Quantum Field Theory	**Exercises No. 5**

[Wednesday 11. May 2010, 12:30-14:00 in D5-153]

Exercise 5.1: Let

$$G_E^{(n)}(\tau_1, ..., \tau_n) \equiv \langle 0|T\{\hat{x}_H(\tau_1) \dots \hat{x}_H(\tau_n)\}|0\rangle \; ,$$
$$G_\beta^{(n)}(\tau_1, ..., \tau_n) \equiv \frac{\mathrm{Sp}\,[e^{-\beta\hat{H}} T\{\hat{x}_H(\tau_1) \dots \hat{x}_H(\tau_n)\}]}{\mathrm{Sp}\,[e^{-\beta\hat{H}}]} \; ,$$

with $0 \le \tau_1, ..., \tau_n \le \beta$. Show that it holds: $\lim_{\beta\to\infty} G_\beta^{(n)}(\tau_1, ..., \tau_n) = G_E^{(n)}(\tau_1, ..., \tau_n)$.

Exercise 5.2: Starting from

$$e^{W(J)} \equiv \int \mathrm{d}\vec{v} \, \exp\Big[-\frac{1}{2} v_i A_{ij} v_j + J_i v_i\Big] = e^{W(0)} \exp\Big[\frac{1}{2} J_i A_{ij}^{-1} J_j\Big] \; ,$$

compute

$$\langle v_m v_n v_o v_p\rangle_0 \equiv \frac{\int \mathrm{d}\vec{v}\; v_m v_n v_o v_p \; \exp\Big[-\frac{1}{2} v_i A_{ij} v_j\Big]}{\int \mathrm{d}\vec{v} \, \exp\Big[-\frac{1}{2} v_i A_{ij} v_j\Big]} \; .$$

Exercise 5.3: Given the integration measure is defined as

$$\int \mathrm{d}\vec{v} \; \equiv \int_{-\infty}^{\infty} \Big[\prod_i \frac{\mathrm{d}v_i}{\sqrt{2\pi}}\Big]$$

what do you obtain for $e^{W(0)}$? [answer: $e^{W(0)} = (\det A)^{-1/2}$].

Exercise 5.4: Consider the four-volume $V = L_0 L_1 L_2 L_3$ with periodic boundary conditions as in lecture.

(a) Show that it holds: $\lim_{V\to\infty} \frac{1}{V} \sum_P \tilde{f}(P) = \int \frac{\mathrm{d}^4 P}{(2\pi)^4} \tilde{f}(P)$.

(b) We define a delta-functions $\bar{\delta}(P)$ through the condition

$$\int_P \bar{\delta}(P - Q) \tilde{f}(P) = \tilde{f}(Q) \; ,$$

with $\int_P = \frac{1}{V} \sum_P$ or $\int_P = \int \frac{\mathrm{d}^4 P}{(2\pi)^4}$. Write down $\bar{\delta}(P)$ for both cases.

(c) Show that the equation $\int_V \mathrm{d}^4 x \, e^{iP\cdot x} = \bar{\delta}(P)$ is valid both for finite and infinite V.