Quantum Field Theory	**Exercises No. 6**

[Wednesday 18. May 2010, 12:30-14:00 in D5-153]

Exercise 6.1:

Show for $n = 2, 4$ that $W[J]$ generates the connected n-point Green functions (use ex. 3.4).

Exercise 6.2:

In lecture we have defined $Z[J]$, $W[J] = \ln Z[J]$, and $\Gamma[\varphi] = W[J] - \int \mathrm{d}^4x\, \varphi(x)J(x)$, where $\varphi(x) = \delta W[J]/\delta J(x)$. Starting from the Schwinger-Dyson equation for $Z[J]$,

$$0 = \left[-\mathcal{L}'_E\left(\frac{\delta}{\delta J(x)}\right) + J(x) \right] Z[J] \; ,$$

check that the following eqs. hold for $W[J]$ and $\Gamma[\varphi]$:

(a) $$\mathcal{L}'_E\left(\frac{\delta W[J]}{\delta J(x)} + \frac{\delta}{\delta J(x)}\right) = J(x) \; ,$$

(b) $$\mathcal{L}'_E\left(\varphi(x) + \int \mathrm{d}^4y\, D[\varphi](x,y)\frac{\delta}{\delta\varphi(y)}\right) = -\frac{\delta\Gamma[\varphi]}{\delta\varphi(x)} \; ,$$

Here $D[\varphi](x,y) \equiv \delta^2 W[J]/\delta J(x)\delta J(y)$ [use $\frac{\delta}{\delta J(x)}\mathbf{1} = 0$].

Exercise 6.3:

Show that the propagator

$$\Delta(x-y) = \int \frac{\mathrm{d}^4P}{(2\pi)^4}\frac{e^{iP\cdot(x-y)}}{P^2+m^2}$$

is finite for $x \neq y$. How does the function behave for $|x-y| \to \infty$? [Hint: choose the right basis for P and use complex analysis.]

Exercise 6.4:

Consider $\mathcal{L}_E \equiv \frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi^3$ from lecture. Write down the Schwinger-Dyson equations in a graphical way, up to including all $\mathcal{O}(g^2)$ terms. Indicate the order of each term in the expansion.