Quantum Field Theory	**Exercises No. 7**

[Wednesday 25. May 2010, 12:30-14:00 in D5-153]

Exercise 7.1:

(a) Determine the following integral

$$I(m^2; d, A) \equiv \int \frac{\mathrm{d}^d P}{(2\pi)^d} \frac{1}{(P^2 + m^2)^A}$$

in dimensional regularisation [hint: use the Euler-Beta function].

(b) Please give $I(m^2; 4 - 2\epsilon, 1)$ for $\epsilon \ll 1$.

Exercise 7.2:

Show that

$$\int dz G_E^{(2)}(x, z) \Gamma_E^{(2)}(z, y) = -\delta^{(4)}(x - y)$$

holds, relating the two-point Green function and the two-point vertex function $\Gamma_E^{(2)}(z, y)$ defined in lecture. Also show that this translates into Fourier space as

$$\tilde{G}_E(p) \tilde{\Gamma}_E(p) = -1.$$

Exercise 7.3:

Many loop integrations can be simplified in the following so-called Feynman parametrisation. Please check that this identity holds:

$$\frac{1}{ab} = \int_0^1 \frac{\mathrm{d}t}{[at + b(1-t)]^2} \ , \quad a, b > 0 \ .$$

Exercise 7.4:

The equation for the β-function is given to leading order by

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \lambda_R = \frac{3}{(4\pi)^2} \lambda_R^2 \ .$$

Verify that

$$\lambda_R(\mu) = \frac{(4\pi)^2}{3 \ln(\mu_0/\mu)}$$

solves this equation, where μ_0 is a (dimensionful) constant.