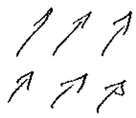
8. Spontaneous Symmetry breaking

- simple examples: – a rod bent under a force

– ground state of a Ferromagnet

$$H = -\sum_{i,j} S_i J_{ij} S_j$$

inv under rotation $O(3)$



$T<T_c$ spins aligned

$T>T_c$ no order

- the same effect can occur in QM or QFT

→ there is an important difference between spontaneous breaking of a <u>global</u> or <u>local</u> symmetry

<u>Example 1)</u> complex scalar field & global symmetry

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \equiv \partial_\mu \phi \partial^\mu \phi^* - V(\phi, \phi^*)$$

- global symmetry $\phi \to e^{i\alpha}\phi$, α const $U(1)$ ($\sim SO(2)$) of kinetic & potential term
- the vacuum satisfies $\langle 0 | \phi(x) | 0 \rangle = 0$

this corresponds to the minimum of the class. potential

(1) $V'(\phi) = m^2 \phi^* + 2\lambda \phi^* (\phi^* \phi) = 0$ has solution $\phi = \phi^* = 0$

<u>Symmetry breaking</u>:

for $m^2 < 0$ (tachyon!) there is a second solution to (1)

$$|\phi|^2 = -\frac{m^2}{2\lambda} \equiv a^2 > 0$$

– in the quantum theory this corresponds to

$$|\langle 0|\phi|0\rangle|^2 = a^2 \neq 0$$ being a non-perturbative effect! (Think of $\phi(x)$ in terms of $a_k, a_k^\dagger$)

the lowest class. pot. energy is not at $\phi = 0$

* we can attribute the non-zero vev to the radial part of $\phi(x) = r(x)e^{i\theta(x)}$ or to one of it's components, $\phi(x) = \tilde{\phi}_1(x) + i\tilde{\phi}_2(x)$ (or any comb.)

say $\phi(x) = a + \frac{\phi_1(x) + i\phi_2(x)}{\sqrt{2}}$ with $\langle 0|\phi_i|0\rangle = 0$

we obtain for $V(\phi,\phi^*) = m^2\left(\left(a+\frac{\phi_1}{\sqrt{2}}\right)^2 + \frac{\phi_2^2}{2}\right)$

$+\lambda\left(a^2 + \sqrt{2}a\phi_1 + \frac{\phi_1^2}{2} + \frac{\phi_2^2}{2}\right)^2$

with $-m^2 = 2\lambda a^2$

$= -2\lambda a^2\left(a^2 + \sqrt{2}a\phi_1 + \frac{1}{2}(\phi_1^2+\phi_2^2)\right)$

$+\lambda\left[a^4 + 2a^2\phi_1^2 + \frac{1}{4}(\phi_1^2+\phi_2^2)^2 + 2a^2\sqrt{2}a\phi_1 + \frac{2a^2}{2}(\phi_1^2+\phi_2^2) + 2\sqrt{2}a\phi_1\frac{1}{2}(\phi_1^2+\phi_2^2)\right]$

$\Rightarrow$ up to constant terms we have

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\phi_1\partial^\mu\phi_1 + \partial_\mu\phi_2\partial^\mu\phi_2\right) - \underbrace{2a^2\lambda\phi_1^2}_{\text{new mass for } \phi_1} - \underbrace{\sqrt{2}a\lambda\phi_1(\phi_1^2+\phi_2^2)}_{\text{new interaction}} - \underbrace{\frac{\lambda}{4}(\phi_1^2+\phi_2^2)^2}_{\text{old interaction}}$$

so ϕ_1 has a standard mass $2a^2\lambda$
ϕ_2 becomes massless

after symmetry breaking

(classically it costs no energy to move in the minimum of the potential) example 2) complex scalar + U(1) gauge field & local symmetry

$$\mathcal{L} = (\partial_\mu + ie A_\mu)\phi\,(\partial^\mu - ie A^\mu)\phi^* - V(\phi,\phi^*) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

which has a local symmetry
$$\boxed{\begin{array}{l}\phi \to e^{i\alpha(x)}\phi = e^{i\alpha(x)}\left(a + \frac{\phi_1}{\sqrt{2}} + i\frac{\phi_2}{\sqrt{2}}\right)\\ A_\mu \to A_\mu - e^{-1}\partial_\mu \alpha(x)\end{array}}$$

- the same analysis of V for $m^2 < 0$ yields extra terms due to the covariant derivative coupling ϕ & A_μ:

$\boxed{\phi(x) = a + \frac{\phi_1}{\sqrt{2}} + i\frac{\phi_2}{\sqrt{2}}}$ gives

mass term for A_μ !?

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + e^2 A_\mu A^\mu\left(a^2 + \sqrt{2}a\phi_1 + \frac{1}{2}(\phi_1^2 + \phi_2^2)\right)$$

$$+ \underbrace{\sqrt{2}\,ea A_\mu\partial^\mu\phi_2} + \sqrt{2}\,eA_\mu\left(\partial_\mu\phi_2\,\phi_1 - \phi_2\partial_\mu\phi_1\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi,\phi^*)$$

has: unusual term: trafo $A_\mu \leftrightarrow \phi_2$ $\Rightarrow$ ϕ_2 unphysical ?!

under infinitesimal gauge trafo
$$\phi_1 \to \phi_1 - \alpha(x)\phi_2$$
$$\phi_2 \to \phi_2 + \alpha(x)\phi_1 + a\alpha(x)$$
inhom trafo as gauge field!

$\Rightarrow$ choose $\alpha(x)$ to make $\phi_2 \equiv 0$
unitary gauge

	symm.	broken symm.	
global:	2 massive scalar $\phi_{1,2}$	1 mass. ϕ_1 + 1 massless ϕ_2	Goldstone phenom.
local:	2 massive scalar $\phi_{1,2}$ + 1 massless photon (2 phys. d.o.f.)	1 massive ϕ_1 + NO ϕ_0 + 1 massive photon (3 phys. d.o.f.)	Higgs phenom.

"photon eats 1 scalar field"

$\rightsquigarrow$ mechanism to provide mass to Z- and W- bosons in standard model

Partial symmetry breaking: [see [R] ch 8.2-3]

- global: same example, with 3 real $\phi_{i=1,2,3}$, $\phi\phi^* = \phi_1^2+\phi_2^2 \to \sum_i \phi_i^2$

$m^2<0$: min of V at $|\phi| = |\phi_1^2+\phi_2^2+\phi_3^2|^{\frac{1}{2}} = a$ $\Rightarrow$ symmetry $SO(3)$

$\rightsquigarrow$ give vev $\neq 0$ to 1 component, say $\phi_3 = \chi + a$

$\rightsquigarrow$ same algebra: $\boxed{m_\chi^2 = 8a^2\lambda}$, $\boxed{m_{\phi_{1,2}} = 0}$ 2 massless ϕ

$\exists$ residual symmetry: rotations among (ϕ_1, ϕ_2) $SO(2)$ (a U(1))

general counting: Goldstone Theorem

G global sym. group before breaking, H subgroup after breaking

$\Rightarrow$ # of massless scalars (Goldstone bosons) = dim G − dim H

$\left(= \dim \left[G/H \right] \right)$

here $3-1=2$

- local: as above 3 scalars + 3 $A_\mu^{i=1,2,3}$ (non-Abelian Higgs phase)

$\longrightarrow$ 1 massive ϕ + 2 massive A_μ^i + 1 massless $\tilde{A}_\mu^i$
(these ate the 2 Goldstones)

* the field content after symmetry breaking is not always obvious after symmetry breaking:

$\longrightarrow$ example chiral symmetry in QCD

- define a helicity basis for quarks $\psi_{\substack{R\\L}} \equiv \underbrace{\frac{1}{2}(1 \pm \gamma_5)}\psi$

(good basis for masses = 0!)

projection operators $\equiv P_{\substack{R\\L}}$ with $P_{\substack{R\\L}}^2 = P_{\substack{R\\L}}$

$P_R P_L = P_L P_R = 0$

$P_R + P_L = \mathbb{1}$

Euclidean $\gamma_5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} = \gamma_5^+$

- the projected 2spinors $\Psi_{R,L}$ are also called Weyl spinors

$$\Psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \quad P_R = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \quad P_L = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} \Rightarrow \psi_R = \begin{pmatrix}\psi_1\\ 0\end{pmatrix}, \ \psi_L = \begin{pmatrix}0\\ \psi_2\end{pmatrix}$$

$$\bar\Psi = \Psi^\dagger \gamma_0 = (\psi_1^\dagger, \psi_2^\dagger)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}\psi_2^\dagger\\ \psi_1^\dagger\end{pmatrix} \Rightarrow \bar\Psi_R = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\bar\Psi = \begin{pmatrix}0\\ \psi_1^\dagger\end{pmatrix}$$

$$\bar\Psi_L = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\bar\Psi = \begin{pmatrix}\psi_2^\dagger\\ 0\end{pmatrix}$$

$$\Rightarrow \underline{\bar\Psi\Psi = \bar\Psi_L\Psi_R + \bar\Psi_R\Psi_L}$$

but $\bar\Psi\gamma_\mu\Psi = \bar\Psi\gamma_\mu(P_R\psi_R + P_L\psi_L) \overset{\{\gamma_5,\gamma_\mu\}=0}{=} \bar\Psi P_L\gamma_\mu\psi_R + \bar\Psi P_R\gamma_\mu\psi_L$

$$= \bar\Psi_R\gamma_\mu\psi_R + \bar\Psi_L\gamma_\mu\psi_L$$

- for a set of N_f flavours $\Psi = \begin{pmatrix}\psi_u\\ \psi_d\\ \psi_s\\ \vdots\end{pmatrix}$ we can do $\underline{\underline{2}}$ indep rotations

$$\boxed{U(N_f)_L \times U(N_f)_R}$$

- in $\mathcal{L}$ the $\underline{\text{kinetic term}}$ $\bar\Psi\gamma_\mu\partial_\mu\Psi = \bar\Psi_L\gamma_\mu\partial_\mu\Psi_L + \bar\Psi_R\gamma_\mu\partial_\mu\Psi_R$ is invariant

- a vev $\neq \langle 0|\bar\Psi\Psi|0\rangle = \langle 0|\bar\Psi_L\Psi_R + \bar\Psi_R\Psi_L|0\rangle$

spont. breaks the symmetry to the diagonal $U(N_f)$ for L and R simultaneously

- the mass term $\underline{\text{explicitly}}$ breaks sym. the same way for deg masses

((the full story is $U_V(1)\times \underset{\uparrow \text{broken by anomaly}}{U_A(1)} \times SU(N_f)_L \times SU(N_f)_R \rightarrow U_V(1)\times SU(N_f)_V$))

Q: $\underline{\text{What are the relevant d.o.f. after breaking}}$?

$N_f = 2$: 3 Pions : $\dim SU(2)_L \times SU(2)_R - \dim SU(2)_V = 6 - 3 = 3$

π^+, π^-, π^0

these are composite fields (Mesons) $\rightarrow$ chiral perturbation theory (see e.g. StrongNet school)