Quantum Field Theory	**Exercises No. 11**

[Wednesday 22. June 2010, 12:30-14:00 in D5-153]

Exercise 11.1: The proper vertex function $\Gamma_\mu(p,q,r)$ is defined by the following Fourier transform:

$$-ig(2\pi)^4\delta^{(4)}(r-p-q)\Gamma_\mu(p,q,r) \equiv \int d^4z d^4x d^4y e^{i(rx-py-qz)} \frac{\delta^3\Gamma[\psi\,\bar\psi, A_\nu]}{\delta\bar\psi(x)\delta\psi(y)\delta A_\mu(z)}\Big|_{\psi=\bar\psi=A=0},$$

with $\Gamma[\psi\,\bar\psi, A_\nu]$ defined in lecture. Check that the Ward identity is satisfied to leading order: express the integrand above through the generating functional $Z[\eta,\bar\eta,J_\nu]$ and the leading order photon and electron propagators.

Exercise 11.2: Draw all possible divergent 1-loop diagrams in QED and give their superficial degree of divergence. What does $D=0$ mean?

Exercise 11.3: Generalize the steps in ex. 7.1 to compute the following integral in Minkowski space ($p^2 = p_0^2 - p_j p_j$):

$$\int \frac{d^dp}{(p^2+2p\cdot q-m^2)^A} = i(-\pi)^{d/2}\frac{\Gamma\Big(A-\frac{d}{2}\Big)}{\Gamma(A)[-q^2-m^2]^{A-d/2}}\ .$$

Use this result to compute the following two integrals:

$$I_1 = \int d^dp \frac{p_\mu}{(p^2+2p\cdot q-m^2)^A}\quad,\quad I_2 = \int d^dp \frac{p_\mu p_\nu}{(p^2+2p\cdot q-m^2)^A}\ .$$

Exercise 11.4: Compute the photon self-energy also called vacuum polarization $\Pi_{\mu\nu}(k)$ in Lorentz gauge $\alpha=1$ using dimensional regularization. Use the Feynman parameterization ex. 7.3 as well as the above ex. 11.3.