Lösungen: Vorkurs Zettel 11

Aufgabe 1

a)

$$f(x,y,z) = yz$$
$$\vec{\nabla} f(x,y,z) = \begin{pmatrix} \partial_x f(x,y,z) \\ \partial_y f(x,y,z) \\ \partial_z f(x,y,z) \end{pmatrix} = \begin{pmatrix} \partial_x yz \\ \partial_y yz \\ \partial_z yz \end{pmatrix} = \begin{pmatrix} 0 \\ z \\ y \end{pmatrix}$$

b)

$$f(\vec{r}) = x^3 + 2xyz$$
$$\vec{\nabla} f(\vec{r}) = \begin{pmatrix} \partial_x (x^3 + 2xyz) \\ \partial_y (x^3 + 2xyz) \\ \partial_z (x^3 + 2xyz) \end{pmatrix} = \begin{pmatrix} 3x^2 + 2yz \\ 2xz \\ 2xy \end{pmatrix}$$

c)

$$f(\vec{r}) = (x^2 + 3xz - z^2) \log(x - z)$$
$$\vec{\nabla} f(\vec{r}) = \begin{pmatrix} \partial_x (x^2 + 3xz - z^2) \log(x-z) \\ \partial_y (x^2 + 3xz - z^2) \log(x-z) \\ \partial_z (x^2 + 3xz - z^2) \log(x-z) \end{pmatrix} = \begin{pmatrix} (2x+3z)\log(x-z) + (x^2+3xz-z^2)\frac{1}{x-z} \\ 0 \\ (3x-2z)\log(x-z) - (x^2+3xz-z^2)\frac{1}{x-z} \end{pmatrix}$$
$$= \log(x-z) \begin{pmatrix} 2x+3z \\ 0 \\ 3x-2z \end{pmatrix} + \frac{x^2+3xz-z^2}{x-z} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

d)

$$f(\vec{r}) = \frac{1}{\|r\|}$$
$$\vec{\nabla} f(\vec{r}) = \begin{pmatrix} \partial_x \frac{1}{\sqrt{x^2+y^2+z^2}} \\ \partial_y \frac{1}{\sqrt{x^2+y^2+z^2}} \\ \partial_z \frac{1}{\sqrt{x^2+y^2+z^2}} \end{pmatrix} = \begin{pmatrix} \frac{2x}{-2\sqrt{x^2+y^2+z^2}^3} \\ \frac{2y}{-2\sqrt{x^2+y^2+z^2}^3} \\ \frac{2z}{-2\sqrt{x^2+y^2+z^2}^3} \end{pmatrix} = -\frac{1}{\|r\|^3} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{\vec{r}}{\|r\|^3}$$

Aufgabe 2

$$h(x,y) = \exp\left[-x^2 - y^2\right]$$
$$\Rightarrow \quad \vec{\nabla} h(x,y) = \begin{pmatrix} -2x\exp\left[-x^2-y^2\right] \\ -2y\exp\left[-x^2-y^2\right] \end{pmatrix} = -2\exp\left[-x^2-y^2\right] \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\vec{\nabla} h(x,y)\Big|_{(1,1)} = -2\exp\left[-2\right]\begin{pmatrix}1\\1\end{pmatrix}$$
$$\vec{\nabla} h(x,y)\Big|_{(1,-1)} = -2\exp\left[-2\right]\begin{pmatrix}1\\-1\end{pmatrix}$$

<u>Höhenlinien:</u>

$$\begin{aligned}
& h_0 = \exp\left[-x_0^2 - y_0^2\right] \\
\Leftrightarrow\quad & \exp\left[\log(h_0)\right] = \exp\left[-x_0^2 - y_0^2\right] \\
\Leftrightarrow\quad & 1 = \exp\left[-x_0^2 - y_0^2 - \log(h_0)\right] \\
\Leftrightarrow\quad & 1 = \exp\left[-x_0^2 - y_0^2 + \log\left(\frac{1}{h_0}\right)\right] \\
\Leftrightarrow\quad & 0 = -x_0^2 - y_0^2 + \log\left(\frac{1}{h_0}\right) \\
\Leftrightarrow\quad & \log\left(\frac{1}{h_0}\right) = x_0^2 + y_0^2
\end{aligned}$$

Die Höhenlinien parametrisieren Kreise.