Quantum Field Theory **Exercises No. 4**

[Wednesday 4. May 2010, 12:30-14:00 in D5-153]

Exercise 4.1:

(a) Consider the definition

$$(2\pi)^4\delta^{(4)}(p_1+...+p_n)\tilde{G}^{(n)}_{T,\mathsf{c}}(p_1,...,p_n) \equiv \int \mathrm{d}^4x_1...\mathrm{d}^4x_n\, G^{(n)}_{T,\mathsf{c}}(x_1,...,x_n)e^{i(p_1\cdot x_1+...+p_n\cdot x_n)} \ .$$

To which property of $G^{(n)}_{T,\mathsf{c}}$ corresponds the presence of the delta-function in front of $\tilde{G}^{(n)}_{T,\mathsf{c}}$?

(b) In general the Green fuction $G^{(n)}_T$ contains also disconnected parts, i.e. $G^{(n)}_T = ... + G^{(m_1)}_T G^{(m_2)}_T$, with $m_1 + m_2 = n$.
How do these parts behave inside the Fourier tranform $\tilde{G}^{(n)}_T$?

Exercise 4.2:

(a) Verify explicitly that the following holds:

$$:\hat{\phi}_I(x_1)\hat{\phi}_I(x_2): \, = \, :\hat{\phi}_I(x_2)\hat{\phi}_I(x_1): \ .$$

(b) Show that within normal ordering one can exchange any two field operators:

$$:...\hat{\phi}_I(x_i)...\hat{\phi}_I(x_j)...: \, = \, :...\hat{\phi}_I(x_j)...\hat{\phi}_I(x_i)...: \ .$$

Exercise 4.3: Check Wick's Theorem to third order:

$$\begin{aligned} T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\hat{\phi}_I(x_3)\} \ = \ & :\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\hat{\phi}_I(x_3): \\ & + :\hat{\phi}_I(x_1): \langle 0|T\{\hat{\phi}_I(x_2)\hat{\phi}_I(x_3)\}|0\rangle \\ & + :\hat{\phi}_I(x_2): \langle 0|T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_3)\}|0\rangle \\ & + :\hat{\phi}_I(x_3): \langle 0|T\{\hat{\phi}_I(x_1)\hat{\phi}_I(x_2)\}|0\rangle \ . \end{aligned}$$

Exercise 4.4: Let $\mathcal{L}_{\mathsf{int}} = -\frac{1}{3!}\lambda\phi^3$. Determine $G^{(3)}_{T,C}(x_1,x_2,x_3)$ up to including order $\mathcal{O}(\lambda^1)$ by counting all diagrams as in lecture. Which diagrams are connected? Check that you counted all contractions.