{K}^0\rangle, \quad \hat{C}|\bar{K}^0\rangle = +|K^0\rangle. \tag{4}$$

Can you find linear combinations, which are eigenvectors of $\hat{C}\hat{P}$?
Which of these states can decay into two pions, which one can decay into three pions, when $\hat{C}\hat{P}$ is conserved? Why can't these reactions occur in QCD?