Blatt 13

1 a) $\int_2^4 (x^2 + 4x + 42)\,dx = \left[\frac{1}{3}x^3 + 2x^2 + 42x\right]_2^4$

$= \frac{380}{3} = 126 + \frac{2}{3}$

b) $\int_{-\frac{1}{\alpha}}^{0} \exp(\alpha x)\,dx = \left[\frac{1}{\alpha}\exp(\alpha x)\right]_{-\frac{1}{\alpha}}^{0}$

$= \frac{1}{\alpha}\cdot 1 - \frac{1}{\alpha}\exp(-1)$

$= \frac{1}{\alpha}\left(1 - \frac{1}{e}\right)$

c) $\int_1^5 \frac{1}{x}\,dx = \left[\ln(x)\right]_1^5 = \ln(5) - \underbrace{\ln(1)}_{=0}$

$= \ln(5)$

2 a) $\int dt\, \dot{x}(t) = x(t) + C$

b) $\int dt\, \dot{x}(t)\, x(t) = \frac{1}{2}(x(t))^2 + C$

c) $\int dq \frac{1}{a+bq} = \frac{1}{b}\ln(a+bq) + C$

$\boxed{3}$ a) $\int_0^a dx \, \frac{1}{x^{1-a}} = \int_0^a dx \; x^{a-1}$

$\overset{a>1}{=} \left[\frac{1}{a} \, x^a \right]_0^a$

$= \frac{a^a}{a} - 0$

$= a^{a-1}$ //

b) $\int_0^1 dx \, (1 - x^2)^2 = \int_0^1 dx \; 1 - 2x^2 + x^4$

$= \left[\frac{1}{5} x^5 - \frac{2}{3} x^3 + x \right]_0^1$

$= \left(\frac{1}{5} - \frac{2}{3} + 1 \right) - 0$

$= \frac{3}{15} + \frac{5}{15} = \frac{8}{15}$ //

c) $\int_0^1 dx \sqrt{1+2x} = \int_0^1 dx \, (1+2x)^{\frac{1}{2}}$

$= \left[\frac{1}{3} (1+2x)^{\frac{3}{2}} \right]_0^1$

$= \frac{1}{3} \cdot 3^{\frac{3}{2}} - \frac{1}{3}$

$= \sqrt{3} - \frac{1}{3}$ //

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a) Sei $n \in \mathbb{N}$. z.z.: $\frac{d}{dx}\sqrt[n]{x} = \frac{1}{n} x^{\frac{1}{n}-1}$

$$\frac{d}{dx}\sqrt[n]{x} = \frac{d}{dx} x^{\frac{1}{n}} = \frac{d}{dx} e^{\frac{1}{n}\ln(x)} = e^{\frac{1}{n}\ln(x)} \cdot \frac{1}{n} x^{-1}$$

$$= e^{\frac{1}{n}\ln(x)} \cdot \frac{1}{n} \cdot e^{-\ln x} = \frac{1}{n} e^{\frac{1}{n}\ln(x) - \ln(x)}$$

$$= \frac{1}{n} e^{\ln(x)\left(\frac{1}{n}-1\right)} = \frac{1}{n}\left(e^{\ln(x)}\right)^{\left(\frac{1}{n}-1\right)}$$

$$= \frac{1}{n} x^{\frac{1}{n}-1} \quad //$$

Äußere Abl.

Innere "

b) $$\frac{d}{dx} x^{\frac{p}{q}} = \frac{d}{dx} e^{\frac{p}{q}\ln(x)} = \frac{p}{q} x^{-1} e^{\frac{p}{q}\ln(x)}$$

$$= \frac{p}{q} e^{-\ln(x)} e^{\frac{p}{q}\ln(x)}$$

$$= \frac{p}{q} e^{\ln(x)\cdot\left(\frac{p}{q}-1\right)}$$

$$= \frac{p}{q}\left(e^{\ln(x)}\right)^{\frac{p}{q}-1}$$

$$= \frac{p}{q} x^{\frac{p}{q}-1} \quad //$$