# Lösungen: Vorkurs Zettel 7

## Additionstheoreme die II

Es gilt:

$$e^{ix} = \cos(x) + i\sin(x).$$

Außerdem:

$$e^{i(x+x')} = e^{ix}e^{ix'}.$$

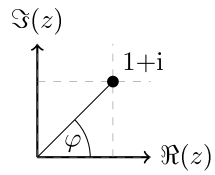
Daraus folgt:

$$\begin{aligned}
& \cos(x+x') + i\sin(x+x') = e^{i(x+x')} = e^{ix}e^{ix'} = (\cos(x) + i\sin(x))\big(\cos(x') + i\sin(x')\big) \\
\Leftrightarrow \quad & \cos(x+x') + i\sin(x+x') = \cos(x)\cos(x') + i\cos(x)\sin(x') + i\sin(x)\cos(x') - \sin(x)\sin(x') \\
& \qquad = \cos(x)\cos(x') - \sin(x)\sin(x') + i\big(\cos(x)\sin(x') + \cos(x')\sin(x)\big)\,.
\end{aligned}$$

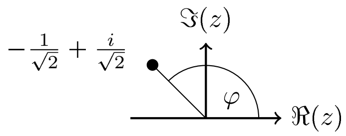
Vergleich von Real- und Imaginärteil liefert die Behauptung.

## Aufgabe 1

## a)




$$\begin{aligned}
& |1+i| = \sqrt{2} \quad \Rightarrow \quad \varphi = \sin^{-1}(\frac{1}{\sqrt{2}}) = \frac{\pi}{4} \\
\Rightarrow \quad & 1 + i = \sqrt{2}e^{i\frac{\pi}{4}}
\end{aligned}$$




$$\begin{aligned}
& \left|-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right| = 1 \quad \Rightarrow \quad \varphi = \pi - \sin^{-1}(\frac{1}{\sqrt{2}}) = \frac{3\pi}{4} \\
\Rightarrow \quad & -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = e^{i\frac{3\pi}{4}}
\end{aligned}$$

### b)

$$3e^{i\frac{2\pi}{3}}\frac{1}{2}e^{i\frac{\pi}{2}} = \frac{3}{2}e^{i(\frac{2\pi}{3}+\frac{\pi}{2})} = \frac{3}{2}e^{i\frac{7\pi}{6}}$$

$$e^{i\frac{\pi}{3}}\frac{3}{5}e^{-i\frac{\pi}{4}} = \frac{3}{2}e^{i\frac{\pi}{12}}$$

## Aufgabe 2

### a)

$$\begin{aligned}
\int_0^4 \mathrm{d}x\,(5-3|x-2|) &= 5\int_0^4 \mathrm{d}x - 3\int_0^4 \mathrm{d}x\,(x-2) \\
&= 20 - 3\left[\int_0^2 (2-x) + \int_2^4 (x-2)\right] \\
&= 20 - 3\left[\int_{-2}^0 \mathrm{d}x\,(2-(x+2)) + \int_0^2 \mathrm{d}x\,(x+2-2)\right] \\
&= 20 - 3\left[\int_{-2}^0 \mathrm{d}x\,(-x) + \int_0^2 \mathrm{d}x\,x\right] \\
&= 20 - 3\left[\int_0^{-2} \mathrm{d}x\,x + \int_0^2 \mathrm{d}x\,x\right] \\
&= 20 - 3\left[\frac{(-2)^2}{2} + \frac{2^2}{2}\right] \\
&= 8
\end{aligned}$$

### b)

$$\begin{aligned}
&\int_0^3 \mathrm{d}x\,\left(1 + \log\left(\frac{\sqrt{1+x^2}+1}{x}\right) + \log\left(\frac{\sqrt{1+x^2}-1}{x}\right)\right) \\
&= \int_0^3 \mathrm{d}x + \int_0^3 \mathrm{d}x\,\log\left(\frac{\left(\sqrt{1+x^2}+1\right)\left(\sqrt{1+x^2}-1\right)}{x^2}\right) \\
&= 3 + \int_0^3 \mathrm{d}x\,\log\left(\frac{1+x^2-1}{x^2}\right) \\
&= 3 + \int_0^3 \mathrm{d}x\,\log(1) \\
&= 3
\end{aligned}$$

## Rätselaufgabe

Zwei mögliche Interpretationen:

$$\int_0^1 ddd = \begin{cases} \int_0^1 \mathrm{d}dd\ = dd\big|_0^1 = 1, \\ \int_0^1 d\,\mathrm{d}d\ = \frac{d^2}{2}\big|_0^1 = \frac{1}{2} \end{cases}$$