[Tutorials on Thursdays: 08-10 in C01-148 and 16-18 in U2-135]

Exercise 11.1: Tadpole integral
We consider the following integral, which is cut-off by the parameter $\Lambda \gg 1$:

$$A(m,\Lambda) \equiv \int_{|\vec{k}|<\Lambda} \frac{d^3\vec{k}}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{1}{k^2 - m^2 + i\varepsilon} ,$$

when $k^2 = k_0^2 - \vec{k}^2$ and $\varepsilon > 0$ is an infinitesimal paramter. How does the integral behave for $\Lambda \gg m$? [Hint: The easiest way to compute the k_0-integral is by employing the residue theorem.]

Exercise 11.2: Bubble integral
Next, we consider with the same notation the integral

$$B(m,q\Lambda) \equiv \int_{|\vec{k}|<\Lambda} \frac{d^3\vec{k}}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk_0}{2\pi} \frac{1}{(k^2 - m^2 + i\varepsilon)((q+k)^2 - m^2 + i\varepsilon)} .$$

How does the integral behave for $\Lambda \gg m, q_0, |\vec{q}|$? [If the computation of the integral is too difficult, suppose that $q^2 \ll m^2$ and use Taylor's Theorem on q^2.]

Exercise 11.3: $SU(2)$-gauge invariance
We consider a pure $SU(2)$-gauge theory with the notation from the lecture, $T_a = \frac{1}{2}\sigma_a$. Show that the Lagrangian $\mathcal{L}_\Phi = (D_\rho \Phi(x))^\dagger D^\rho \Phi(x)$ with the covariant derivative $D_\rho = \partial_\rho + igT_a W_\rho^a(x)$ is invariant under the following local transformations:

$$\Phi(x) \to \Phi(x)' = U(x)\Phi(x) \quad \text{mit} \quad U(x) \equiv \exp[-i\beta_a(x)T_a] \quad \text{und} \quad \beta_{a=1,2,3}(x) \in \mathbb{R},$$

$$T_a W_\rho^a(x) \to T_a W_\rho^a(x)' = U(x) T_a W_\rho^a(x) U(x)^\dagger + \frac{i}{g}(\partial_\rho U(x))U(x)^\dagger.$$

Next, prove the transformation $D_\rho \Phi(x) \to D'_\rho \Phi(x)' = U(x) D_\rho \Phi(x)$, from which follows the invariance of $\mathcal{L}_\Phi$. [The invariance of the kinetic term $\mathcal{L}_W = -\frac{1}{4} F_{\mu\nu}^a F^{a\,\mu\nu}$ from W_μ^a can be proved with infinitesimal transformations. This will be shown in the tutorials.]

Exercise 11.4: Weak hyper charge
Verify the table for the weak hyper charge Y_W which was given to you in the lecture: Plug in the known electrical charges Q of the leptons, quarks and of the higgs boson and the z-components I_z^W of $SU(2)_L$ and invoke the formula $Q = I_z^W + Y_W$ in the end.