

Lösungen: Vorkurs Zettel 3

The Difference between Knowledge and Understanding

a)

$$\begin{aligned}f(x) &= x^2 \\f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\&= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{2hx + h^2}{h} \\&= \lim_{h \rightarrow 0} 2x + h \\&= 2x\end{aligned}$$

b)

$$\begin{aligned}f(x) &= \cos(x) \\f'(x) &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h \rightarrow 0} \frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h} \\&= \cos(x) \underbrace{\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h}}_{=0} - \sin(x) \underbrace{\lim_{h \rightarrow 0} \frac{\sin(h)}{h}}_{=1} \\&= -\sin(x)\end{aligned}$$

Aufgabe 1

a)

$$\begin{aligned}f(x) &= x^8 \\ \frac{d}{dx} f(x) &= \frac{d}{dx} x^8 = 8x^7\end{aligned}$$

b)

$$h(x) = x^3 x^5 = x^8 \quad \Rightarrow \quad \text{siehe a)}$$

c)

$$\begin{aligned}g(x) &= \sin^3(x) \\ \frac{d}{dx} g(x) &= 3 \left(\frac{d}{dx} \sin(x) \right) \sin^2(x) \\ &= 3 \sin^2(x) \cos(x)\end{aligned}$$

d)

$$\begin{aligned}
 i(x) &= \sin^2(x) + \cos(x^2) \\
 \frac{d}{dx} i(x) &= \frac{d}{dx} \sin^2(x) + \frac{d}{dx} \cos(x^2) \\
 &= 2 \sin(x) \cos(x) - 2x \sin(x^2) \\
 &= 2(\sin(x) \cos(x) - x \sin(x^2))
 \end{aligned}$$

e)

$$\begin{aligned}
 j(x) &= x^2 \sin^2(x) \\
 \frac{d}{dx} j(x) &= \left(\frac{d}{dx} x^2 \right) \sin^2(x) + \left(\frac{d}{dx} \sin^2(x) \right) x^2 \\
 &= 2x \sin^2(x) + x^2 2 \sin(x) \cos(x) \\
 &= 2x \sin(x) (\sin(x) + x \cos(x))
 \end{aligned}$$

f)

$$\begin{aligned}
 h(x) &= \cos^3(x) \sin(a) \\
 \frac{d}{dx} h(x) &= \frac{d}{dx} \sin(a) \cos^3(x) = \sin(a) \frac{d}{dx} \cos^3(x) \\
 &= -3 \sin(a) \sin(x) \cos^2(x)
 \end{aligned}$$

Aufgabe 2

a)

$$\begin{aligned}
 \frac{d}{dx} x^n &= \frac{d}{dx} \underbrace{x \times x \times \dots \times x}_{n\text{-Mal}} \\
 &= \overbrace{\underbrace{x \times \dots \times x}_{(n-1)\text{-Mal}} \times \frac{d}{dx} x + \dots + \underbrace{x \times \dots \times x}_{(n-1)\text{-Mal}} \times \frac{d}{dx} x}_{n\text{-Mal}} \\
 &= nx^{n-1}
 \end{aligned}$$

b)

$$\begin{aligned}
 p_n(x) &= a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0, \quad a_0, \dots, a_n \in \mathbb{C} \\
 \frac{d}{dx} p_n(x) &= a_n \frac{d}{dx} x^n + a_{n-1} \frac{d}{dx} x^{n-1} + \dots + a_1 \frac{d}{dx} x + \frac{d}{dx} a_0 \\
 &\stackrel{a)}{=} na_n x^{n-1} + (n-1)a_{n-1} x^{n-2} + \dots + 2a_2 x + a_1
 \end{aligned}$$

Aufgabe 3

a)

$$f(x) = 5x^2x^{3+2} = 5x^7$$

$$f'(x) = 5 \cdot 7 \cdot x^6 = 35x^6$$

b)

$$h(x) = \frac{1}{2x+1} = f(g(x)),$$

mit $f(x) = \frac{1}{x}$, $g(x) = 2x+1$

$$h'(x) = f(g(x))' = g'(x)f'(g(x))$$

$$= -\frac{2}{(2x+1)^2}$$

c)

$$f(x) = \sqrt{1 + \sin(x^3)} = g(h(i(x))),$$

mit $g(x) = \sqrt{x}$, $h(x) = 1 + \sin(x)$, $i(x) = x^3$

$$f'(x) = g(h(i(x)))' = h'(i(x))g'(h(i(x)))$$

$$= i'(x)h'(i(x))g'(h(i(x)))$$

$$= 3x^2 \cos(x^3) \frac{1}{2} \frac{1}{\sqrt{1 + \sin(x^3)}}$$

$$= \frac{3}{2} \frac{x^2 \cos(x^3)}{\sqrt{1 + \sin(x^3)}}$$

d)

$$h(x) = \frac{x^2 + 1}{x + 2} = \frac{u(x)}{v(x)}$$

$$h'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$$

$$= \frac{2x(x+2) - (x^2+1)}{(x+2)^2}$$

$$= \frac{2x^2 + 4x - x^2 - 1}{(x+2)^2} = \frac{x^2 + 4x - 1}{(x+2)^2}$$

Rätsel

$$s_J = 400 \text{ m}, \quad v_H = 3v_J$$

$$s_J = v_J \cdot t_J \Leftrightarrow t_J = \frac{s_J}{v_J}$$

$$\Rightarrow s_H = v_H \cdot t_J = v_H \cdot \frac{s_J}{v_J} = 3v_J \cdot \frac{s_J}{v_J} = 3s_J$$

$$= 1200 \text{ m}$$