Quantum Field Theory	**Exercises No. 3**

[Wednesday 20. April 2010, 12:30-14:00 in D5-153]

Exercise 3.1: Show that the following holds:

$$\text{(a)} \qquad \int_{-\infty}^{\infty} \frac{\mathrm{d}\tilde{k}}{2\pi} \frac{e^{i\tilde{k}\tau}}{\tilde{k}^2 + E^2} = \frac{1}{2E}\Big[\theta(-\tau)e^{E\tau} + \theta(\tau)e^{-E\tau}\Big] \ ,$$

$$\text{(b)} \qquad \lim_{\epsilon\to 0^+} \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} \frac{e^{ikt}}{k^2 - E^2 + i\epsilon} = \frac{-i}{2E}\Big[\theta(-t)e^{iEt} + \theta(t)e^{-iEt}\Big] \ .$$

Hint: use Cauchy's integral formula.

Exercise 3.2: The Fourier transformation of the Schwinger propagator is given by

$$\tilde{G}_E(\tilde{p}^0;\vec{p}) = \frac{1}{(\tilde{p}^0)^2 + E_{\vec{p}}^2} \ ,$$

and the spectral function is defined as

$$\tilde{\rho}(p^0;\vec{p}) = \frac{\pi}{2E_{\vec{p}}}\Big[\delta(p^0 - E_{\vec{p}}) - \delta(p^0 + E_{\vec{p}})\Big] \ .$$

Demonstrate that the following relation holds:

$$\tilde{\rho}(p^0,\vec{p}) = \mathrm{Im}\, \tilde{G}_E(\tilde{p}^0 \to -i(p^0 + i0^+);\vec{p}) \ .$$

Hint: recall different definitions of the delta-function.

Exercise 3.3: Let $\hat{U}_I(t,t_0)$ be the time evolution operator as defined in lecture.

(a) Using its definition express $\hat{U}_I(t,t_0)$ in terms of the following operators

$$e^{i\hat{H}t}, \ e^{i\hat{H}t_0}, \ e^{i\hat{H}_0 t}, \ e^{i\hat{H}_0 t_0}$$

.

(b) Demonstrate that for any times t_1, t_2 it holds $\hat{U}_I^\dagger(t_1,t_2) = \hat{U}_I(t_2,t_1)$.

(c) Show that $\hat{U}_I(t_1,t_2)\hat{U}_I(t_2,t_3) = \hat{U}_I(t_1,t_3)$ holds for $t_1 \geq t_2 \geq t_3$.

(d) Show that $\hat{U}_I(t_1,t_3)\hat{U}_I^\dagger(t_2,t_3) = \hat{U}_I(t_1,t_2)$ holds for $t_1 \geq t_2 \geq t_3$.

(e) Using these results show that the scattering matrix $\hat{S}$ defined in lecture is unitary

$$\hat{S}^\dagger\hat{S} = \hat{S}\hat{S}^\dagger = \mathbb{1} \ .$$

Exercise 3.4: Write down the connected part of the 4-point function $G_{T,C}^{(4)}(x_1,x_2,x_3,x_4)$.