[Tutorials on Thursdays: 08-10 in C01-148 and 16-18 in U2-135]

Exercise 12.1: Gauge symmetry and Higgs boson

1. Directly after spontaneous symmetry breaking the Higgs doublet is given by

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ v + \phi_0(x) + i\phi_3(x) \end{pmatrix}.$$

   Construct the $SU(2)$-matrix $U(x) = \exp[-i\beta_a(x)\sigma_a/v]$ for $v \gg 1$, so that

$$U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \phi_0(x) \end{pmatrix}.$$

   [Hint: You are allowed to expand the matrix $U(x)$ in powers of $1/v$.]

2. How does the charge conjugate of the Higgs $\tilde{\Phi}(x) = i\sigma_2\Phi(x)^*$ transform under the $SU(2)_L$ transformation from Exercise 11.3? Which role does $i\sigma_2$ play?

3. Prove that the Yukawa interactions $\bar{Q}'_{1,L}\tilde{\Phi}(x)\psi_{u,R}$ and $\bar{Q}'_{1,L}\Phi(x)\psi_{d,R}$ are invariant under $U(1)_Y$-gauge transformations (and therefore also their Hermitean conjugates). Are these terms also invariant under $SU(2)_L$ transformations?

Exercise 12.2: Weinberg angle and vector boson masses

1. Use the value of the weak fine structure constant $g_W^2 = 4\sqrt{2}G_F m_W^2$, which was given to you in the lecture, and the value of the elementary charge e from the electromagnetic fine structure constant $\alpha_{em} = e^2/4\pi = 1/137$, to find $\sin\theta_W$ starting from $e = g_W \sin\theta_W$. Compare this value to the one from the Particle Data Booklet.

2. Which value do you find for the ratio m_Z/m_W? Compare this value to the one from the Particle Data Booklet.

Aufgabe 12.3: Phase of the Yukawa coupling

Consider the mass term of the following form:

$$\mathcal{L}_{m_u} = -\frac{v}{\sqrt{v}}[h_u \bar{u}_L u_R + h_u^* \bar{u}_R u_L].$$

Find a phase transformation of the quark field u such that h_u can be chosen as a real coupling constant. [This transformation is also called chiral transformation.]