Quantum Field Theory **Exercises No. 8**

[Wednesday 1. June 2010, 12:30-14:00 in D5-153]

Exercise 8.1: Let us define a Fock-space for Fermions in analogy to Bosons:

$$|\vec{k}_1...\vec{k}_n; s_1...s_n\rangle = \hat{a}^{\dagger(s_1)}_{\vec{k}_1} \dots \hat{a}^{\dagger(s_n)}_{\vec{k}_n}|0\rangle \ .$$

Please check the Pauli-principle:

$$|\vec{k}_1...\vec{k}_i...\vec{k}_j...\vec{k}_n; s_1...s_i...s_j...s_n\rangle = -|\vec{k}_1...\vec{k}_j...\vec{k}_i...\vec{k}_n; s_1...s_j...s_i...s_n\rangle \ .$$

Compute $\langle \vec{p}_1, \vec{p}_2; t_1, t_2 | \vec{k}_1, \vec{k}_2; s_1, s_2\rangle$ and compare to the bosonic result from ex. 2.4.

Exercise 8.2: Verify that the anti-commutation relations for creation and anihilation operators given in lecture imply the following equal time canonical anti-commutation relations

$$\left\{\hat{\psi}_\alpha(x^0, \vec{x}), \hat{\Pi}_\beta(x^0, \vec{y})\right\} = i\delta^{(3)}(\vec{x} - \vec{y})\delta_{\alpha\beta} \ , \quad \alpha, \beta = 1, ..., 4 \ ,$$

where $\hat{\Pi}_\beta \equiv i\hat{\psi}^\dagger_\beta = i\hat{\bar{\psi}}_\alpha \gamma^0_{\alpha\beta}$ [hint: use the completeness relation for the spinors $u_\alpha(\vec{p}, s)$, $v_\beta(\vec{p}, s)$, and substitute $\vec{p} \to -\vec{p}$ at some point].

Exercise 8.3: Let M be a 2×2 matrix with bosonic entries, and define the two vectors $c^{*T} \equiv (c_1^* \ , c_2^*)$, $c^T \equiv (c_1 \ , c_2)$ with fermionic Grassmann variables. Compute the following Grassmann integral with the bilinear "action" $S_E = c^{*T} M c$:

$$I(M) \equiv \int \mathrm{d}c_1^* \mathrm{d}c_1 \mathrm{d}c_2^* \mathrm{d}c_2 \ \exp(-S_E) \ .$$

Compare the answer to ex 5.3 (what happens when you consider Gaussian integrals over complex bosonic variables there?).

Exercise 8.4: Compute the following averages with S_E defined in the previous ex. 8.3:

(a) $\langle c_1 c_1^* \rangle \equiv \dfrac{\int \mathrm{d}c_1^* \mathrm{d}c_1 \mathrm{d}c_2^* \mathrm{d}c_2 \ c_1 c_1^* \ \exp(-S_E)}{\int \mathrm{d}c_1^* \mathrm{d}c_1 \mathrm{d}c_2^* \mathrm{d}c_2 \ \exp(-S_E)}$,

(b) $\langle c_1 c_2^* \rangle$,

(c) $\langle c_2 c_1^* \rangle$,

(d) $\langle c_2 c_2^* \rangle$.

Can you find a compact way of generating these averages?