Quantum Field Theory	**Exercises No. 12**

[Wednesday 06. July 2010, 12:30-14:00 in D5-153]

Exercise 12.1: Starting from the definitions $i)[T^a, T^b] = if^{abc}T^c$, $ii)\mathrm{Tr}[T^aT^b] = \delta^{ab}/2$, $iii)D_\mu(A) = \mathbf{1}_{N_c\times N_c}\,\partial_\mu - igT^aA^a_\mu$, $\mathcal{D}^{ab}_\mu(A) = \delta^{ab}\partial_\mu + gf^{acb}A^c_\mu$, show that the following holds

$$\int \mathrm{d}^4x\ \bar{c}^a\Big\{-\mathcal{D}^{ac}_\mu(B)\mathcal{D}^{cb}_\mu(A+B)\Big\}c^b = \int \mathrm{d}^4x\ 2\,\mathrm{Tr}\left\{[D_\mu(B),\bar{c}][D_\mu(A+B),c]\right\}\,,$$

where $c = c^aT^a$, $\bar{c} = \bar{c}^aT^a$.

Exercise 12.2: Let

$$\begin{aligned}\Pi_{\mu\nu}(Q) \;&=\; \frac{g^2N_c}{2}\int\frac{\mathrm{d}^dP}{(2\pi)^d}\frac{1}{P^2(P+Q)^2}\Big[4(d-2)P_\mu P_\nu + 2(d-2)(P_\mu Q_\nu + P_\nu Q_\mu)\\ &\quad +(d-10)Q_\mu Q_\nu + 8\delta_{\mu\nu}Q^2 + 2(2-d)\delta_{\mu\nu}(P+Q)^2\Big]\,.\end{aligned}$$

Verify that the following relation holds: $Q_\mu Q_\nu \Pi_{\mu\nu}(Q) = 0$.

Exercise 12.3: Show that in $d = 4 - 2\epsilon$ dimensions it holds (cf. page 57)

$$\int\frac{\mathrm{d}^dP}{(2\pi)^d}\frac{1}{P^2(P+Q)^2} = \frac{\mu^{-2\epsilon}}{(4\pi)^2}\left[\frac{1}{\epsilon} + \ln\frac{\mu^2}{Q^2} + \mathcal{O}(1)\right]\,.$$

Exercise 12.4: Using the definitions in exercise 12.1 above show the following:

(a) Jacobi-identity [Einstein summation convention]

$$f^{abe}f^{cde} + f^{cae}f^{bde} + f^{bce}f^{ade} = 0\,.$$

(b) From $ii)$ the following representation holds

$$f^{abc} = -2i\mathrm{Tr}\Big[[T^a, T^b]T^c\Big].$$

Also show that this implies that the structure constants are totally antisymmetric.

c) The adjoint matrix representation of the generators $(T^b)_{ac} = if^{abc}$ satisfies the commutation relation $i)$.