Quantum Field Theory	**Exercises No. 9**

[Wednesday 8. June 2010, 12:30-14:00 in D5-153]

Exercise 9.1: We define expectation values as

$$\langle ... \rangle_0 \equiv \frac{\int \{\prod_i \mathrm{d}c_i^* \mathrm{d}c_i\} (...) \exp(-\sum_{p,q} c_p^* A_{pq} c_q)}{\int \{\prod_i \mathrm{d}c_i^* \mathrm{d}c_i\} \exp(-\sum_{p,q} c_p^* A_{pq} c_q)} ,$$

with c_j^*, c_k being Grassmann variables.

What do you get for $\langle c_k c_l c_m c_n \rangle_0$, $\langle c_k c_l c_m c_n^* \rangle_0$, and $\langle c_k c_l c_m^* c_n^* \rangle_0$?

Exercise 9.2: Verify the Schwinger propagator for the Dirac field

$$\langle 0 | T\{\hat{\psi}_\alpha(x) \hat{\bar{\psi}}_\beta(y)\} | 0 \rangle = \int \frac{d^4P}{(2\pi)^4} e^{iP\cdot(x-y)} \frac{[-i\gamma_\mu^E P_\mu + m\mathbf{1}]_{\alpha\beta}}{P^2 + m^2} .$$

Exercise 9.3: Given pure Yang-Mills theory with the Lagrange density

$$\mathcal{L}_M \equiv -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} .$$

What are the classical equations of motion?

Exercise 9.4: Let us consider $C(x) \equiv \mathrm{Tr}[F_{\mu\nu}(x) F_{\rho\sigma}(x)] \epsilon^{\mu\nu\rho\sigma}$, where $\epsilon^{\mu\nu\rho\sigma}$ is the totally antisymmetric 4-dimensional Levi-Civita tensor (=+1, if $\mu\nu\rho\sigma$ is an even permutation of 0123, =-1 if it is an odd permutation, and =0 otherwise).

(a) Show that $C(x)$ is gauge invariant.

(b) Demonstrate that $C(x)$ is Lorentz invariante for proper Lorentz transformations Λ with $\det \Lambda = +1$.

(c) Verify that $C(x)$ satisfies

$$\begin{aligned} C(x) &= \partial_\mu K^\mu(x) , \\ K^\mu(x) &= 2\epsilon^{\mu\nu\rho\sigma} \Big(A^a_\nu \partial_\rho A^a_\sigma + \frac{g}{3} f^{abc} A^a_\nu A^b_\rho A^c_\sigma \Big) . \end{aligned}$$

Hence it is a surface (or topological) term ofter integration over $\int \mathrm{d}^4x$.