Quantum Field Theory	**Exercises No. 1**

[Wednesday 06. April 2010, 12:30-14:00]

Exercise 1.1: Consider a generic Lagrange density $\mathcal{L}(\phi, \partial_\mu \phi)$ that is not explicitly x- dependent.

(a) Verify that $\mathcal{L}$ is invariant under the substitution

$$x^\mu \to x^{\mu\prime} = x^\mu + \omega^\mu \ , \quad \phi(x) \to \phi'(x') = \phi(x) \tag{1}$$

where ω^μ is x-independent.

(b) What are the corresponding Noether currents j^μ_ν?

(c) In this case we denote by $j^\mu_\nu \equiv -T^\mu{}_\nu$. Show that $T^{00} = \mathcal{H}$.

Exercise 1.2: Let $\partial_\mu j^\mu_i = 0$. Under which conditions is the "charge" $Q_i \equiv \int_V \mathrm{d}^3\vec{x}\, j^0_i$ a conserved quantity?

Exercise 1.3: Consider the Lorentz transformation $x^\mu \to x^{\mu\prime} \equiv \Lambda^\mu{}_\nu x^\nu$, where $\eta^{\alpha\beta}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta^{\mu\nu}$. We assume that field ϕ remains invariant: $\phi(x) \to \phi'(x') \equiv \phi(x)$.

(a) Show that $S = \int \mathrm{d}^4x\, \mathcal{L}$ with $\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$ remains invariant under this Lorentz transformation.

(b) Demonstrate that an infinitesimal $\Lambda^\mu{}_\alpha$ can be written in the form $\Lambda^\mu{}_\alpha = \delta^\mu{}_\alpha + \epsilon^\mu{}_\alpha$, where $\epsilon^{\mu\nu} = -\epsilon^{\nu\mu}$.

(c) What are the corresponding Noether currents?

Exercise 1.4: Let us consider the following Lagrange density for the two fields $\phi_{1,2}$

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi_a\partial_\mu\phi_a - V(\phi_a^2) \ , \tag{2}$$

(summation convention!) and the substitution

$$\delta\phi_1 \equiv -\phi_2\delta\omega \ , \quad \delta\phi_2 \equiv \phi_1\delta\omega \ . \tag{3}$$

Verify that this is a symmetry. What is the Noether current?