

Symmetries in Physics

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lectures Tuesdays 10:15 - 11:45 in 06-135

Fridays 12:30 - 14:00 — " —

exercises Mondays 14:15 - 15:45 — " —

• script & exercises → Ma-Phy group page → teaching

Literature:

group theory part: "Groups, representations & physics"

H.F. Jones 2. Ed Bristol 1998

FB17: QD80+QD130 j77(2) (also 1. Ed)

otherwise specified in lecture

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- * Lagrange formalism & conserved charges p. 4
 - Symmetry transformations and generators
 - Noether's Theorem
 - Coleman-Mandula Theorem
- * conformal symmetry in d dimensions p. 13
 - conformal group in $d > 2$

II Inner Symmetries - Group Theory p. 17

- * definitions, examples and discrete groups p. 17
 - S_n permutation, C_n cyclic, D_n dihedral group
 - Cayley's Theorem: finite group \cong subgroup of S_n
- * general properties of groups & maps p. 24
 - conjugation, equiv. var., conj. class, coset
 - Theorem of Lagrange $\dim H$ divides $\dim G$
 - normal subgroups $gHg^{-1} = H$, quotient groups: G/H , H norm. subgroup
 - reps, group homomorphisms, kernel & image group
- * group representations p. 32
 - matrix reps, equiv. of reps, irreps
- * groups acting on vector spaces p. 37
 - linear bases, similarity, G -module, submodule
 - linear indep., Gram-Schmidt, unitarity, Hermiticity
 - Maschke's Theorem
- * Properties of irreducible representations p. 46
 - 2 Schur lemmas: $D(g)B = B D(g) \Rightarrow B = \lambda \mathbb{1}$, $D(g)B = D(g)B \Rightarrow B = 0$
 - Fundamental Orthogonality Theorem $B = \sum_g O^{(\mu)}(g) \lambda O^{(\nu)}(g^{-1}) = \delta_{\mu\nu} S^{\mu\nu}$
 - Orthogonality of characters $\frac{1}{|G|} \sum_{g \in G} \chi^{(\mu)}(g) \chi^{(\nu)}(g)^* = \delta_{\mu\nu}$
 - # of inequiv. irreps = # of conj. classes
 - decomposition of irreps
 - the regular rep. $S_n = \sum \chi$
- * characters of irreps & character tables p. 56
- * direct products of reps and their decomp p. 59

* Continuous groups p. 61

- examples $(S)U(N)$, $(S)O(N)$, $SO(2) \cong U(1)$
- Clebsch-Gordan series for $U(N)$

* Lie-groups and Lie-algebras p. 69

- unitary matrix reps $D = \exp \left[i \sum_{a=1}^n \theta_a T_a \right]$
- $SU(2)$, $SO(2)$, $SO(3) \cong SU(2)/\mathbb{Z}_2$ $SO(3) = \exp \left[i \sum_{a=1}^3 \theta_a T_a \right]$
- irreps of $SO(3)$, orth. of char.

* properties of generators and structure constants p. 80

- examples for reps of Lie-alg.
- Casimir op., Cartan metric

* Sub-alg., ideal and (semi-)simple Lie algebras p. 83

The Cartan basis of a Lie-algebra p. 87

- roots and root vectors
- scalar products in the Cartan basis, Cartan-Weyl basis
- quantisation of the root vectors $2 \frac{\alpha \cdot \beta}{\alpha^2} = n = 0, 1, 2, 3$
- graphical rep of the roots & Weyl reflections, example $r=2$

* Dynkin diagrams p. 99

- pos., neg. and simple roots
- Chevalley basis and classification of semi-simple Lie-alg.
- representations and weights, root lattice, highest weight

* The group $SU(N)$: irreps and Young diagrams p. 103

- invariants and high dim. reps

III Coordinate Trafos: Lorentz and Poincaré group p. 115

* Lorentz group, $SO(3,1)$ and $SL(2, \mathbb{C})$, topology

- generators of $SO(3,1)$, irreps of $\mathfrak{so}(3,1)$

* The Poincaré group p. 125

- generators, reps and Casimir ops.
- massive reps $m \neq 0$
- massless reps $m = 0$

* Extension of Poincaré: Susy and conformal symmetry p. 133

- superspace

* Superfield formalism p. 137: chiral multiplet