# Lösungen: Vorkurs Zettel 6

## Aufgabe 1

### a)

$$(3+4i)+(2-3i) = 3+2+(4-3)i = 5+i$$

### b)

$$i+(0.5-2i) = 0.5+(1-2)i = 0.5-i$$

### c)

$$(0.3-4i)-(1-2i) = 0.3-1+(-4+2)i = -0.7-2i$$

### d)

$$(1+i)-(4-i) = 1-4+(1+1)i = -3+2i$$

### e)

$$(3+4i)(2-3i) = 6-9i+8i-12i^2 = 6+12-i = 18-i$$

### f)

$$(2-3i)(2.5+3i) = 5+6i-7.5i-9i^2 = 5+9-1.5i = 14-1.5i$$

### g)

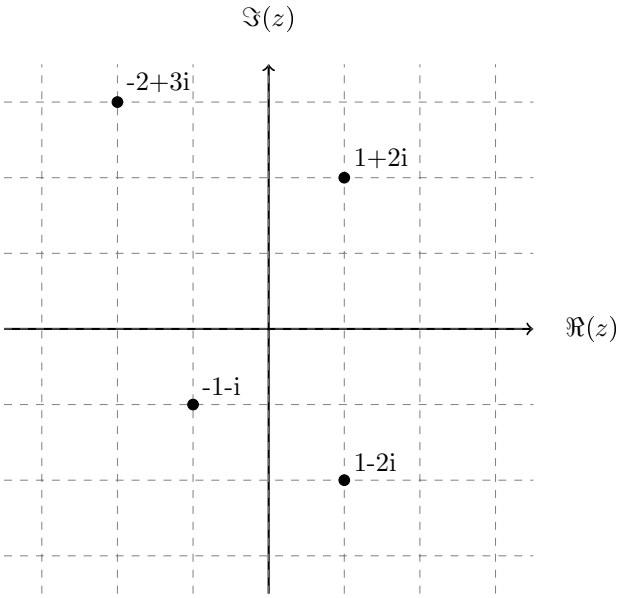
$$\frac{1}{1-3i} = \frac{1+3i}{(1-3i)(1+3i)} = \frac{1+3i}{10}$$

### h)

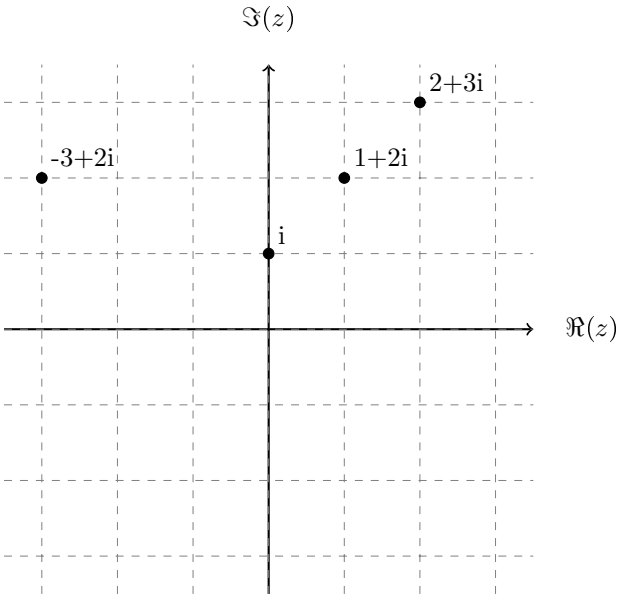
$$\frac{2+2i}{1+2i} = \frac{(2+2i)(1-2i)}{(1+2i)(1-2i)} = \frac{2-4i+2i-4i^2}{5} = \frac{6-2i}{5}$$

## Aufgabe 2

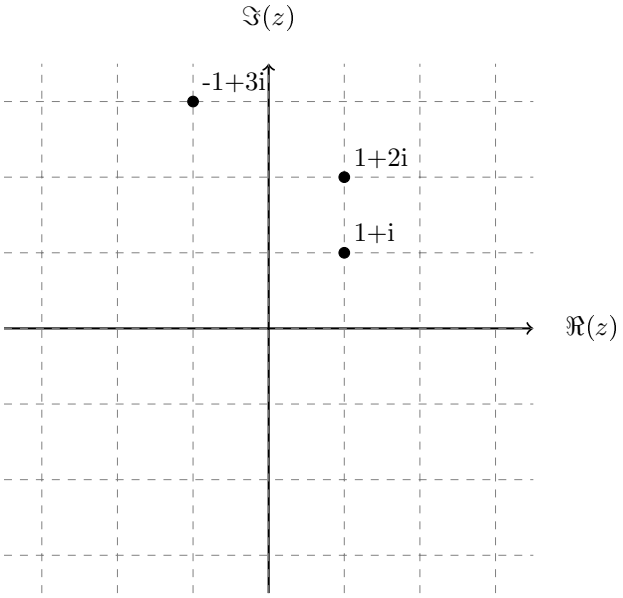
### a)




### b)



## c)




## Aufgabe 3

$$\begin{aligned} &\text{Ges.:} \quad z = \sqrt{-i} \\ &\Rightarrow \quad z^2 = -i = a^2 + 2iab - b^2 \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad a^2 - b^2 = 0 \quad &\wedge \quad 2iab = -i \\ \Leftrightarrow \quad a^2 = b^2 \quad &\wedge \quad ab = -\frac{1}{2} \\ \Rightarrow \quad a = \pm b \quad &\Rightarrow \quad \pm b^2 = -\frac{1}{2} \end{aligned}$$

1. Fall:

$$\begin{aligned} b^2 &= -\frac{1}{2} \\ \Rightarrow \quad b &= \pm\frac{i}{\sqrt{2}} \\ \Rightarrow \quad a &= \frac{i}{\sqrt{2}} \\ \Rightarrow \quad z_{1/2} &= \mp\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \end{aligned}$$

2.Fall:

$$\begin{aligned} -b^2 &= -\frac{1}{2} \\ \Rightarrow \quad b &= \pm\frac{1}{\sqrt{2}} \\ \Rightarrow \quad a &= \frac{1}{\sqrt{2}} \\ \Rightarrow \quad z_{3/4} &= \frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}} \end{aligned}$$

Test:

$$\begin{aligned} z_1^2 = -i, \quad z_2^2 = z_3^2 = i, \quad z_4^2 = -i \\ \Rightarrow \quad z_{1/2} = \mp\frac{1}{\sqrt{2}} \pm \frac{i}{\sqrt{2}} \end{aligned}$$

## Aufgabe 4

$$R_\Omega := R, \quad R_L := i\omega L, \quad \in \mathbb{R}_C := \frac{1}{i\omega C}$$

- $$\begin{aligned} R_G &= R_\Omega + R_L + R_C \\ &= R + i\left(\omega L + \frac{1}{i^2\omega C}\right) \\ &= R + i\left(\omega L - \frac{1}{\omega C}\right) \end{aligned}$$

  Diese Gleichung repräsentiert den Gesamtwiderstand einer Reihenschaltung von Ohm'schen Widerstand, Spule und Kondensator.

- $$\begin{aligned} \frac{1}{R_G} &= \frac{1}{R_\Omega} + \frac{1}{R_L} + \frac{1}{R_C} \\ &= \frac{R_L R_C}{R_\Omega R_L R_C} + \frac{R_\Omega R_C}{R_\Omega R_L R_C} + \frac{R_\Omega R_L}{R_\Omega R_L R_C} \\ &= \frac{R_L R_C + R_\Omega R_C + R_\Omega R_L}{R_\Omega R_L R_C} \\ \Leftrightarrow \quad R_G &= \frac{R_\Omega R_L R_C}{R_L R_C + R_\Omega R_C + R_\Omega R_L} \\ &= \frac{Ri\omega L\frac{1}{i\omega C}}{\frac{i\omega L}{i\omega C} + \frac{R}{i\omega C} + Ri\omega L} \\ &= R\frac{L}{C}\frac{1}{\frac{L}{C} + Ri\left(\omega L - \frac{1}{\omega C}\right)} \\ &= \frac{1}{\frac{1}{R} + i\left(\frac{\omega}{C} - \frac{L}{\omega}\right)} \\ &= \frac{\frac{1}{R} - i\left(\frac{\omega}{C} - \frac{L}{\omega}\right)}{\frac{1}{R^2} + \left(\frac{\omega}{C} - \frac{L}{\omega}\right)^2} \end{aligned}$$

  Diese Gleichung repräsentiert den Gesamtwiderstand in einer Parallelschaltung von Ohm'schen Widerstand, Spule und Kondensator.

## Zusatzaufgabe

$$\begin{aligned} & z = \sqrt[3]{-27} = 3 \cdot \sqrt[3]{-1} =: 3w \\ \Rightarrow \quad & w^3 = -1 \end{aligned}$$

$$\begin{aligned} & w^3 = (a + i\,b)^3 = a^3 + 3ia^2b - 3ab^2 - ib^3 = -1 \\ \Rightarrow \quad & a^3 - 3ab^2 = -1 \quad \wedge \quad 3a^2b - b^3 = 0 \end{aligned}$$

1. Fall: $b = 0$: Aus $b = 0$ folgt sofort $a = w_1 = -1$.
2. Fall: $b \neq 0$:

$$\begin{aligned} & b^3 = 3a^2b \\ \Leftrightarrow \quad & b^2 = 3a^2 \\ \Rightarrow \quad & b = \sqrt{3}a \\ \Rightarrow \quad & -1 = a^3 - 9a^3 \\ \Leftrightarrow \quad & -1 = -8a^3 \\ \Leftrightarrow \quad & a = \frac{1}{2} \quad \wedge \quad b = \pm\frac{\sqrt{3}}{2} \end{aligned}$$

$$\Rightarrow \quad w_1 = -1, \quad w_{2/3} = \frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

Daraus folgt sofort:

$$z_1 = -3, \quad z_{2/3} = \frac{3}{2} \pm \frac{3\sqrt{3}i}{2}$$